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Trading options based on percentile estimates

Master's Thesis

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Dedicated to my family & friends,

especially to my grandfather Stathis...

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Abstract

Contrary to the ubiquitous practice of option price estimation for trading strategies, that focuses on modelling and forecasting volatility of the underlying asset, mostly through stochastic processes, we embrace a deterministic approach. Our aim is to investigate whether profits can be achieved, in index options' trading, by predicting, through deterministic percentile predictors, the future price of the underlying asset, which is utilized as the option's exercise price. The Black and Scholes (1973) formula is employed for obtaining the theoretical prices of those options. Trading strategies of percentile predictors are comparatively evaluated on the basis of the cumulative profits achieved, under a certain trading rule. Over a fifteen-year period data sample of options on the S&P500 index, of both bullish and bearish percentile predictors, we provide empirical evidence that the percentile predictors on index options' trading strategies are profitable when applied opposingly to the current trading norm, where they seem to follow the trend of the index.

Keywords: options, index option trading, speculative strategies, percentile predictors, S&P500, VIX

1. Introduction

Options are a class of financial derivatives. The latter are contracts, or securities, the value of which depends upon the value of another asset. An option is a contract that gives its owner the right, not the obligation, to buy or sell a set quantity of an asset, such as a stock, at a fixed price on, or up to, a given future date.

Options' trading, a branch of financial derivatives markets, has been greatly expanded, in terms of the traded assets' worth, in recent years. Investors and portfolio managers utilize options for both hedging and speculation purposes. Numerous trading strategies exist, providing alternatives for all motives and circumstances. From an academic point of view, since the 1950s, the study of finance has been transformed into a high-status enterprise with the theoretical account of options, dating from the start of the 1970s, at its' core, as well as been transformative of the corresponding markets (MacKenzie D. 2006).

The cornerstone of financial literature around option theory is the precise and comprehensive option pricing models proposed by Black and Scholes (1973) and Merton (1973)[1]. Their pioneering work on option pricing, set aside former complexities, such as an investor's risk aversion and expectation of stock price movement, thus providing a legitimate framework for derivatives markets, on the basis of an arbitrage mechanism.

The correct pricing of options is crucial for the efficiency of hedging strategies, optimal portfolio decisions and risk management. Hence, financial institutions, funds, investors and portfolio managers seek accurate estimations of options' prices. The Black-Scholes (BS) formula is able to provide them, under the prerequisite that an estimation of future volatility of the underlying asset, which is used as an input parameter, is available. As Degiannakis S., Filis G. and Hassani H. (2018) state, the single most important component for pricing option contracts, is forecasting volatility.

In terms of technical analysis, parametric and non-parametric techniques have

[1] In 1997, Robert C. Merton and Myron S. Scholes were awarded the 1997 Nobel Prize in Economics for their work in finding a new method to determine the value of derivatives. Fisher Black died in 1995 and although the Nobel Prize is not given posthumously, the Nobel committee acknowledged Black's role in the Black-Scholes-Merton model.

been developed for modelling and forecasting time series. A great part of the literature on characterizing and modelling financial time series, such as stock prices or index prices, revolves around the parametric class of Autoregressive Conditional Heteroskedasticity (ARCH) models, introduced by Engle (1982), and their generalized form (GARCH), which follow a stochastic process, proposed by Bollersev (1986). As far as non-parametric techniques are concerned, the literature is rather limited, although combinations of non-parametric and parametric techniques provide evidence of improved forecasts, as shown in the work of Degiannakis et al. (2018).

Early and recent studies, such as those of Chiras and Manaster (1978) and Degiannakis (2008) among others, provide evidence that implied volatility indices are better predictors of future volatility compared to the aforementioned parametric models.

In this study, estimations of options' prices for trading strategies are approached from a deterministic perspective. Percentile points of the empirical distribution of log-returns of an asset are employed as unbiased predictors of its future price. The future price of the asset, obtained from the percentile predictors, is utilized as the exercise price of an option on that asset, the theoretical price of which is given by the Black-Scholes formula. Percentile predictors' trading strategies on index options are comparatively evaluated on the basis of their cumulative profits over a certain period.

In particular, the underlying asset of the options used in this essay is the Standard and Poor's 500 (S&P500 or SPX) index. The CBOE Volatility Index (VIX), representing the current forward (30 calendar days, or 22 trading days) looking volatility of the S&P500, is utilized as the volatility input parameter for the BS formula. The empirical investigation, covering a range of fifteen years from January 2nd 2004 up to July 18th 2019, provides evidence that percentile predictors' trading strategies on index options are profitable when applied opposingly to the trading norm. The methodology is explained in detail subsequently.

The rest of this essay is structured as follows: in chapter 2, a historical reflection and description of the discussed subjects is presented. In chapter 3, the data used in the empirical research are described, while chapter 4 presents the methodology that was followed, and chapter 5 investigates the empirical results; chapter 6 concludes. Tables and Figures are presented in appendixes A and B.

2. Historical reflection and description of the discussed concepts

2.1. Derivatives

Financial derivatives can be seen as secondary securities, whose value depends upon (is derived from) the value of the primary security, or asset, that they are linked to. As McDonald (2013) simply, yet comprehensively describes it, a derivative is a financial instrument that has a value determined by the price of something else.

Even though financial products like these have made headlines due to the great losses of financial institutions, funds and corporations, related to them, they are a useful and everyday part of business[2]. Derivatives can also be thought of as a bet on the price of something, but if you own that something, or you are willing to acquire it, this bet provides insurance against undesired outcomes concerning the price. They can be used either to hedge against inherent, or potential, risk of an investment, or to simply speculate from the change of price. A financial derivative's contract cannot be characterized as a speculative or risk-reducing (hedging) one, without knowing who is using it, and how. Epigrammatically, some of the underlying motives of the use of financial derivatives are[3]:

i. Risk management
ii. Speculation
iii. Reduced transaction costs
iv. Regulatory arbitrage[4]

Futures, options, swaps, and forwards are just some of the potentially infinite financial derivatives. For the purpose of this essay, it is unnecessary to analyze other derivatives than options.

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[2] "Derivatives sometimes make headlines. Prior to the financial crisis in 2008, there were a number of well-known derivatives-related losses: Procter & Gamble lost $150 million in 1994, Barings Bank lost $1.3 billion in 1995, Long-Term Capital Management lost $3.5 billion in 1998, the hedge fund Amaranth lost $6 billion in 2006, Société Générale lost €5 billion in 2008. During the crisis in 2008 the Federal Reserve loaned $85 billion to AIG in conjunction with AIG's losses on credit default swaps. In the wake of the financial crisis, a significant portion of the Dodd-Frank Wall Street Reform and Consumer Protection Act of 2010 pertained to derivatives". (McDonald (2013, p.1))

[3] For a more thorough insight on the motives of using financial derivatives see McDonald (2013, p.11 13).

[4] Arbitrage refers to the investing opportunity where profit is certain, or else risk-free.

2.2. Derivatives markets

Derivatives, as defined earlier, such as options, have been trading since at least the seventeenth century (MacKenzie D. 2006). In modern times, transactions of financial products usually take place in organized exchanges, although traders have the option to circumvent them by trading directly with a dealer, for several reasons[5].

Financial derivatives began trading formally in 1972, when futures on seven currencies became available for trading by the Chicago Mercantile Exchange (CME). Until then, regulators considered financial derivatives as being dangerously close to gambling, consequently, the formation of a market around such products was not feasible, mostly due to their inefficient pricing mechanism.

Risk is an inherent component of life in general, as well as in business in particular. It arises from both natural and unnatural events, like physical disasters and wars, political conflicts or economic decisions that affect markets and their participants. When price-risk, resulting from any cause, in a market increases, the introduction of derivatives in that market is often observed. As McDonald (2013) states: “the link between price variability and the development of derivatives markets is natural – there is no need to manage risk when there is no risk” (p. 6). Thus, financial derivatives markets serve the economy in the way that they allow *risk-sharing mechanisms* to be actualized.

2.3. Options

Options are contracts that give their owner the right, but do not impose the obligation, to buy or sell an asset at a fixed price (strike, or exercise price) on, or up to, a given future date. They are one of the two classes[6] of financial derivatives discussed in the chapter above. The main segregation of options stems from whether they give the right to buy or sell the underlying asset; on the first case they are named *calls*, on the second *puts*.

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[5]These transactions are said to occur *over-the-counter* (OTC). The reasons a trader might prefer OTC trading are namely to avoid high fees and transaction costs, to trade custom financial products or to avoid the uncertainty that the announcement of a large sale would bring on the market. It is worth mentioning that for many categories of financial products, the value of OTC trading is greater than the value traded on exchanges.

[6] The other class of financial derivative products is called *locks* or *lock derivative products*. Their name stands for the imposed obligation of the respective parties to abide by the agreed-upon terms over the life of the contract.

Some key terms in describing options are:

Exercise price: The exercise price, or strike price of an option is the price at which the owner of the option will either buy, or sell the asset, depending on whether it is a call or a put respectively.

Exercise: The exercise of an option is the act of paying the exercise price to receive the asset, or selling it at the exercise price, like above.

Expiration: The expiration of the option is the date by, or upon, which the option must be either exercised or it becomes worthless.

Exercise style: The exercise style of the option defines the time at which the option can be exercised. If an option can be exercised at any point up to its expiration, it is called an "American" option; if it can be exercised only on the day of expiration, it is called "European". If the option can be exercised only during specified periods of its lifetime, it is called "Bermudan".

Unit of trading; Contract size: The unit of trading, or contract size, of an option is the amount of the underlying asset that is subject to being purchased or sold upon the exercise of a single option contract.

Another way to describe options is by their degree of *moneyness*. This term refers to the payoff of the option if it were exercised immediately. An **in-the-money** option is one that if it were exercised immediately, the payoff would be positive. However, considering the price paid for the option itself, a positive payoff does not necessary mean that profit would be earned. A call option with an exercise price less than the asset price and a put with an exercise price greater than the asset price, are in-the-money options. An **out-of-the-money** option is one with a negative payoff if exercised immediately. If the exercise price of a call option is greater than the asset price and the exercise price of a put option is less than the price of the asset, they are both out-of-the-money. An **at-the-money** option is one that its exercise price is approximately equal to that of the asset.

The underlying asset of an option might be a non-financial asset, such as the weather, the outcome of elections in a country, the winner of an Olympic event and so on[7]. Financial underlying assets of options are usually stocks, commodities, currencies, stock indexes, interest rates, exchange rates and other financial products, such as futures.

Index options are options to buy or sell the value of the underlying index. They are a simple tool used by investors, traders and speculators to profit on the general direction of the underlying index while putting very little capital at risk.

When an option is exercised, the delivery of the underlying asset takes place. This seems natural for stocks or commodities, but another process is needed when the physical delivery is either not possible, or has significant transaction costs. Commodities can be physically delivered; stocks and futures are delivered by updating the ownership records and currencies or exchange rates can be delivered by making the necessary transactions in a corresponding bank account. The complexity of the process required to calculate the spot price of each stock of a broad stock index, accompanied by the relevant transaction fees, has caused the **cash-settlement** of such options, which is a financial method of settlement where the two parties make a net cash payment[8]. Thus, index options are always cash-settled[9].

An important feature of options is their **time to expiration**, or **time to maturity**. It is the period between the issue of the option and its expiration date. The time to expiration varies from days up to a year. This feature is relevant to the motives of the option holder and the strategy s/he follows.

Perhaps the most important characteristic of an option is its price, or as market practitioners refer to it, the option's *premium*. The price of an option is essential for the efficiency of the interested parties' strategies. Arbitrage conditions should not exist in an efficient market. Thus, the *fair* value of an option improves market efficiency. The mechanisms and the assumptions of option pricing are presented as detailed below.

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[7] MacDonald (2013) in his book "Derivatives Markets", presents an extreme example of a non-financial asset of a future derivative, proposed by the U.S. Pentagon. The underlying asset was the occurrence of an event, in particular a terrorist attack on the U.S. (p. 28).

[8] Cash-settlement often occurs in other assets settlements for speculation purposes, or to avoid the additional costs of a physical delivery, such as transportation costs, or delivery insurance.

[9] The *unit of trading* of cash-settled options, like stock index options, is determined by the *multiplier* that is fixed by the market that the option is traded in ("Characteristics and Risks of standardized options", p. 8).

In this essay, European style options of the S&P500 index with a monthly periodicity[10] (*standard* options) were employed, to investigate the profitability of certain index options' trading strategies.

2.4. Pricing options

The central question in option pricing revolves around the explanation of their cost and the parameters that determine it. In contrast to *lock* derivatives, options give the right to back away from the agreement imposed by the contract. Hence, the pricing of options should embody this attribute. As McDonald (2013) states: "The principal question in option pricing is: *How do you value the right to back away from a commitment?*" (p. 265).

Until a more complete theory of option pricing was developed, market practitioners used to price options based on rules of thumb. Certain parameters were, and still are, intuitively suggested to influence the cost of options, such as: the current price of the underlying asset; the exercise price of the option; the option's time to maturity; the level of interest rates; the volatility of the price of the underlying asset. however, this list alone is not enough to form a precise option pricing mechanism.

Key developments in option pricing theory took place in the U.S. from late 1950s onwards but the solutions provided, required parameters, the values of which were extremely hard to determine, such as an investor's expectations of returns of the underlying asset and the degree of the investor's risk-aversion (MacKenzie 2006).

2.5. The Black-Scholes-Merton model

In 1973, the work of Fisher Black and Myron Scholes on option theory, accompanied by the additional input from Robert Merton[11], revolutionized both the theory and practice of finance. The model that they proposed, refers to stock options and is based on certain assumptions:

- Stocks can be bought or sold at any point in time without incurring transaction costs or causing market prices to move.

[10] The time to maturity of the options we examined does not exceed 25 trading days.
[11] Merton (1973) developed the Black-Scholes model to adjust for dividend paying stocks and American style options.

- Stock prices fluctuate log-normally.
- Stock price volatility remains constant.
- Stocks do not pay dividends.
- Risk-free interest rate remains constant.
- Options can be exercised only on the day of expiration (European style).
- Short-selling (sale of a borrowed asset) incurs no financial penalty.
- There are no taxes.
- There are no transaction costs.

Under these assumptions, the construction of a costless self-financing portfolio of an option and a continuously adjusted position in the stock, which can replicate the payoff of the option, is possible. In this approach, option prices adjust to eliminate any arbitrage opportunities. The Black-Scholes model can be extended to price any security whose payoffs depend on the prices of other securities. The importance of the Black-Scholes model for financial economics is given by McDonald (2013): "This methodology is important not only for pricing European call options; it provides the intellectual foundation for pricing virtually all derivatives, and also underpins the risk-management practices of modern financial institutions"(p. 627).

This dynamic strategy reduces to a partial differential equation subject to a set of boundary conditions that are determined by the specific terms of the option, the famous Black-Scholes (BS) option pricing equation, or BS formula. The success of this equation stems from the fact that it harmoniously links the option's price, the underlying asset's price, the asset's volatility, the riskless interest rate and time, to increase market efficiency by eliminating any arbitrage opportunities.

While previous option pricing models required input parameters the values of which had to be empirically estimated, or determined by judgement, the BS formula was parsimonious in this respect, but even it required an estimation of the asset's volatility, since it is *future* volatility that matters to the price of an option (MacKenzie 2006). The parameters required by the BS formula to derive the theoretical fair price of an option are:

i. The underlying asset's current price.

ii. The option’s exercise price.
iii. The option’s time to maturity.
iv. The risk-free interest rate.
v. The expected volatility of the underlying asset through the lifetime of the option.

Since the only estimation required for the input parameters of the formula is the one of volatility, a great part of the literature is successfully involved in obtaining accurate predictions of it.

In this research, the BS formula was employed to derive the theoretical price of options which were used to comparatively evaluate the trading strategies on index options that were investigated.

3. Data description

Daily data of two indices are employed for the empirical investigation of this study; Standard and Poor's 500 (S&P500 or SPX) with 4165 observations and the CBOE Volatility index (VIX) with 3912 observations. SPX and VIX are used as the asset's price and volatility inputs[12], respectively, in the BS formula. The data cover a range from the 2nd of January, 2003 and 2004, respectively, up to the 19th of July, 2019 (i.e. 4165 and 3912 trading days)[13]. Data of the Effective Federal Funds Rate were also employed, as the risk-free interest rate input for the BS formula, for the same period as VIX[14]. Expiration dates of S&P500 option were obtained from option expiration calendars and used to derive the time to expiration. The data were collected from CBOE, the Federal Reserve Bank of St. Louis (fred.stlouisfed.org) and marketwatch.com.

The S&P 500 index is a market-capitalization-weighted index of the 500 largest U.S. publicly traded companies in all sectors and is widely regarded by investors as the best gauge of large-capitalized U.S. equities and the tendency of the U.S. economy in general. It is a float-weighted index, meaning company market capitalizations are adjusted by the number of shares available for public trading. Considered as the leading indicator of the U.S. stock market, the S&P 500 was the basis for the creation of the first benchmark index to measure the market's expectation of future volatility. The CBOE (Chicago Board Options Exchange) introduced the VIX index in 1993, which is a real-time market index that represent the market's expectation of 30 days ahead volatility, derived from the price inputs of the S&P 500 options[15].

Descriptive statistics of the selected indices and the logarithmic returns of the S&P500 are presented in Table 1. The log-returns of S&P500, in which we are more interested as we shall see further on, do not follow the normal distribution according to the Jarque-Bera statistic and its corresponding p-value. The distribution of log-

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[12] For robustness purposes, front month future contract prices were also used as the volatility input in the BS formula.

[13] The SPX index observations exceed those of the other input parameters by 253, as those were needed to obtain the empirical distribution of log-returns of the last trading year, as it is described below.

14 The Effective Federal Funds Rate was given in monthly frequency, consequently it was adjusted to match the daily data by applying the monthly value on every trading day of that month.

[15] For more information see the VIX white paper at http://www.cboe.com/micro/vix/vixwhite.pdf

returns is platykurtic due to the large positive value of kurtosis, while skewness is negative and differs from the zero-value imposed by the normal distribution. The histogram of log returns is presented in Figure 1. Graphical representations of the S&P500, the VIX and the log-returns of S&P500 are presented in Figure 2, Figure 3 and Figure 4 respectively. The joint graphical representation of S&P500, VIX and log returns of S&P500 is presented in Figure 5.

Table 1: Descriptive statistics of S&P500, VIX and the logarithmic returns of S&P500

	LOG RETURNS	SPX	VIX
Mean	0.000288	1603.976	0.185215
Median	0.000690	1388.170	0.160600
Maximum	0.109572	3025.860	0.808600
Minimum	-0.094695	676.5300	0.091400
Std. Dev.	0.011393	571.2701	0.085540
Skewness	-0.353113	0.794730	2.632252
Kurtosis	14.62252	2.521495	12.83537
Jarque-Bera	23551.67	478.6273	21617.93
Probability	0.000000	0.000000	0.000000
Sum	1.202573	6686977.	772.1608
Sum Sq. Dev.	0.540970	1.36E+09	30.49782
Observations	4168	4169	4169

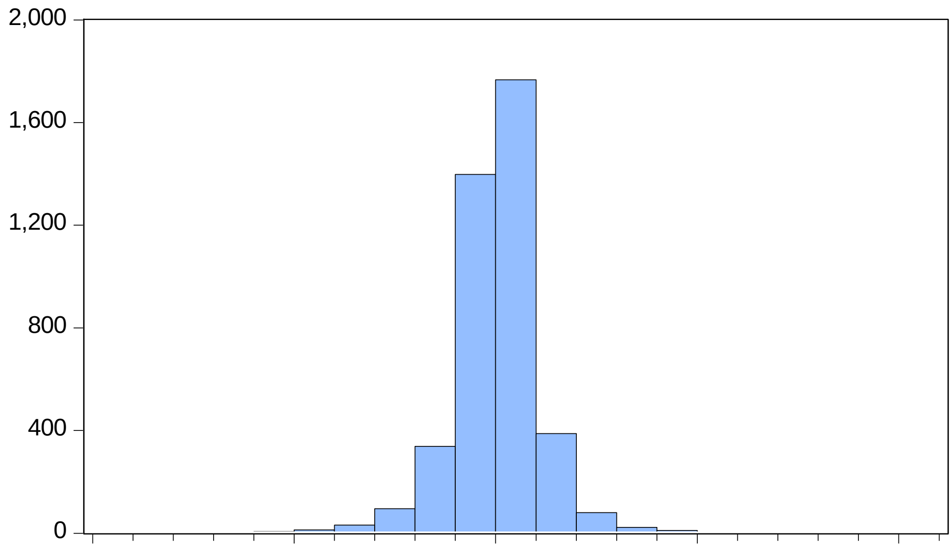

Figure 1. Histogram of the log-returns of S&P500

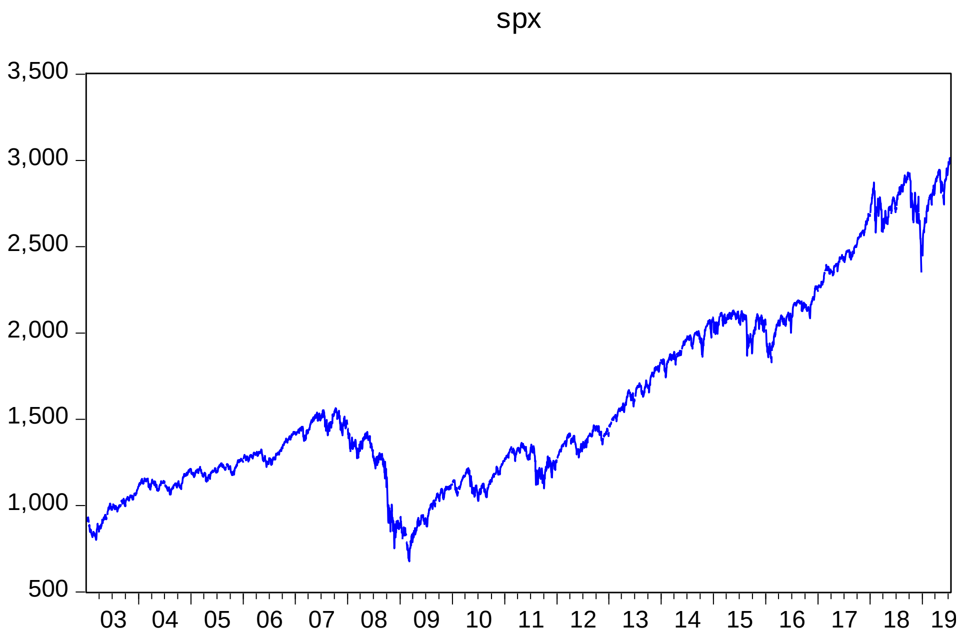

Figure 2. S&P500 from 2nd January 2003 up to 18th July 2019

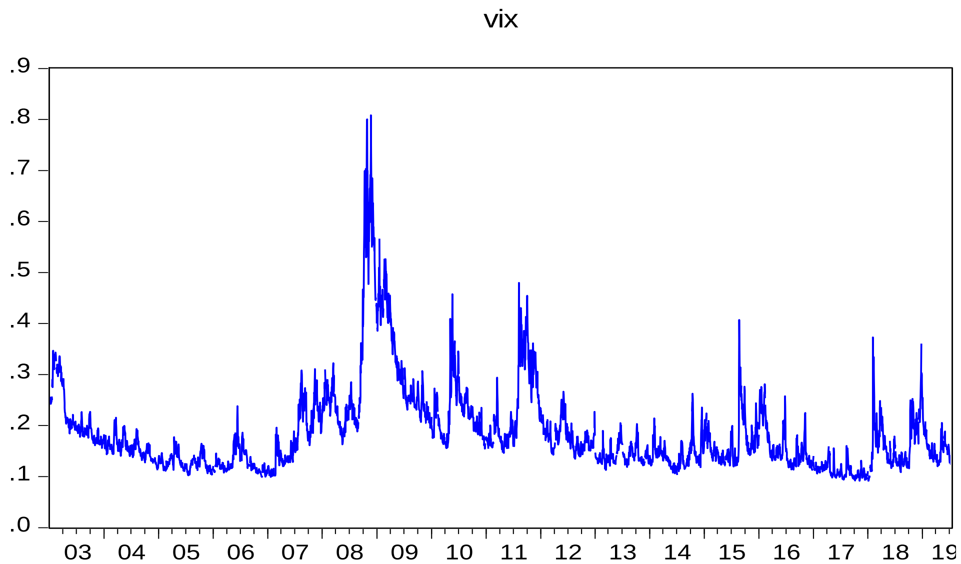

Figure 3. VIX from 2nd January 2003 up to 18th July 2019

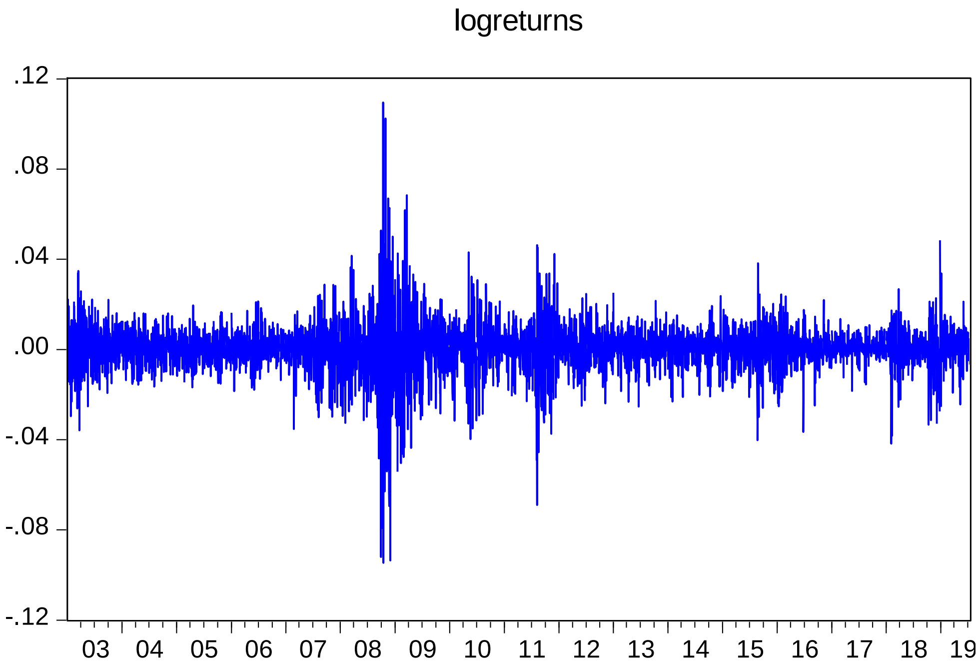

Figure 4. S&P500 log-returns from 3rd January 2003 up to 18th July 2019

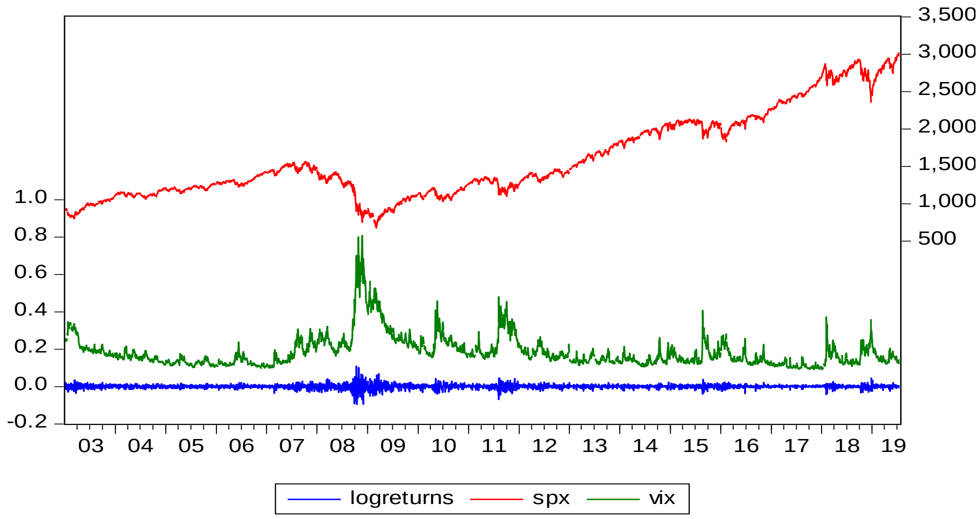

Figure 5. Joint graph of VIX, S&P500 and log-returns of S&P500

4. Methodology

Option price estimations, utilized either for hedging or speculation purposes, are commonly approached by modelling and forecasting volatility of the underlying asset. In terms of technical analysis, ARCH and GARCH models, stochastic volatility models, realized volatility models, as well as non-parametric, or semi parametric techniques are employed to obtain accurate volatility forecasts (Degiannakis et al. 2018), even though volatility indexes are considered as better predictors of future volatility, as seen in the work of Chiras and Manaster (1978) and Degiannakis et al. (2008) among others.

This study embraces a deterministic approach in option pricing, with respect to the option's exercise price. Since volatility indexes provide adequate estimates of future volatility, we focus on estimates of the asset's future price, that is the corresponding option's exercise price. The question imposed is: Considering trading options, can profits be achieved by predicting their exercise price through deterministic predictors?

There is a plethora of alternatives to choose from, primary among asset classes with different nature and behavior in the global economic system, the range of the options' time to maturity and the variety of relevant trading strategies, before we can answer this question.

In order to narrow our options, we adopt a simple trading rule to build our strategy; **options are purchased, with the intention to be exercised**. Furthermore, we focus on standard options which have a monthly expiration periodicity[16], with no more than twenty-five trading days to maturity. Considering the underlying asset of the option, we choose stock index options instead of other asset classes such as currencies, commodities, stocks or other financial derivatives.

Index options are options to buy or sell the value of the underlying index, they are always cash-settled and are typically European style. They are a simple tool used by investors, traders and speculators to profit on the general direction of the

[16] Standard options have a monthly expiration periodicity and usually expire on the third Friday of each month. For more information about S&P500 options you can visit www.cboe.com or any other brokerage website.

underlying index while putting very little capital at risk. In this study, the Standard and Poor's 500 index, considered as the leading indicator of the U.S. stock market and the best gauge of the U.S. economy in general, is used as the underlying asset of options employed for comparatively evaluating the trading strategies in question.

Percentile points of the empirical distribution of the past trading year of the asset's log-returns, are used as predictors of the future value of the asset. These future values, derived from the percentile predictors, are utilized as the exercise price of options on the asset.

The BS formula, derived from the Black & Scholes model, is the most used tool, among traders, to calculate the theoretical price of European options. The daily prices of options obtained from it, are used to determine the daily returns of each percentile predictor's strategy, on the basis of their cumulative profits over a specified period and under a trading rule.

4.1. Percentile points as Predictors

The basic elements of this research are percentile points of the empirical distribution of the log-returns of a financial asset. Since log-returns represent the percentage change of the asset's value, the aforementioned percentile points are used as predictors of the asset's future value. In this way, we achieve a constant[17] criterion for our predictions, which is the desired percentile of the aforementioned empirical distribution. Percentile predictors are unbiased because they use current available information by employing in-sample data and thus, not relying on producing recursive forecasts by utilizing stochastic processes.

The percentile predictors are obtained as follows:

Let $[Y_t]_{t=0}^{T} = \left[log\left(\frac{P_t}{P_{t-1}}\right)\right]_{t=1}^{T}$ refer to the log-return series, where P_t is the closing price of the asset of the trading day t. The empirical distribution of Y_t of the last n trading days, denoted as $F_{(n)t}(Y_t)$, is defined as:

[17] The percentile point is on a specific percentile of the empirical distribution across the data set.

$$F_{(n)t}(Y_t) = \frac{1}{n+1}\sum_{i=1}^{n} 1_{Y_i < Y_t} \quad (1)$$

where n is the size of the rolling sample $\left[Y_{(n)t}\right]_{t-n}^{t}$ over the data time frame t = 1,…,T, for $t \geq n$, and $1_{Y_i < Y_t}$ is an indicator function that is equal to 1 if $Y_i \leq Y_t$ and 0 otherwise.

The desired percentile to be evaluated is denoted as k[18]. Then, the percentile point of the k^{th} percentile of $F_{(n)t}(Y_t)$, which is the percentile predictor, is denoted as $f_{(n)t,k}(Y_t, k)$.

The asset's future price prediction is later utilized as the exercise price of options on the asset. The exercise price of the option, $E_{t,k}$ [19], for the trading day t and percentile predictor k, is given by:

$$E_{t,k} = f_{(n)t,k}(Y_t, k) \times P_t, \text{ where } t \geq n \quad (2)$$

Since the log-returns of the asset express its' percentage change, the percentile predictor, as defined above, indicates a value placed over k% of those changes, of the past n trading days. By multiplying the percentile predictor at time t, with the closing price of the asset at time t, we obtain a plausible, based on available information, estimation of the future price of the asset, used as the exercise price of the option purchased on trading day t.

In this research, the S&P 500 index, denoted as SPX, is used as the underlying asset, mentioned above as P, where SPX_t is the closing price of the index of trading day t. We assume that a sample size of $n = 252$ trading days is adequate for the empirical distribution of Y_t. We also consider six percentiles to be evaluated, three on each tail of the distribution. In particular the percentiles in question are the 1%, 5%,

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[18] The vector of percentiles to be evaluated is $[k]_1^J$ with dimensions $1 \times J$, where J is the number of percentiles and $0{,}1 \leq k_j \leq 0{,}99$.

[19] The array of exercise prices $E_{(T-n)\times J}$, that contains J exercise prices for each trading day t, is denoted as:

$$E_{(T-n)\times J} = \begin{bmatrix} E_{t-n,1} & \cdots & E_{t-n,J} \\ \vdots & \ddots & \vdots \\ E_{T,1} & \cdots & E_{T,J} \end{bmatrix}$$

10%, 90%,95% and 99%[20].

For instance, by employing the percentile predictor $f_{252|t,99\%}$, we get consistent estimations of the asset's future price throughout the trading period, resulting from the percentage change that lies above 99% of the past trading year's percentage changes.

4.2. BS formula price estimations

In this study, the Black-Scholes formula is employed to estimate the theoretical prices of the options, which are utilized to comparatively evaluate the profitability of the percentile predictors' trading strategies. The Black-Scholes mathematical model developed, under certain assumptions, for pricing an option contract, has been extensively mentioned, studied and extended in the existing literature. For the purpose of this research, there is no need for in-depth analysis of the underlying mathematical processes. In short, the BS option formula is calculated by multiplying the underlying asset's price by the cumulative standard normal probability distribution function. Thereafter, the net present value of the exercise price multiplied by the cumulative standard normal distribution is subtracted from the resulting value of the previous calculation for *call* options, whereas the opposite subtraction takes place for *put* options.

In general, according to the basic BS model, the price of a call and a put option denoted, respectively, by PR^{call} and PR^{put} are given by:

$$PR^{call} = P \cdot N(d_1) - E \cdot e^{r \cdot \tau} \cdot N(d_2) \tag{3}$$

$$PR^{put} = -P \cdot N(d_1) + E \cdot e^{r \cdot \tau} \cdot N(d_2) \tag{4}$$

where $d_1 = \frac{\log(P/E) + \left(r + \frac{1}{2}\left(\frac{\sigma_P^2}{2}\right)\right)\tau}{\sigma_P \sqrt{\tau}}$ and $d_2 = d_1 - \sigma_P\sqrt{\tau}$. Here, P refers to the underlying asset's current price, E is the option's exercise price, r is the compounded

[20] Thus, the vector of the percentiles under evaluation is:

$$[k_j]_{j=1}^{6} = [0{,}01_1 \quad 0{,}05_2 \quad 0{,}10_3 \quad 0.90_4 \quad 0{,}95_5 \quad 0{,}99_6]$$

risk-free interest rate, τ[21] is the time to expiration, $N(.)$ is the cumulative standard normal distribution function and σ_P is the standard deviation of log-returns of the asset P. The BS formula requires five input variables as mentioned above.

Expected volatility is implied by solving the BS formula for σ, using the option's market price and its' corresponding exercise price. As mentioned earlier, implied volatility indices have long been considered as better predictors of future volatility, supported by evidence from recent studies.

The VIX index, derived from the prices of the SPX options, is a competent approximation of current forward-looking volatility of SPX. Since we use options of SPX, VIX, is suitable to be employed as the volatility input for the BS formula, in order to produce estimations of options' prices closest to the ones observed in the market. For robustness purposes, we also used the daily prices of front month VIX future contracts, but no qualitive improvement was observed.

In this study, we consider four out of five inputs for the BS formula as given. The remaining one is the exercise price input, accounting for the profitability of the percentile predictors' trading strategies, which is constructed as shown previously. We denote the BS formula inputs that we employ as:

i. SPX_t is the current price of the underlying asset, which is the SPX index, on trading day t.
ii. $E_{t,k}$ is the exercise price derived from the percentile predictor of the k^{th} percentile, $f_{(n)t,k}(Y_t, k)$, on trading day t.
iii. r_t is the compounded risk-free interest rate on trading day t, in our case the Effective Federal Funds Rate.
iv. τ_t is the time to expiration of the option on trading day t.
v. V_t is the expected volatility on trading day t, which is the closing price of the VIX index on that day.

For the comparative evaluation of the percentile predictors' trading strategies, the BS formula provides a number of option prices for each trading day t equal to the

[21] τ is given in trading days, which means that weekends and trading holidays are not included. The BS formula measures the time to expiration in years. We transform τ, to be consistent with the BS formula, considering 252 trading days per annum as $\tau/252$.

number of percentile predictors we wish to evaluate. Thus, for every percentile predictor, equations (3) and (4) are formed as:

$$PR_{t,k}^{call} = SPX_t N(d_1) - E_{t,k} e^{r_t \tau_t} N(d_2) \quad (5)$$

$$PR_{t,k}^{put} = -SPX_t N(d_1) + E_{t,k} e^{r_t \tau_t} N(d_2) \quad (6)$$

where $d_1 = \frac{\log\left(\frac{SPX_t}{E_{t,k}}\right) + \left(r_t + \frac{1}{2}\left(\frac{V_t}{2}\right)\right)\tau_t}{V_t \sqrt{\tau_t}}$, $d_2 = d_1 - V_t\sqrt{\tau_t}$, and $PR_{t,k}^{call(put)}$ refers to the call(put) option price of percentile predictor k, on trading day t.

4.3. Comparative evaluation of Percentile Predictors' Trading Strategies

The simple trading rule we devised – **options are exercised** - determines the manner and time that profits are earned. Profits or losses cannot be known prior to the day of expiration. On the day of expiration, if the exercise price of the call(put) option is under(above) the asset price[22], the option is exercised. The profit, assuming the option is exercised, is the difference between the exercise price and the index price on that day, minus the option's price, while the loss is limited to the premium initially paid. In mathematical notation:

$$R_{t,k}^{call}\left(P_{t+\tau_t}, E_{t,k}, PR_{t,k}^{call}\right) = \begin{cases} -PR_{t,k}^{call}, & if\ P_{t+\tau_t} \leq E_{t,k} \\ P_{t+\tau_t} - \left(E_{t,k} + PR_{t,k}^{call}\right), & if\ P_{t+\tau_t} > E_{t,k} \end{cases} \quad (7)$$

$$R_{t,k}^{put}\left(P_{t+\tau_t}, E_{t,k}, PR_{t,k}^{call}\right) = \begin{cases} -PR_{t,k}^{put}, & if\ P_{t+\tau_t} \geq E_{t,k} \\ E_{t,k} - \left(P_{t+\tau_t} + PR_{t,k}^{put}\right), & if\ P_{t+\tau_t} < E_{t,k} \end{cases} \quad (8)$$

Where $R_{t,k}^{call(put)}$ is the profit(revenue) or loss of the call(put) option of the percentile predictor $f_{(n)t,k}(Y_t, k)$, on trading day t, P is the price of the asset and $t + \tau_t$ is an indicator providing the asset's price on the day of expiration. In our analysis, $P =$

[22] In our research, we employ the closing price of the S&P500 on the day of expiration for convenience. The value actually considered to determine whether an option on S&P500 is exercised or not, is the exercise-settlement value, SET, which is calculated using the opening sales price in the primary market of each component security on the expiration date. The exercise-settlement amount is equal to the difference between the exercise-settlement value and the exercise price of the option, multiplied by $100. For more information see SPX Options Product Specifications in http://www.cboe.com/products/stock-index-options-spx-rut-msci-ftse/s-p-500-index-options/s-p-500-options-with-a-m-settlement-spx/spx-options-specs.

SPX.

The percentile predictors' trading strategies are comparatively evaluated on the basis of the cumulative profits achieved throughout the period that is examined. Cumulative profits of call(put) options for each percentile predictor, denoted as $CR_{t,k}^{call(put)}$, are given by:

$$CR_{t,k}^{call(put)} = R_{t,k}^{call(put)} + CR_{t-1,k}^{call(put)} \quad (9)$$

The values needed for the comparative evaluation of the percentile predictors' trading strategies, for any given period over the data time frame, are obtained by equation (9), such as t is the last trading day of that period. The trading strategy with the greatest value of $CR_{t,k}^{call(put)}$ is considered the most profitable one.

5. Empirical findings

The profitability of percentile predictors' trading strategies on SPX index options was comparatively evaluated, on the basis of their cumulative profits, over a fifteen-year period from January 2nd 2004 up to July 18th 2019. Six percentile predictors were examined, derived from the percentiles 1%, 5%, 10%, 90%, 95% and 99% of the empirical distribution of the last 252 trading days of the log-returns of SPX_t for each trading day t. The BS formula was employed to provide daily estimations of the theoretical prices of options for each percentile predictor's trading strategy, by utilizing the exercise prices provided by the percentile predictors. Consequently, the profitability of the percentile predictors on index options' trading strategies was comparatively evaluated on the basis of the cumulative profits achieved under the trading rule that options are exercised.

It is important to note the norm of option trading behavior according to the current and the anticipated movement of the underlying asset's price. A *bullish* trader, anticipating a rise in the price of the asset, will purchase a call option with exercise price greater than the current price, in hope of an even greater rise in the price, enough to cover the premium and make profits. On the other hand, a *bearish* trader will purchase a put option with exercise price lower than the current price of the asset, anticipating an even greater drop of the price, as above.

The mean of the log-returns series, Y_t^T, $\bar{Y}$ wages around zero, which means that the percentile predictors represent a bullish anticipation on the price of SPX, for percentiles above the median, and a bearish one otherwise. Hence, options' trading strategies of percentile predictors can be described as *bullish call*, *bullish put*, *bearish call* and *bearish put*. Three bullish and tree bearish percentile predictors were employed in order to compare the effectiveness of various percentiles on each strategy.

Considering that SPX was in an uptrend throughout the period we examine, as seen in Figure 2, – except a relatively small period from around late 2007 up to early 2009 -, bullish call options' strategies were expected to be profitable, according to the aforementioned trading norm. Surprisingly, this is not the case as seen in Table 2 below.

Table 2: Cumulative Returns of Percentile Predictors' trading strategies on index options over the period January 2nd 2003 – July 18th 2019

	Bearish strategies			Bullish strategies		
Percentile Predictors	1%	5%	10%	90%	95%	99%
Call	$11.683,12	$5.708,78	$1.868,42	-$14.019,65	-$15.040,50	-$15.523,46
Put	-$5.860,49	-$11.803,17	-$15.625,93	-$31.443,09	-$32.452,81	-$32.908,04

The amounts are in hundreds of U.S. dollars

Bearish call strategies were able to make profits across all the percentile predictors that were examined. The 1% percentile predictor bearish call strategy earned $1.168.312,00 at the end of the period we examined, while more moderate percentile predictors of the same strategy provided less earnings; $570.878,00 for the 5% percentile predictor and $186.842,00 for the 1% percentile predictor. On the other hand, bullish call strategies suffered losses from $1.401.965,00 for the 90% percentile predictor and $1.504.050,00 for the 95% percentile predictor, up to $1.552.346,00 for the 99% percentile predictor. Both bearish and bullish strategies of put options suffered losses across all the percentile predictors, as it was expected. For the bearish put strategy of the 1% percentile predictor, a loss of $586.049,00 was reported, while losses of $1.180.317,00 and $1.562.593,00 were reported for the 5% and 10% percentile predictors respectively. The bullish put strategy losses were even greater; $3.144.309,00 loss for the 90% percentile predictor, $3.245.281,00 loss for the 95% percentile predictor and $3.290.804,00 loss for the 99% percentile predictor.

The course of the cumulative returns, over the time frame we examined, of the bearish call strategy of the 1% percentile predictor and the bullish call strategy of the 99% percentile predictor, are presented in Figure 6 and Figure 7 respectively[23]. The bullish call strategy seems to have a smooth course while the bearish call strategy

[23] The graphical representations of the courses of cumulative results of the rest trading strategies across all the percentile predictors that were examined are presented in Appendix A in page 38.

seems to have a rough one. In Figure 8, the course of the cumulative profits of the bearish call strategy of the 1% percentile predictor is presented alongside the course of the price of the S&P500 index, where the former seems to follow the latter.

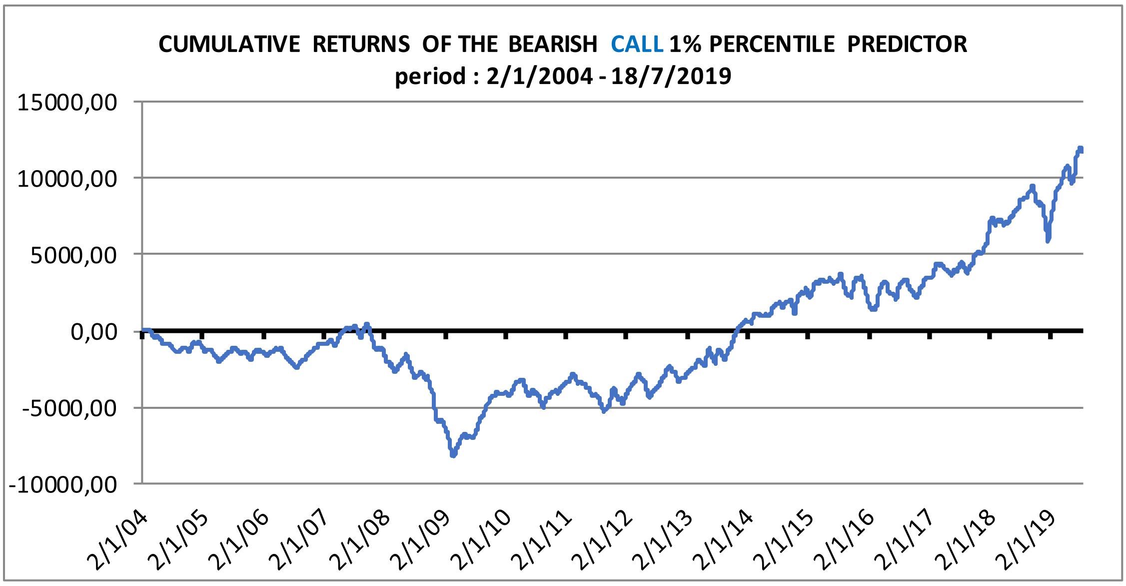

Figure 6. Cumulative Returns of the 1% Percentile Predictor' Bearish Call trading strategy

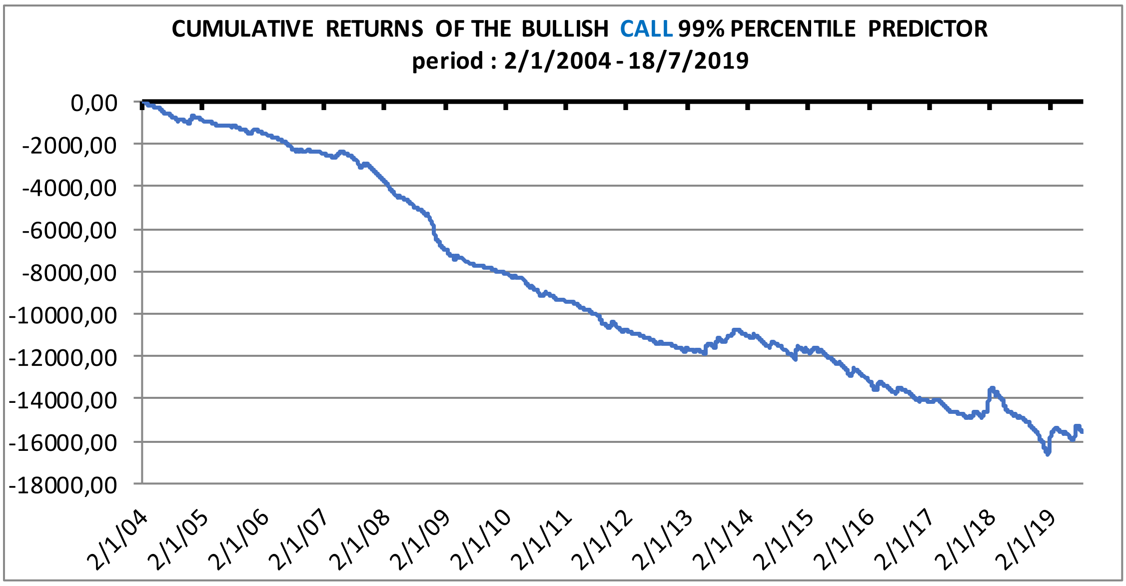

Figure 7. Cumulative Returns of the 99% Bullish Percentile Predictor's Call trading strategy

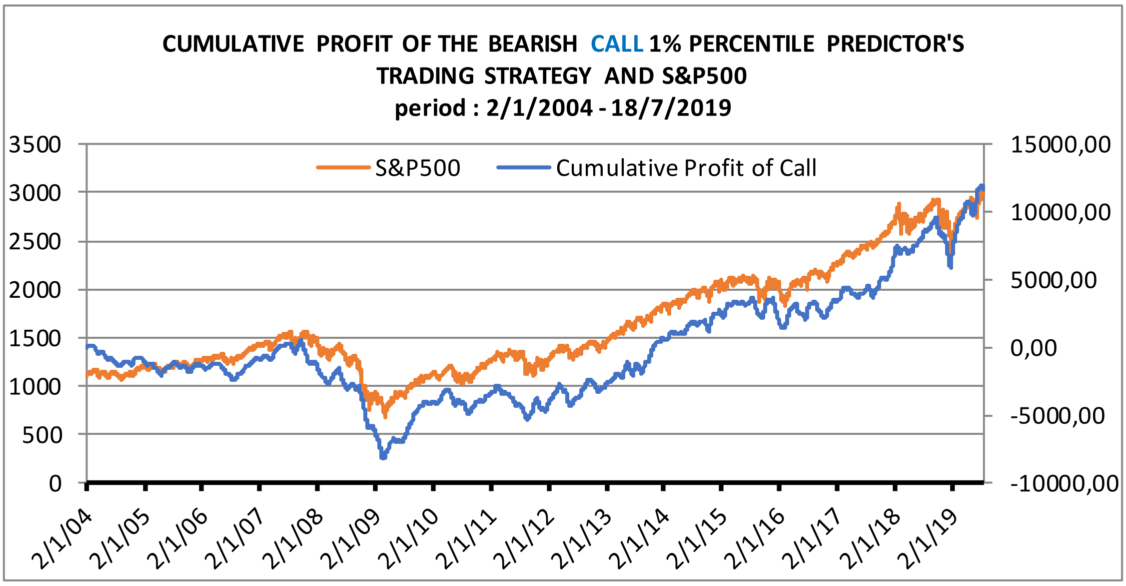

Figure 8. Cumulative Returns of the 1% Bearish Percentile Predictor's trading strategy and the S&P500 index

These results provide empirical evidence that percentile predictors on index options' trading strategies are profitable when applied contrary to the trading norm with respect to the underlying asset's trend. Also, *extreme* percentile predictors, derived from percentiles on the far-ends of the empirical distribution's tails, such as the 1% and 99%, outperform the moderate ones. Furthermore, bearish call options' trading strategies of percentile predictors seem to follow the trend of the underlying asset.

For robustness purposes, front month future contracts' daily prices of VIX were employed as the volatility input for the BS formula but no qualitive improvement was observed[24]. Relevant Figures are presented in 9. APPENDIX B: Cumulative returns of Percentile Predictors' strategies on index options of the S&P500, using VIX Futures' daily prices as the volatility input in the BS formula.

[24] See Table 3: Cumulative Returns of Percentile Predictors' trading strategies using VIX Futures' daily prices as the volatility input in the BS formula
over the period January 2nd 2003 – July 18th 2019s

6. Conclusions

Options are financial instruments which are widely used by market participants for both hedging and speculation purposes. The correct pricing of options has triggered a lot of interest in the financial literature and transformed both the theory and practice of finance on the intellectual foundation of the Black-Scholes model, which not only provides the framework for pricing all derivatives but also underpins risk-management practices and valuation under uncertainty. On this basis, numerous methods have been proposed in order to limit risk by modelling and forecasting financial time series, mostly through stochastic processes. In this context, market efficiency has improved by limiting both risk and opportunities for speculation.

This study examined whether a deterministic approach on pricing options could be employed for speculation purposes. Percentile points of the past trading year's empirical distribution of log-returns of the underlying asset were utilized as deterministic predictors of a potential future price of the asset. These percentile predictors reflect either a bullish or a bearish trading strategy to be implemented. The percentile predictors on index options' trading strategies were comparatively evaluated on the basis of the cumulative profits of options which utilized the predicted future price of the asset as the exercise price input parameter.

The results of the empirical investigation of the percentile predictor trading strategies on options of the S&P500 index over a period of fifteen years, from January 2nd 2004 up to July 18th 2019, suggest that within this framework profits can be earned if a trader acts contrary to the trading norm. In other words, when the index value is in an uptrend, bearish call percentile predictors' strategies, especially of extreme percentile predictors, are profitable and the course of their cumulative profits seem to follow the trend of the index. However, conclusions cannot be drawn in the case of a downtrend of the index's value.

These findings are very interesting in the context that there is allowance for speculation in option trading which opposes the trading norm and the ubiquitous practice of pricing options. Further research in this direction is needed considering the range of options' time horizons to maturity and the variety of the underlying assets' classes. In addition, further investigation on the effect of the rate of the asset's returns on the efficiency of the trading strategies in question, their performance in downtrend periods and the comparative evaluation of various percentiles could provide enlightening findings.

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8. APPENDIX A: Cumulative returns of Percentile Predictors' trading strategies on index options of the S&P500, using VIX as the volatility input in the BS formula

8.1. Cumulative returns of Bearish Call Percentile Predictors' trading strategies

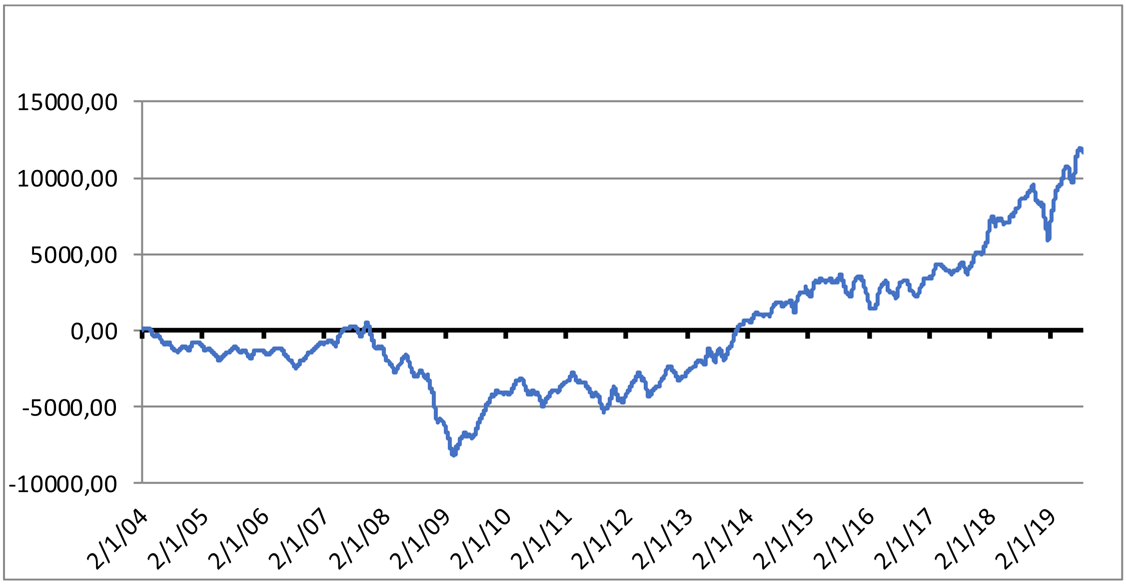

Figure 9. Cumulative Returns of the 1% Percentile Predictor's Call trading strategy

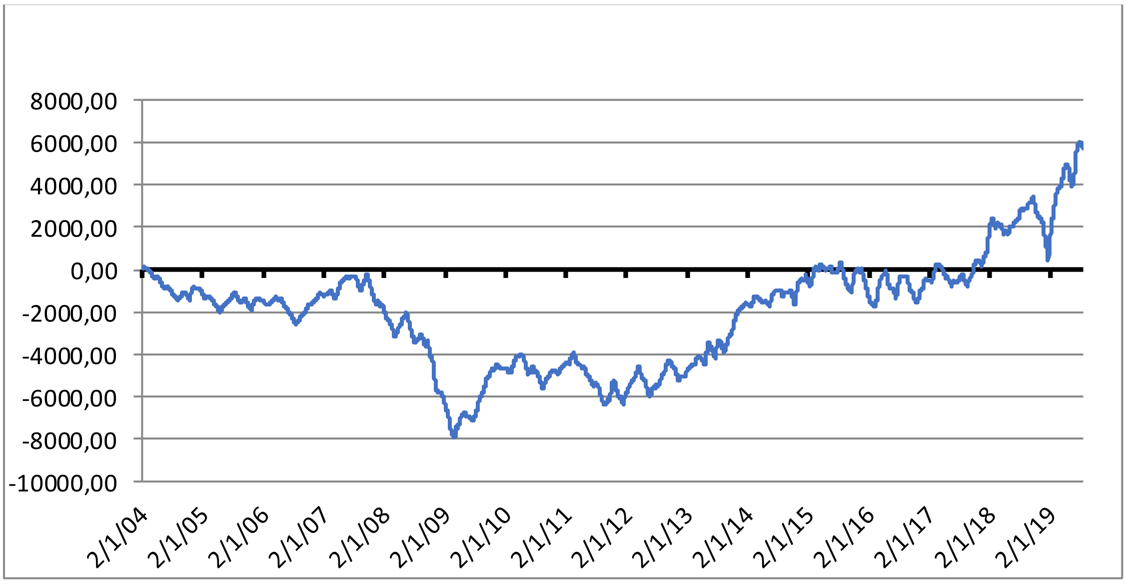

Figure 10. Cumulative Returns of the 5% Percentile Predictor's Call trading strategy

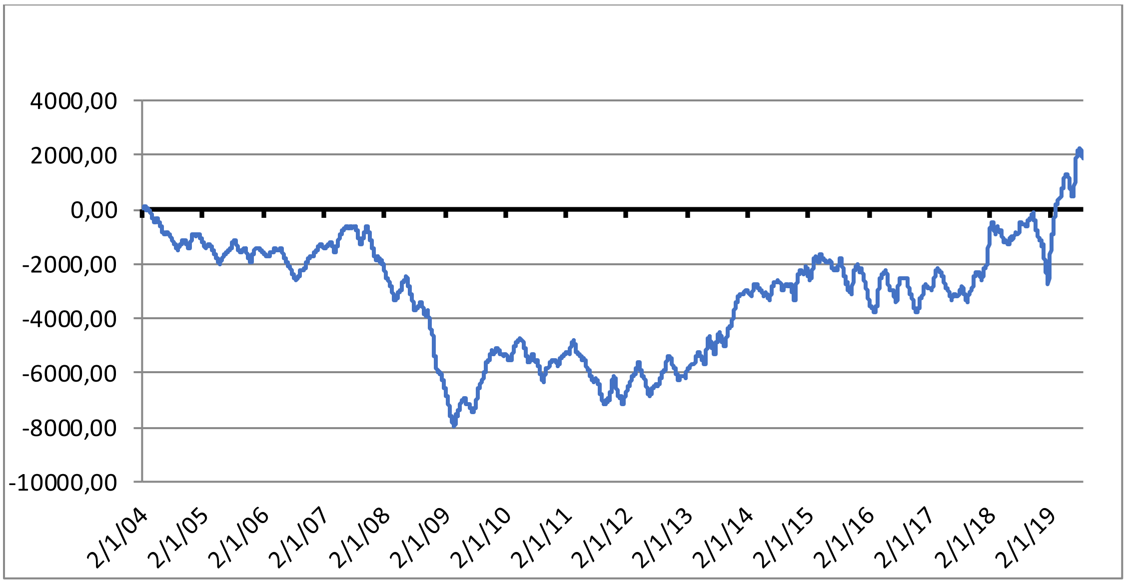

Figure 11. Cumulative Returns of the 10% Percentile Predictor's Call trading strategy

8.2. Cumulative Returns of Bearish Put Percentile Predictors' trading strategies

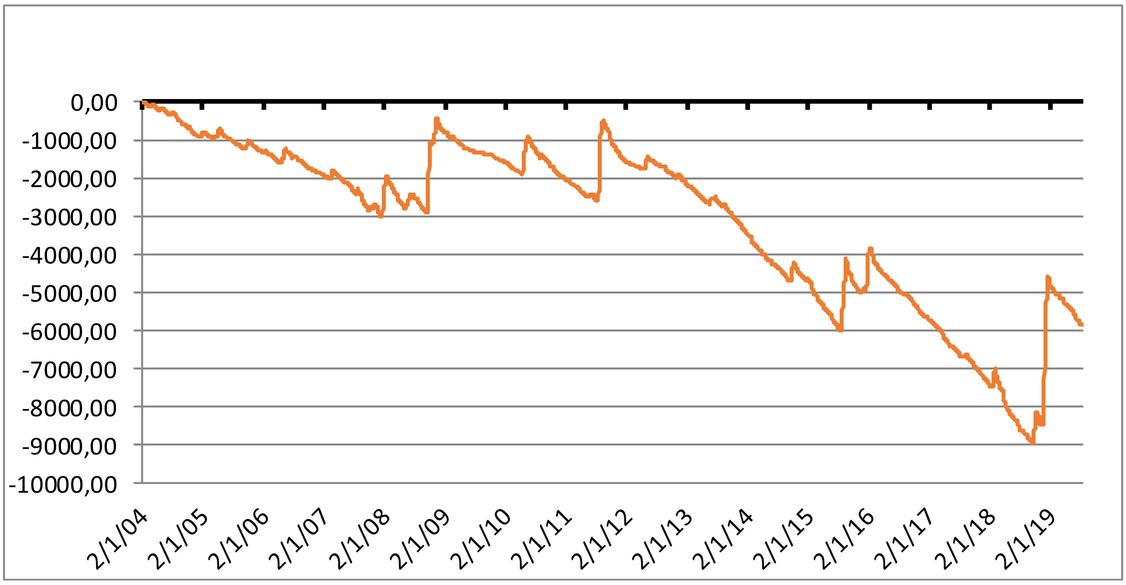

Figure 12. Cumulative Returns of the 1% Percentile Predictor's Put trading strategy

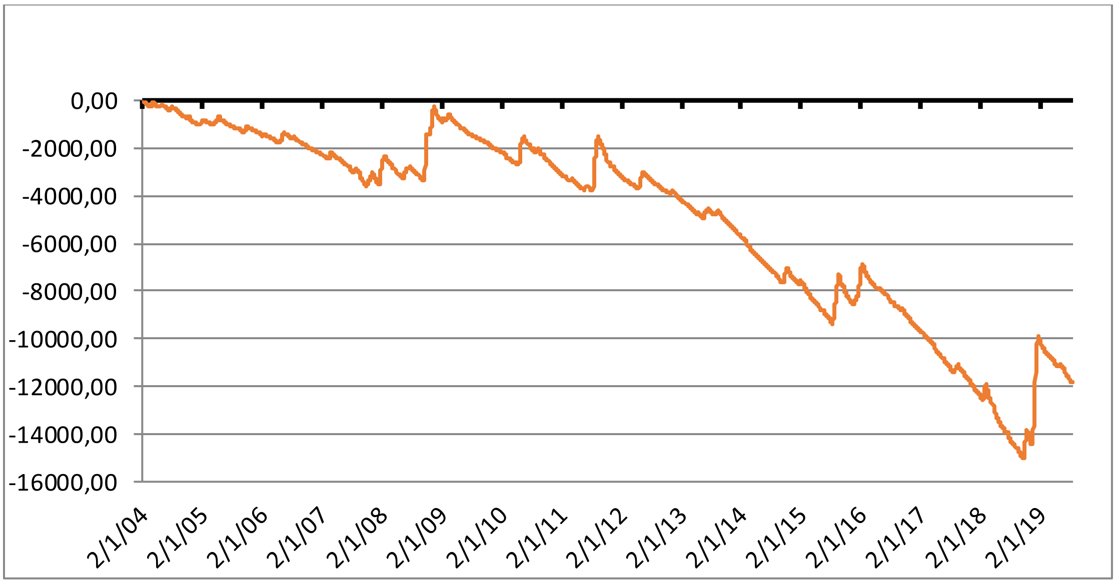

Figure 13. Cumulative Returns of the 5% Percentile Predictor's Put trading strategy

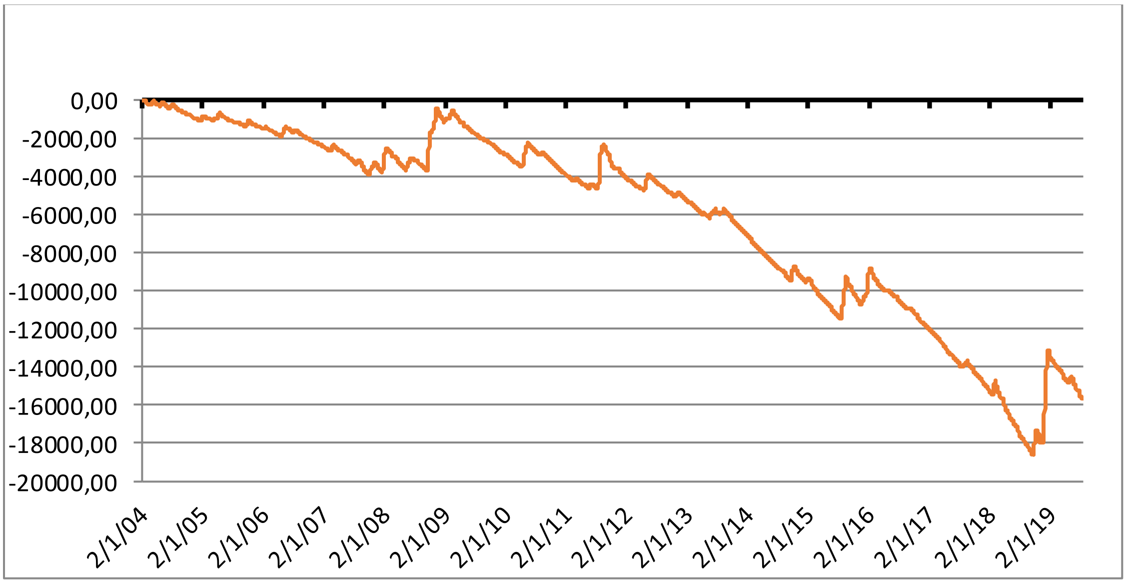

Figure 14. Cumulative Returns of the 10% Percentile Predictor's Put trading strategy

8.3. Cumulative returns of Bullish Call Percentile Predictors' trading strategies

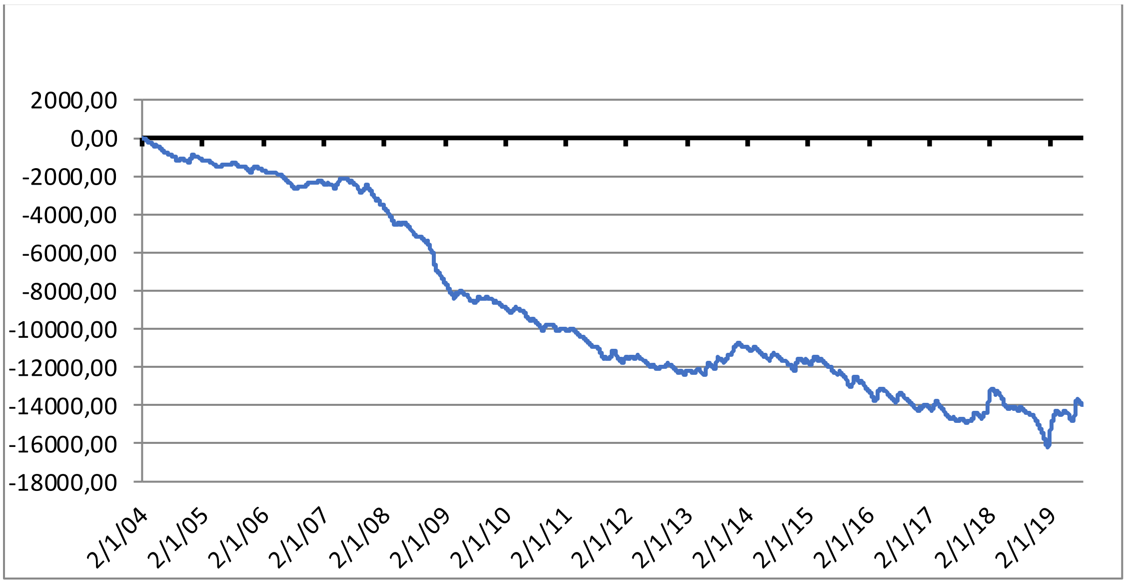

Figure 15. Cumulative Returns of the 90% Percentile Predictor's Call trading strategy

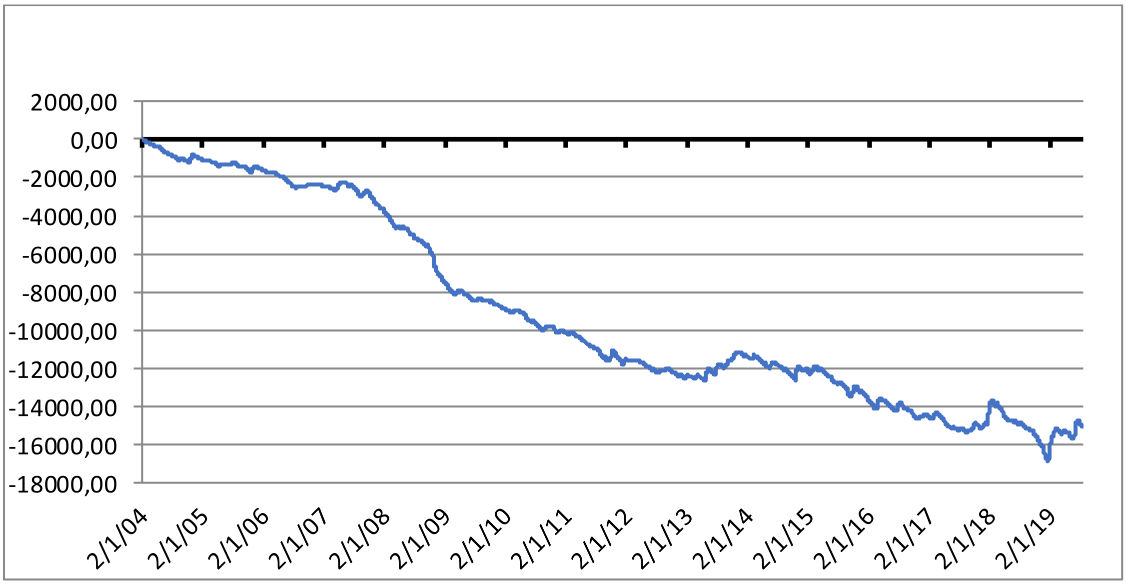

Figure 16. Cumulative Returns of the 95% Percentile Predictor's Call trading strategy

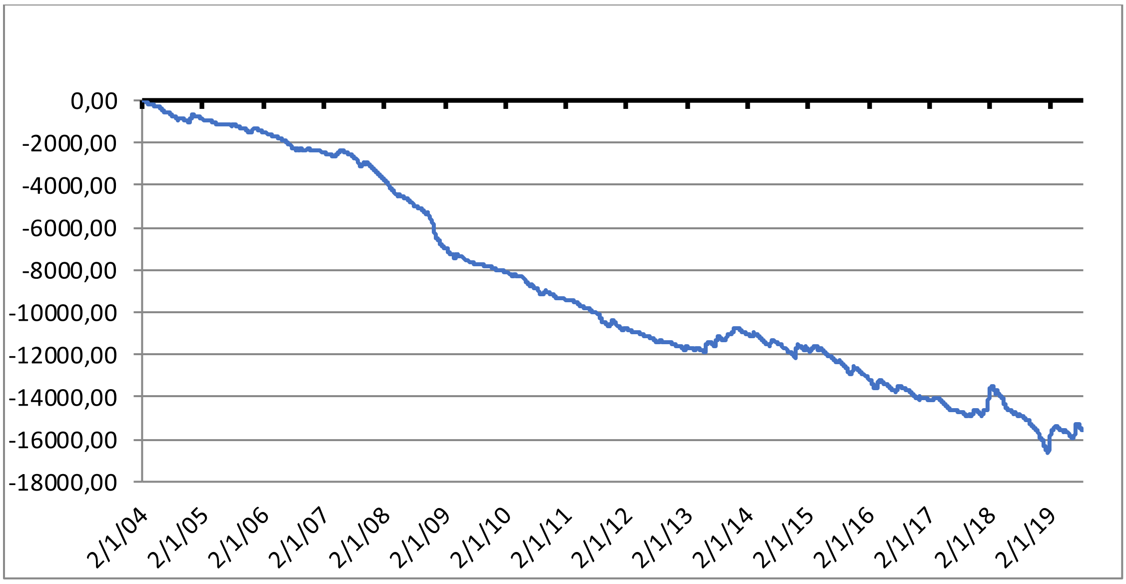

Figure 17. Cumulative Returns of the 99% Percentile Predictor's Call trading strategy

8.4. Cumulative Returns of Bullish Put Percentile Predictors' trading strategies

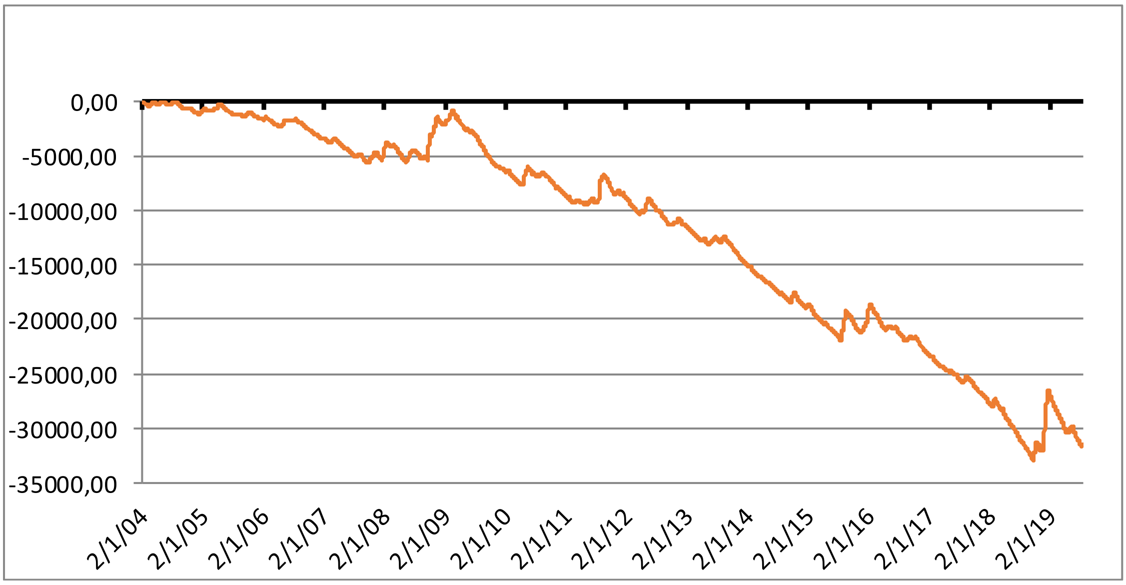

Figure 18. Cumulative Returns of the 90% Percentile Predictor's Put trading strategy

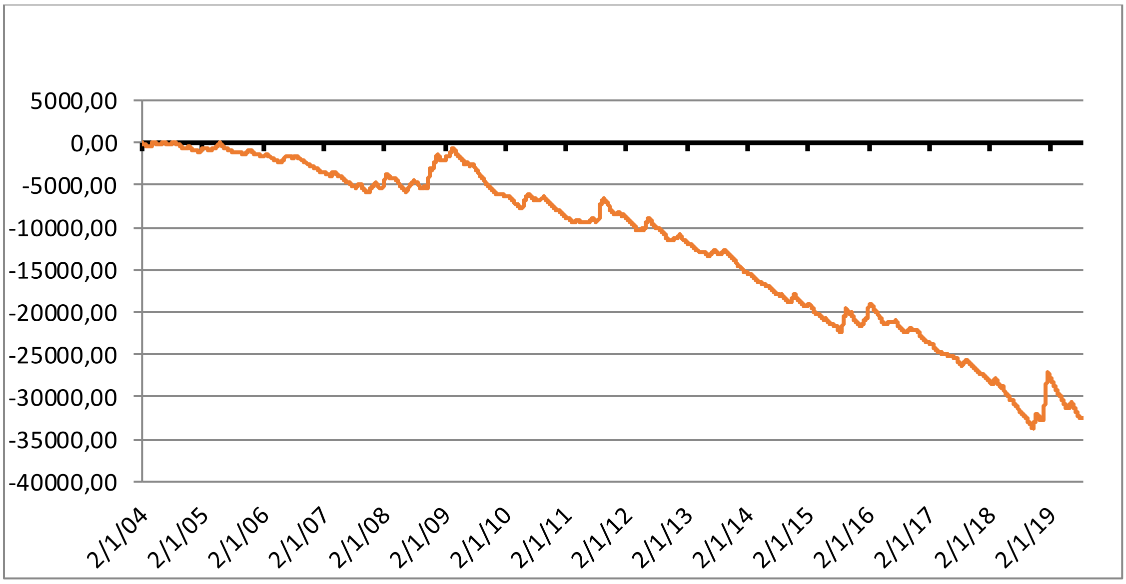

Figure 19. Cumulative Returns of the 95% Percentile Predictor's Put trading strategy

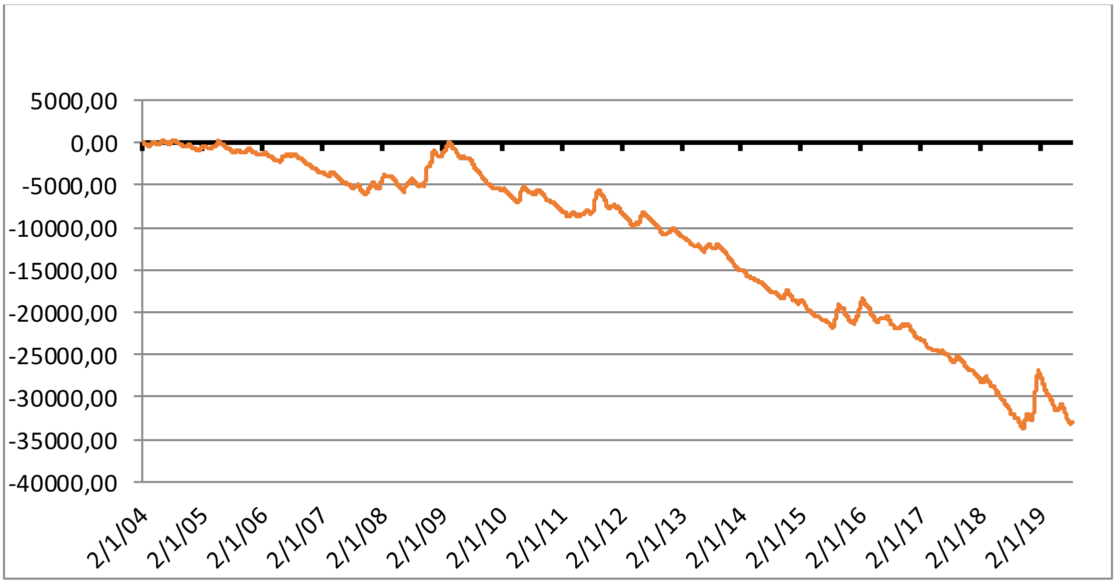

Figure 20. Cumulative Returns of the 99% Percentile Predictor's Put trading strategy

9. APPENDIX B: Cumulative returns of Percentile Predictors' strategies on index options of the S&P500, using VIX Futures' daily prices as the volatility input in the BS formula.

Table 3: Cumulative Returns of Percentile Predictors' trading strategies using VIX Futures' daily prices as the volatility input in the BS formula over the period January 2nd 2003 – July 18th 2019s

	Bearish strategies			Bullish strategies		
percentiles	1%	5%	10%	90%	95%	99%
Calls	$9.539,37	$3.014,51	-$1.053,69	-$16.978,16	-$17.853,42	-$17.949,81
Puts	-$8.004,24	-$14.497,43	-$18.548,04	-$34.401,61	-$35.265,74	-$35.334,40

9.1. Cumulative returns of Bearish Call Percentile Predictors' trading strategies

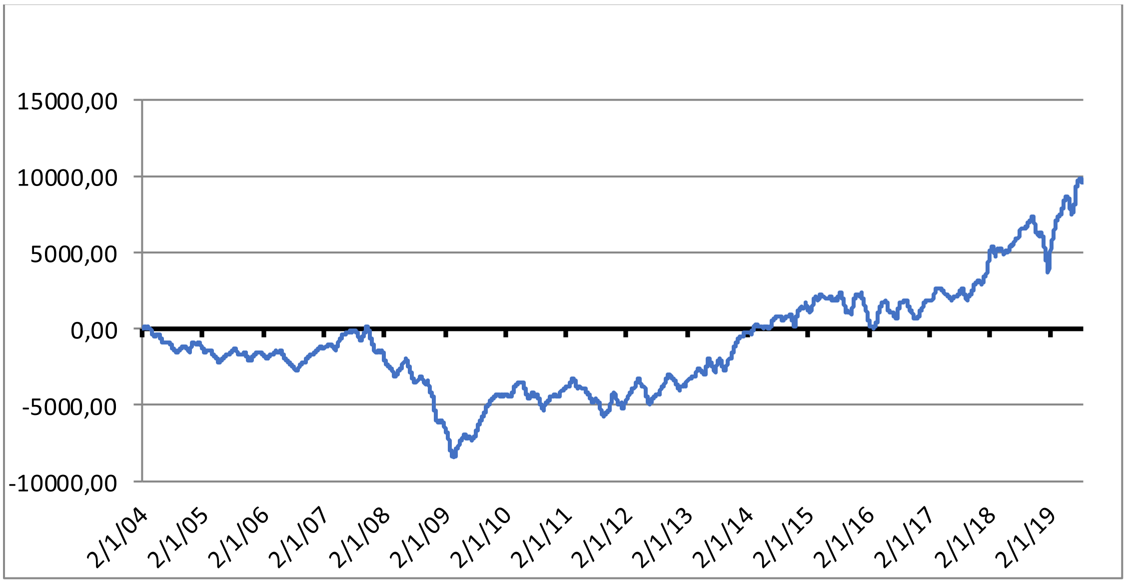

Figure 21. Cumulative Returns of the 1% Percentile Predictor's Call trading strategy

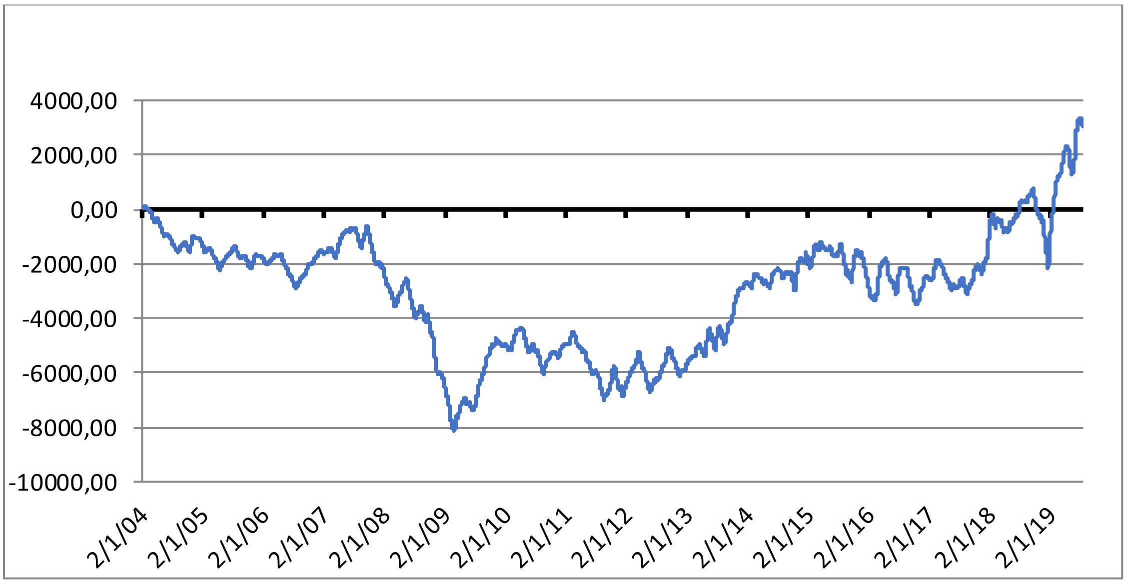

Figure 22. Cumulative Returns of the 5% Percentile Predictor's Call trading strategy

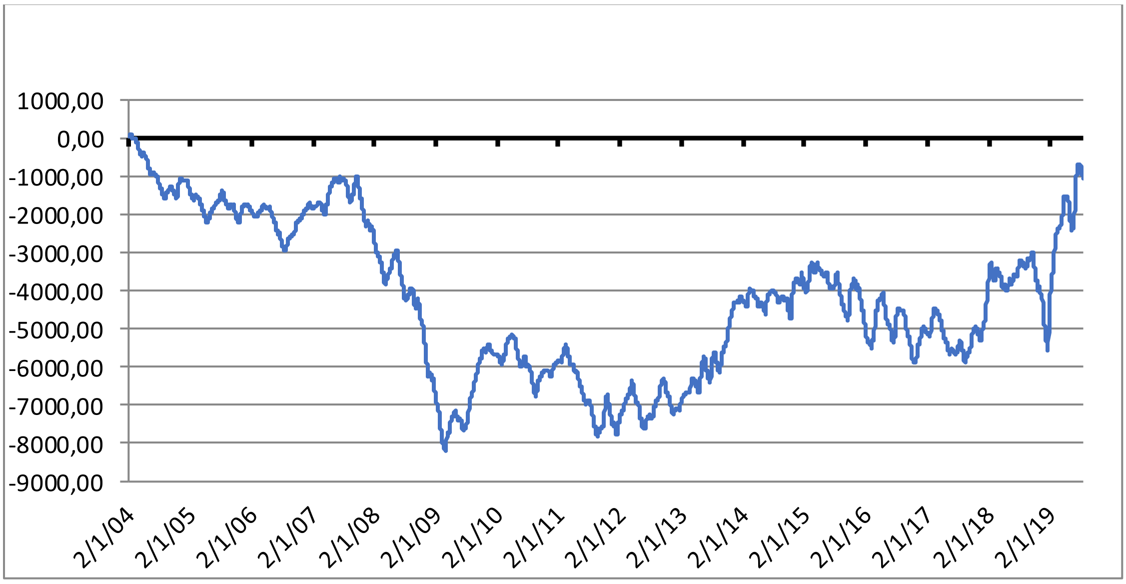

Figure 23. Cumulative Returns of the 10% Percentile Predictor's Call trading strategy

9.2. Cumulative returns of Bearish Put Percentile Predictors' trading strategies

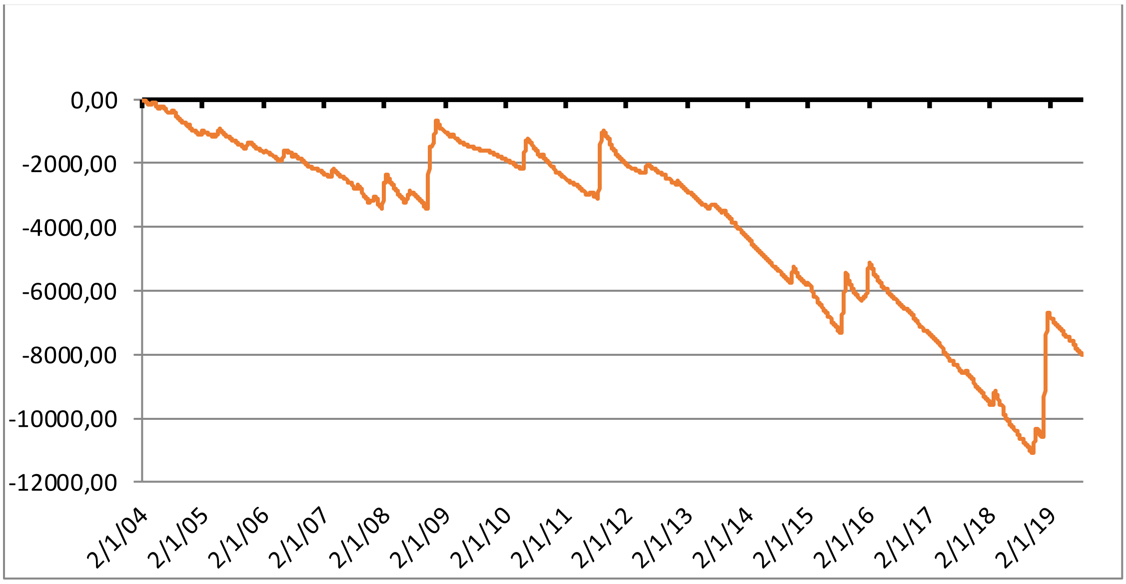

Figure 24. Cumulative Returns of the 1% Percentile Predictor's Put trading strategy

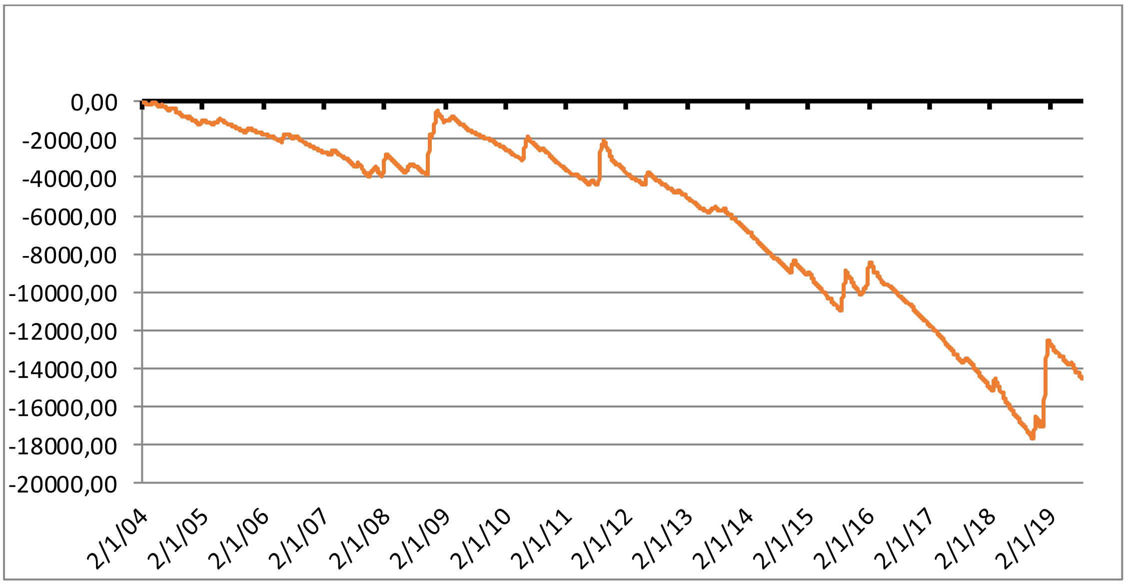

Figure 25. Cumulative Returns of the 5% Percentile Predictor's Put trading strategy

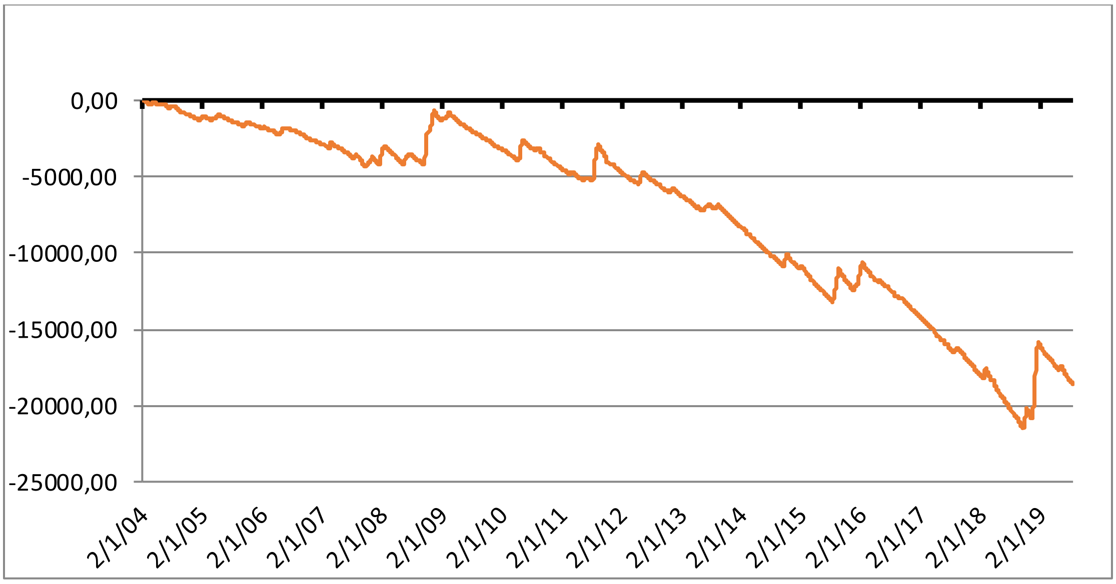

Figure 26. Cumulative Returns of the 10% Percentile Predictor's Put trading strategy

9.3. Cumulative Returns of Bullish Call Percentile Predictors' trading strategies

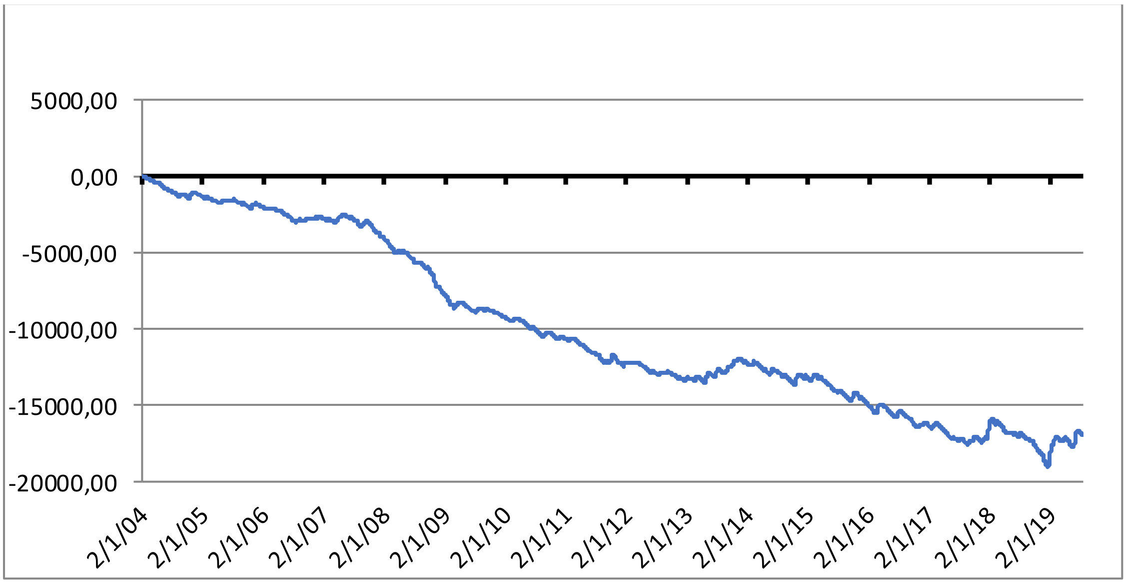

Figure 27. Cumulative Returns of the 90% Percentile Predictor's Call trading strategy

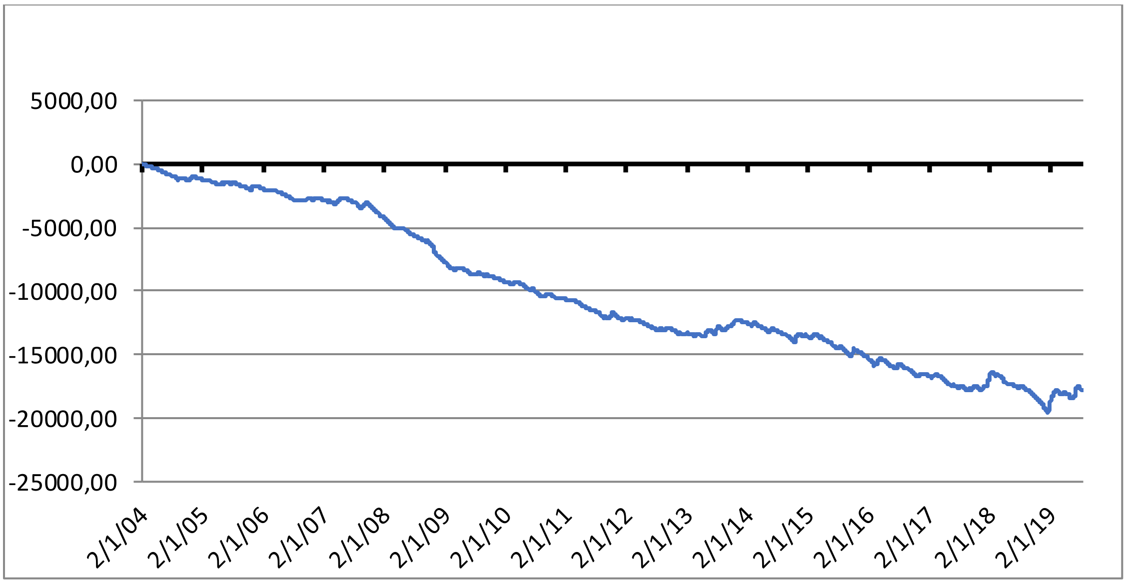

Figure 28. Cumulative Returns of the 95% Percentile Predictor's Call trading strategy

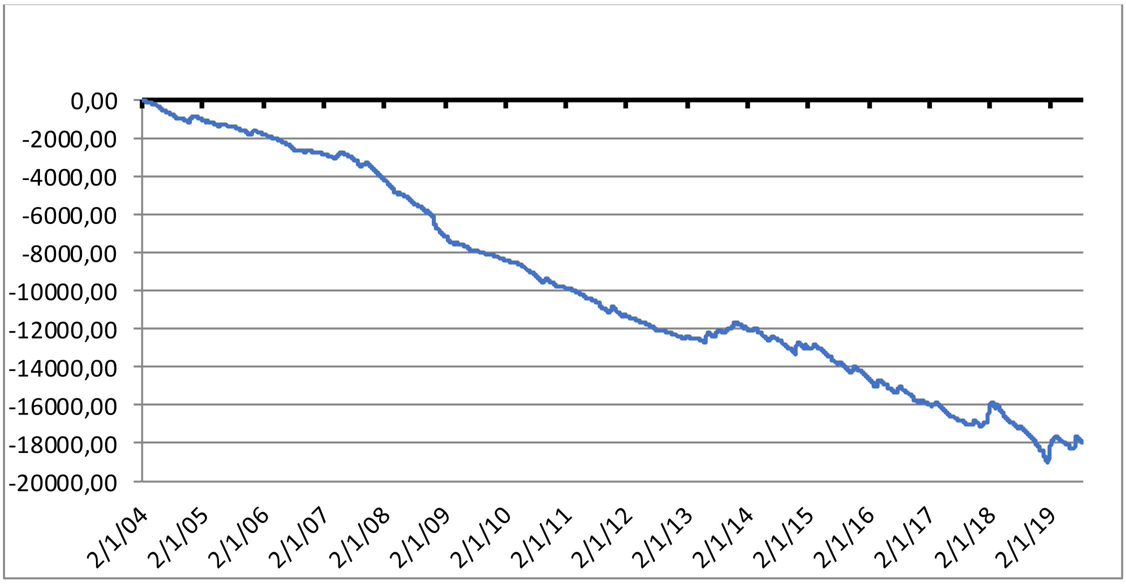

Figure 29. Cumulative Returns of the 99% Percentile Predictor's Call trading strategy

9.4. Cumulative returns of Bullish Put Percentile Predictors' trading strategies

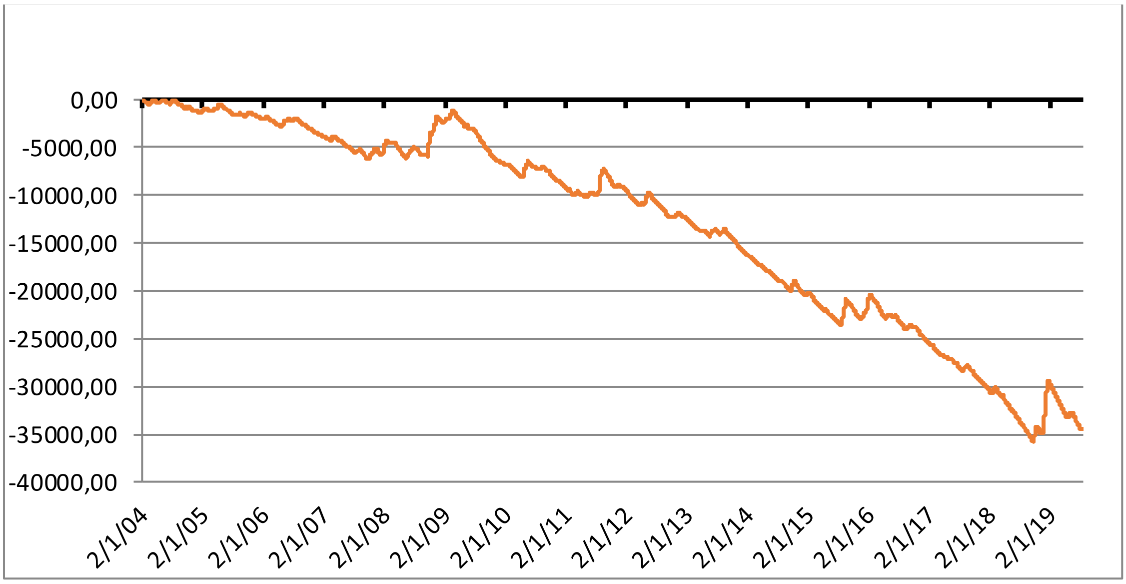

Figure 30. Cumulative Returns of the 90% Percentile Predictor's Put trading strategy

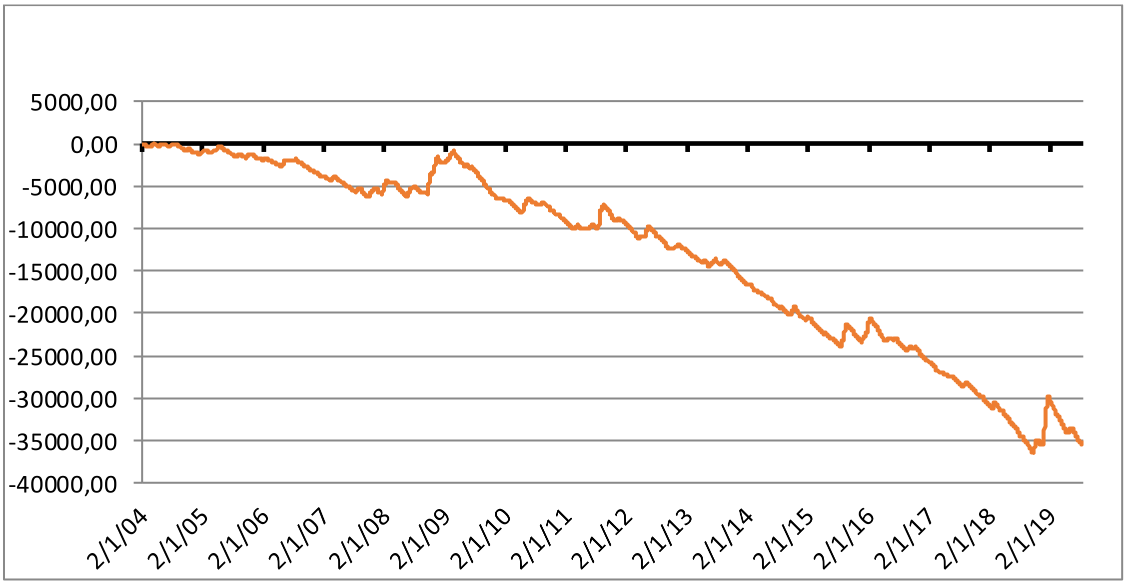

Figure 31. Cumulative Returns of the 95% Percentile Predictor's Put trading strategy

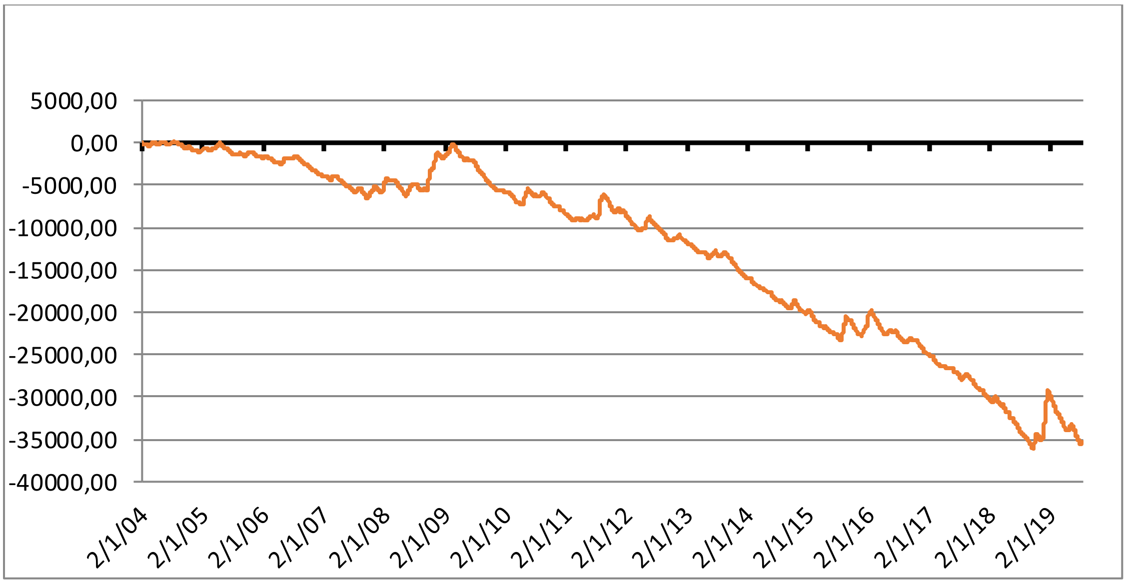

Figure 32. Cumulative Returns of the 99% Percentile Predictor's Put trading strategy