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PANTEION UNIVERSITY OF SOCIAL AND POLITICAL SCIENCES

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Trading Futures of Implied Volatility Index

ΔΙΠΛΩΜΑΤΙΚΗ ΕΡΓΑΣΙΑ

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Αθήνα,2019

Τριμελής Επιτροπή

ΝΤΕΓΙΑΝΝΑΚΗΣ ΣΤΑΥΡΟΣ, ΑΝΑΠΛΗΡΩΤΗΣ ΚΑΘΗΓΗΤΗΣ ΠΑΝΤΕΙΟΥ ΠΑΝΕΠΙΣΤΗΜΙΟΥ(ΕΠΙΒΛΕΠΩΝ)

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Ευχαριστίες

Ευχαριστώ θερμά τον επιβλέποντα καθηγητή μου κύριο Ντεγιαννάκη Σταύρο, καθώς και τον υποψήφιο διδάκτορα κύριο Δελή Παναγιώτη για όλη την υποστήριξη που μου παρείχαν με τις συμβουλές τους και τις υποδείξεις τους κατά την διάρκεια εκπόνησης της διπλωματικής εργασίας .

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Abstract

This paper studies the trading of the implied volatility index VIX and its futures. In order to do so we conduct forecasts on the index itself and its futures. For this study we use heterogeneous autoregressive models also known as HAR models, in order to deal with inherent financial data difficulties. We use said models to forecast both the VIX index and its futures. Specifically regarding the forecasting of the index futures we use two types of data, two time-series that are based on the expiration date of the product and another one that uses the highest trading volume criterion. We conclude that it is better to trade the index's futures (based on the investing.com chain) based on one day ahead forecasts of said futures.

1. Introduction

In this text we discuss the general idea behind this paper. Futures of the S&P 500 implied volatility index are a stock exchange product. Therefore conducting forecasts on it might be of economic value if said forecasts are used in conjunction with trading the futures. Before we delve deeper into the process followed it is important that we established the general background.

VIX

The VIX index is created by the Chicago Board Options Exchange, CBOE, and is an index that represents the market's expectation of 30-day forward looking volatility. Essentially it is an index of implied volatility. Many studies consider implied volatility, first noted by Latané and Rendleman (1976) ,as an accurate predictor of future volatility. According to Whaley (2008),it is an index that measures volatility and not price. Furthermore, it is a financial benchmark designed to be an up-to-the-minute market estimate of expected volatility of the S&P 500 index. Another way to use the VIX would be for speculation purposes since people can bet on the index's movements. Last but not least, VIX can be used as a hedge in order to protect a portfolio against a market crush. The latter can be achieved when for example someone buys calls while simultaneously assuming the opposite position (sells puts) of the same expiration date. This is known as the "reverse collar" strategy.

It is calculated by using the mid-point of real-time S&P 500 index(SPX) option bid and ask quotes. The general formula used for the calculation of VIX according to the official CBOE website is:

$$\sigma^2 = \frac{2}{T}\sum_i \frac{\Delta K_i}{K_i^2} e^{RT} Q(K_i) - \frac{1}{T}\left[\frac{F}{K} - 1\right]^2 \tag{1}$$

where:

σ stands for $\frac{VIX}{100}$

R is risk free interest rate to expiration

F is the forward index level and is derived from index option prices

T stands for time to expiration

K_0 is for the first strike below the index level F

K_i is the strike price of i^{th} out-of-the-money option; a call if $K_i > K_0$ and a put if $K_i < K_0$ and a both call and put if $K_i = K_0$

ΔK_i is the interval between strike prices , technically half the difference between the strike on either side of ΔK_i

$$\Delta K_i = \frac{K_{i+1} - K_{i-1}}{2} \tag{2}$$

$Q(K_i)$ stands for the midpoint of the bid and ask spread option with strike K_i

The VIX index is also known as the "fear index" or "fear gauge" since option prices tend to change in times of turbulence and therefore investors might use the VIX in order to get an idea of what do people think, or expect to happen in the underlying market.

VIX Futures

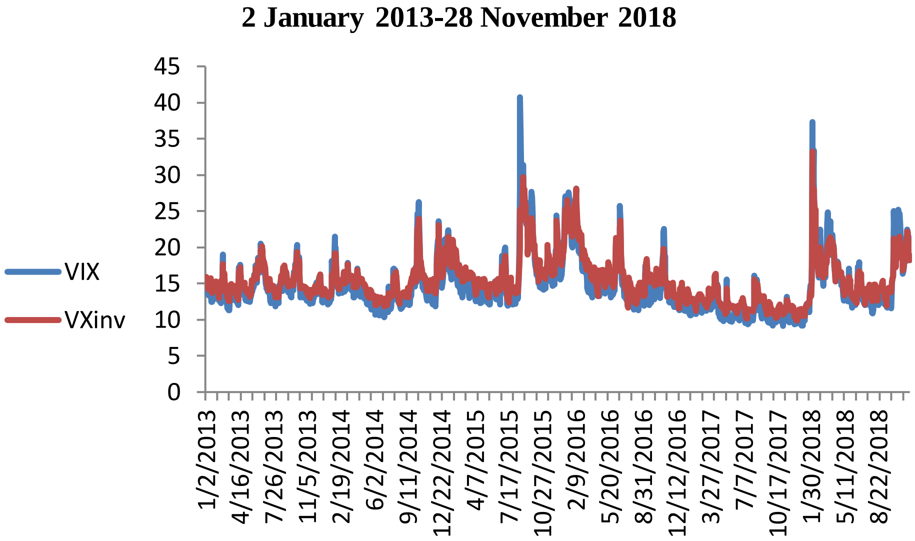
It is important to be mentioned that the index of implied volatility(VIX) is not a product that can directly be traded. However, on March, 2004 the CBOE introduced the VIX futures and therefore transformed the VIX into a security, thus making it a stock exchange tradable product. In general, futures are contracts that obligate the buyer to purchase or the seller to sell an asset at a specific future date and price that have been pre-determined. In the case of VIX futures (denoted as VX) one can trade volatility independent of the level of stock prices. The characteristics of the VIX futures are as follows according to the official CBOE website:

- The contract multiplier for each VX contract is $1000
- The minimum price interval is 0.05 points. This means a $50 change per contract
- The final settlement value for a VX contract is calculated using A.M settled SPX options.[1] More specifically the final settlement value is a special opening quotation of the VIX index, which is calculated from the sequence of opening prices for regular trading hours of the SPX options used to calculate the index on the final settlement date.
- The final settlement date for a VIX future contract is on the Wednesday that is 30 days prior to the third Friday of the calendar month immediately following the month in which the contract expires.

It should be noted that as other stock exchange products VIX futures are subject to official holidays and special CBOE holidays as well.

[1] Only SPX options that expire on Friday are used. The difference between A.M and P.M settled options is that A.M options stop trading at Thursday evening but their settlement value is settled on Friday, therefore the trader is vulnerable during overnight.

Figure1: Figure of the VIX index and VIX futures



In the figure above we see that our futures chain moves in the same way the index moves, a finding which agrees with the literature.

HAR models

Financial data are hard to be used in conjunction with standard econometrics models. For example, one serious problem is the strong persistence of the returns (squared or absolute), a persistence that lasts for long time periods. Another problem is that financial data show scaling and multiscaling behaviors. As a result, standard stochastic volatility models cannot cope with those problems. Hence, econometricians tried to solve the abovementioned problems by using fractional difference operators in order to achieve long- memory volatility, Corsi(2009). Fractional difference operators have been used in the FIGARCH models of returns or ARFIMA models of realized volatility and were able to achieve long- memory in a parsimonious way. Still, this approach came with some problems of its own since imposing a set infinite-dimensional restrictions on the infinity variable lags is a mathematic trick which lacks any economic interpretation, Corsi (2009) .Furthermore, by applying the fractional difference operator we end up losing a lot of observations due to the long build-up period that is needed. Last but not least, models such as these cannot reproduce the multiscaling behavior of the empirical data.

Fulvio Corsi (2009) proposed an additive cascade model of volatility components defined over different time periods (both long and short), a model which is generated by the actions of different types of market participants. As he states the model "*is able to reproduce the main empirical features observed in the data while remaining parsimonious and easy to estimate*". He named the model Heterogeneous

Autoregressive model of Realized Volatility (HAR-RV). The idea behind the model comes from the Heterogeneous Market Hypothesis (Müller and Dacorogna and Dav and Pictet and Olsen and Ward, 1993). According to this hypothesis there is heterogeneity across traders as opposed to the idea of a homogeneous market in which, all participants react to the news in the same way. More specifically, in financial markets heterogeneity exists for different reasons like: various time horizons, different ways of processing information, geographical locations etc. Corsi (2009) focused on the heterogeneity that arises from the difference in time horizons. The main idea is that agents with different time horizons perceive and react differently and thus create different types of volatility components. In more detail, Corsi(2009) used three primary volatility components: the short-term traders with daily or higher trading volatility, the medium-term traders who typically rebalance their positions weekly and the long-term agents whose characteristic time is one or more months. Therefore we end up with the following model:

$$RV_{t+1d}^{d} = c + \beta^{d} RV_{t}^{d} + \beta^{w} RV_{t}^{w} + \beta^{m} RV_{t}^{m} + \omega_{t+1d} \tag{3}$$

where RV_{t+1d}^{d} is the ex post volatility estimation and ω_{t+1d} is a volatility innovation.

This type of models are extensively used in the literature as they are easy to estimate, are believed to predict future volatility and can capture certain features of financial data, such as long memory (Andersen, 2007; Busch, 2011).

By running the model above for each of our cases we obtain various outputs which lead to the conclusion: The highest economic profit is gained when we trade the index's futures (based on the Investing.com chain) based on one day ahead forecasts of said futures.

The rest of the paper is organized as follows. The VIX and real life section contains some real life examples of the index. The data section contains a detailed description of the data. The methodology section describes the procedure we followed in order to reach our results. In the results section we speak of the outputs we got in detail. The final section concludes the study.

2. VIX and Real Life

The index of implied volatility of S&P 500 options, VIX, has become a staple component of trading in the years after its release as the index’s up and down movements relate to trader's behavior. Various factors like the underlying market, news etc. affect the VIX and there is a big discussion as to what really makes it tick. Below we mention some real life events and their impact on the markets and the VIX.

The return of the "tariff man"

On May,5,2019 president Donald Trump threatened to boost the existing tariffs on China via his twitter account. As a result, volatility soared on Monday, May 6,2019. The VIX index jumped up around 46 percent peaking at around 18.80 points. This increase in the index reflects concerns amidst the traders that the new tariffs will cause the trade talks between China and the U.S to crumble. Until April 23, hedge funds were net short about 178,000 VIX futures contracts, the largest position on record according to CFTC data. This aggressive betting against the VIX was evidence of confidence among traders. It should be mentioned that the index's upward movement was the highest since February's 2018 market turmoil. Furthermore, China's benchmark stock indexes (CSI 300, Shangai composite index) plunged and experienced their biggest drop in three years on that Monday. Similar news from across Europe as well since Germany's DAX, France's CAC 40 and Spain's IBEX 35 dropped. In the U.S, the Dow Jones Industrial Average dropped as much as 471 points. While people view these changes in different ways, David Spika, president of Guidestone Capital Management says:" *combined with optimism surrounding Friday's labor report and recent 1Q GDP surprise, i doubt these comments move the needle for more than a day or two*", whereas Andrew Tilton, chief Asia-Pacific economist at Goldman Sachs Group Inc states that:"*trade has been put to the side by many market participants*", it does not change the fact that Chinese stocks tumbled and so did yuan.

February's Volmaggedon

On February,5,2018 the volatility index VIX jumped from 18 points to 37 in one trading day, the highest level since 2015. This marks an over one-hundred percent increase of the index, the highest noted ever one-day jump .The previous week a market selloff had sparked and as a result affected various global markets. As a result, the Dow Jones Industrial Average fell 1175 points on that day its biggest one-day drop ever. The S&P500 declined by 4.1 percent. Furthermore, the investors betting against the index VIX had severe loses(billions) as products that trade at the inverse of VIX (XIV, SVXY) crumbled. More specifically, XIV opened at $99 on Monday and at Tuesday's opening it had fallen to $7.35. As Nick Colas co-founder of Data Trek states:"*it's kind of a design flaw to issue a product that can mathematically go to zero in a day."* The trading of those products was halted temporarily. It should be mentioned that until that moment volatility has been historically low something that caused complacency among investors. While there are a lot of opinions as to what caused this event later referred to as "Volpocalypse" or "Volmaggedon" since the markets are a complex thing, a lot of people seem to think that the complacency of people believing that the tame conditions of the previous year would continue and the collapse of the overcrowded "short volatility trade" had a major impact.

The British referendum

On June,23,2016 the UK held its referendum. The question was whether or not to leave the EU. The result of the referendum was in favor of leaving. As a result, both European and American markets suffered, a usual phenomenon given that markets are linked to each other. The following day the VIX rose from 17.25 to 25.76 showing an increased uncertainty. The underlying S&P 500 fell 3.6%(the biggest decrease in 10 months) and NASDAQ decreased by 4.1% (its biggest decrease since 2011). On Britain's side now, the FTSE 100 plunged more than 500 points as the markets opened, although it gradually recovered over the day following the bank of England's statement:"*we are willing to take all necessary steps to fulfill its responsibilities*." Furthermore, sterling fall from $1.50 against the US dollar to $1.33 and eventually closed to $1.368. Consequently, there was an increased demand for protection and many turned to gold in order to find a safe haven. As a result, the price of gold jumped more than 5%.

The Greek referendum

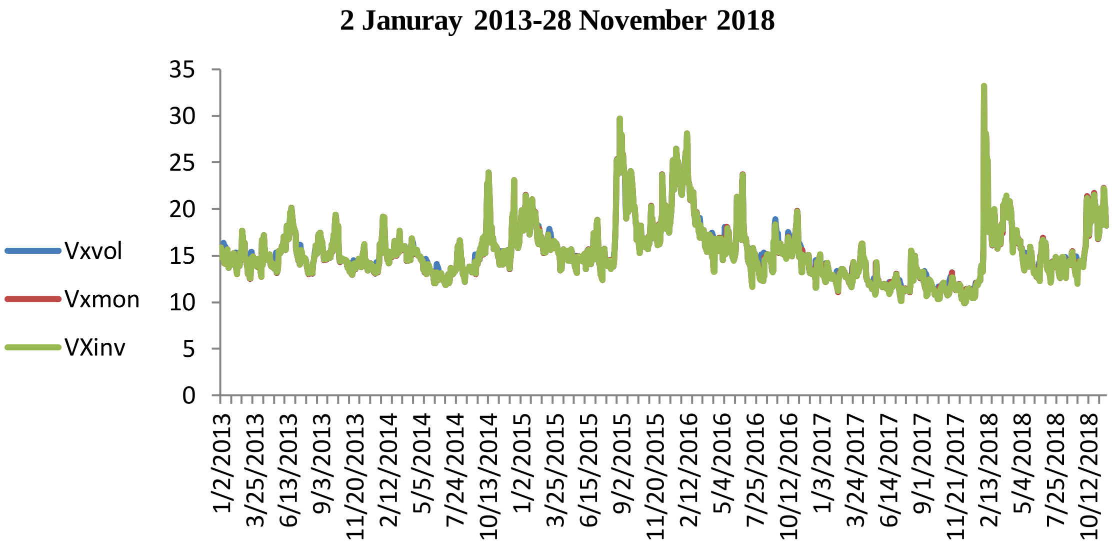
On June,27,2015 the Greek prime minister Alexis Tsipras announced a referendum in order for Greece to decide whether to accept the bailout conditions in the country's debt crisis, conditions that were jointly proposed by the International Monetary Fund (IMF), the European Central Bank (ECB) and the European Commission (EC). The referendum day would be July,5, 2015.The VIX had closed at 14,02 on 6/26/2015. On Monday(6/29/2015) when the markets opened after the negotiations between Europe and Greece had crumbled and the announcement of the referendum the VIX rose and finally settled on 18.85 a jump of around 34%. The S&P 500 declined by 2% for the first time in the year placing it in the red. The Dow Jones Industrial Average fell by 350 points or 2%. S&P downgraded Greece to CCC-. The rating agency stated in a Reuters report that: *"the probability of the country exiting the euro zone is now 50 percent."* The referendum's result was in favor of "No". Still, despite the turmoil a lot of people believed that Greece would not leave the euro zone. Some traders thought that given that the VIX tends to decrease after an increase now it might be a good time to short the index. Indeed, the index's previous increase was followed by a decrease, reaching 16.09 on 7/7/2015 and then it increased again for a couple of days reaching 19.97 on 7/9/2015.

3. Data

The VIX index and futures closing and highest trading volume prices were obtained from the CBOE for the period of January the 2nd, 2013 to the 28th of November,2018. We obtained the closing prices of the futures for the same period from the Investing.com as well.

Due the nature of VIX futures there are multiple contracts available for each training day, more specifically on each date there nine contracts available, the one of the current month and then eight more contracts, one for each of the eight forward looking months. To construct the time series we use two different criteria in order to pick one contract for each day. The time-series based on data from the Investing.com uses the nearest month of expiration criterion[2]. One of the two time-series that are based on CBOE data is made with the nearest month of expiration criterion as well[3]. According to this criterion we "jump" to the next month on the third Wednesday of every month. The third time-series is based on CBOE data but uses a different criterion, that of the highest trading volume. In this case we "jump" to the next month when there is a significant change in the trading volume[4].

Figure2: Figure of the three different futures chains



In the figure above we see how the three different future chains progress through time. The three chains progress in the same way and are almost identical with only a few and small differences, result of the different criteria and datasets that were used. Below we use a few more figures. While the various ranges of dates used are still between the range of 2 January 2013-28 November 2018, they are chosen randomly in order to depict better said small differences.

[2] VXinv is the notation we use for the Investing.com time-series based on the nearest month of expiration criterion.
[3] VXmon is the notation we use for the CBOE time-series based on the nearest month of expiration criterion.
[4] VXvol is the notation we use for the CBOE time-series based on the highest trading volume criterion.

Figure2a: Figure of the three different futures chains

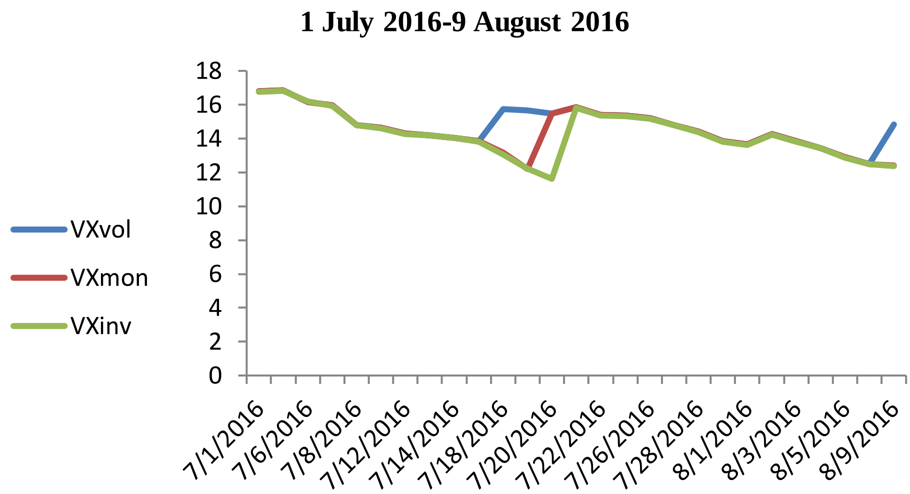


Figure2b: Figure of the three different futures chains

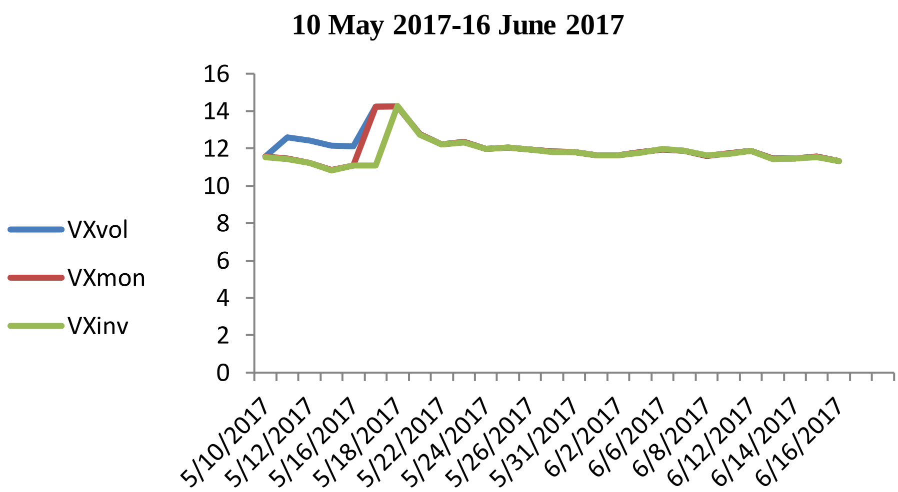


Figure2c: Figure of the three different futures chains

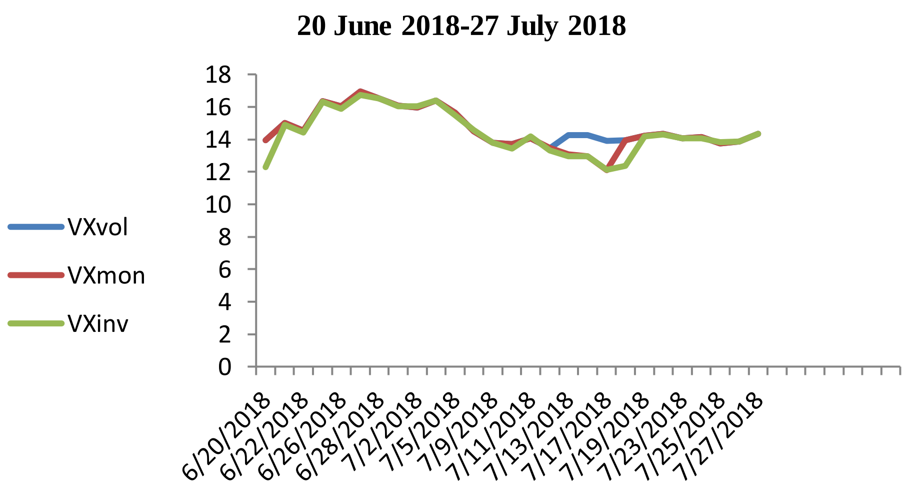


Table 1: Descriptive statistics of the implied volatility index, VIX futures investing time-series, VIX futures CBOE month, VIX futures CBOE volume.

	VIX(2nd Jan,2013-28thNov,2018)	VXinv(2nd Jan,2013-28thNov,2018)	VXmon(2nd Jan,2013-28thNov,2018)	VXvol(2nd Jan,2013-28thNov,2018)
Mean	14,63905977	15,35081934	15,3725319	15,48218939
Standard Error	0,098124121	0,080224358	0,079424394	0,075302489
Median	13,69	14,75	14,75	14,9
Mode	12,64	14,55	13,95	14,2
Standard Deviation	3,786370674	3,09566244	3,064793793	2,905739507
Sample Variance	14,33660288	9,583125941	9,392960991	8,443322085
Kurtosis	4,795885402	2,662902445	2,728838304	2,631752479
Skewness	1,756216707	1,342762051	1,354788838	1,27028477
Range	31,6	23,32	23,32	23,32
Minimum	9,14	9,88	9,88	9,88
Maximum	40,74	33,2	33,2	33,2
Count	1489	1489	1489	1489

Table 1 presents the descriptive statistics of the VIX index and the three different time-series we used. The mean value of the implied volatility index is 14,63 while its standard error is 3,78. Notice how the values of the time-series made with the same criterion are close to each other or even identical whereas the values of the time-series made with the highest trading volume criterion have a bigger difference compared to the ones made with the criterion of the nearest month of expiration.

4. Methodology

Estimation process

The models used in this paper are Heterogeneous Autoregressive Models also known as HAR models. The HAR model is being considered by the literature (i.e. Degiannakis and Filis, 2017, Sévi, 2014) as a prominent framework in predicting volatility accurately. On the left hand sign we have the logarithm of the dependent variable, which varies depending on the case and on the right hand sign we have the logarithm of the same variable in different time lags as well as the model's errors. The logarithmic transformation is used extensively in the literature (i.e Degiannakis and Livada and Panas, 2008; Degiannakis, Filis and Hassani, 2015; Degiannakis and Filis, 2018) as it is a monotone transformation and therefore a change in log(x) is approximately the same (percentage) change in x.[5] Furthermore, by using the natural logarithm the variables in question follow a distribution that is more approximately normal. We estimated the parameters using the OLS method.

The models used are the following:

$$log(VIX_t) = w_0^{(t)} + w_1^{(t)} log(VIX_{t-1}) + w_2^{(t)} \left(5^{-1} \sum_{k=1}^{5} log(VIX_{t-k})\right) + w_3^{(t)} \left(22^{-1} \sum_{k=1}^{22} log(VIX_{t-k})\right) + \varepsilon_t \tag{4}$$

$$log(VXmon_t) = w_0^{(t)} + w_1^{(t)} log(VXmon_{t-1}) + w_2^{(t)} \left(5^{-1} \sum_{k=1}^{5} log(VXmon_{t-k})\right) + w_3^{(t)} \left(22^{-1} \sum_{k=1}^{22} log(VXmon_{t-k})\right) + \varepsilon_t \tag{5}$$

$$log(VXvol_t) = w_0^{(t)} + w_1^{(t)} log(VXvol_{t-1}) + w_2^{(t)} \left(5^{-1} \sum_{k=1}^{5} log(VXvol_{t-k})\right) + w_3^{(t)} \left(22^{-1} \sum_{k=1}^{22} log(VXvol_{t-k})\right) + \varepsilon_t \tag{6}$$

[5] log(x) refers to the natural algorithm with a base of e≈2.71828

$$log(VXinv_t) = w_0^{(t)} + w_1^{(t)} log(VXinv_{t-1}) + w_2^{(t)} \left(5^{-1} \sum_{k=1}^{5} log(VXinv_{t-k})\right) + w_3^{(t)} \left(22^{-1} \sum_{k=1}^{22} log(VXinv_{t-k})\right) + \varepsilon_t \tag{7}$$

where $w_0^{(t)}, w_1^{(t)}, w_2^{(t)}$ and $w_3^{(t)}$ denote the rolling parameters to be estimated and $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$.

Forecasting Process

After estimating the parameters we conduct our forecasts. The initial sample period is 500 days. We use the remaining 968 days as our out-of-sample period. The HAR models used for 1-day-ahead forecasts are as follows:

$$VIX_{t+1|t} = \exp\left(\hat{w}_0^{(t)} + \hat{w}_1^{(t)} log(VIX_t) + \hat{w}_2^{(t)} \left(5^{-1} \sum_{k=1}^{5} log(VIX_{t-k+1})\right) + \hat{w}_3^{(t)} \left(22^{-1} \sum_{k=1}^{22} log(VIX_{t-k+1})\right) + 1/2\hat{\sigma}_\varepsilon^2\right) \tag{8}$$

$$VXmon_{t+1|t} = \exp\left(\hat{w}_0^{(t)} + \hat{w}_1^{(t)} log(VXmon_t) + \hat{w}_2^{(t)} \left(5^{-1} \sum_{k=1}^{5} log(VXmon_{t-k+1})\right) + \hat{w}_3^{(t)} \left(22^{-1} \sum_{k=1}^{22} log(VXmon_{t-k+1})\right) + 1/2\hat{\sigma}_\varepsilon^2\right) \tag{9}$$

$$VXvol_{t+1|t} = \exp\left(\hat{w}_0^{(t)} + \hat{w}_1^{(t)} log(VXvol_t) + \hat{w}_2^{(t)} \left(5^{-1} \sum_{k=1}^{5} log(VXvol_{t-k+1})\right) + \hat{w}_3^{(t)} \left(22^{-1} \sum_{k=1}^{22} log(VXvol_{t-k+1})\right) + 1/2\hat{\sigma}_\varepsilon^2\right) \tag{10}$$

$$VXinv_{t+1|t} = \exp\left(\hat{w}_0^{(t)} + \hat{w}_1^{(t)} log(VX_{invt}) + \hat{w}_2^{(t)}\left(5^{-1}\sum_{k=1}^{5} log(VXinv_{t-k+1})\right) + \hat{w}_3^{(t)}\left(22^{-1}\sum_{k=1}^{22} log(VXinv_{t-k+1})\right) + 1/2\hat{\sigma}_\varepsilon^2\right) \quad (11)$$

For multiple days ahead($s \geq 1$) the models used are as follows:

$$VIX_{t+s|t} = \exp\left(\widehat{w}_0^{(t)} + \widehat{w}_1^{(t)} \log(VIX_{t+s-1|t}) + \widehat{w}_2 \left(s^{-1}\sum_{k=1}^{s-1} \log(VIX_{(t-k+s)|t}) + (5-s)^{-1}\sum_{k=s}^{5} \log(VIX_{(t-k+s)})\right) + \widehat{w}_3\left(s^{-1}\sum_{k=1}^{s-1} \log(VIX_{t-k+s|t}) + (22-s)^{-1}\sum_{k=s}^{22} \log(VIX_{t-k+s})\right) + 1/2\hat{\sigma}_\varepsilon^2\right) \quad (12)$$

$$VXmon_{t+s|t} = \exp\left(\widehat{w}_0^{(t)} + \widehat{w}_1^{(t)} \log(VXmon_{t+s-1|t}) + \widehat{w}_2 \left(s^{-1}\sum_{k=1}^{s-1} \log(VXmon_{(t-k+s)|t}) + (5-s)^{-1}\sum_{k=s}^{5} \log(VXmon_{t-k+s})\right) + \widehat{w}_3\left(s^{-1}\sum_{k=1}^{s-1} \log(VXmon_{t-k+s|t}) + (22-s)^{-1}\sum_{k=s}^{22} \log(VXmon_{t-k+s})\right) + 1/2\hat{\sigma}_\varepsilon^2\right) \quad (13)$$

$$
\begin{aligned}
VXvol_{t+s|t} = \exp\Bigg(& \widehat{w}_0^{(t)} + \widehat{w}_1^{(t)} \log\left(VXvol_{t+s-1|t}\right) \\
& + \widehat{w}_2 \left(s^{-1} \sum_{k=1}^{s-1} \log\left(VXvol_{(t-k+s)|t}\right) \right. \\
& \left. + (5-s)^{-1} \sum_{k=s}^{5} \log\left(VXvol_{(t-k+s)}\right) \right) \\
& + \widehat{w}_3 \left(s^{-1} \sum_{k=1}^{s-1} \log\left(VXvol_{t-k+s|t}\right) \right. \\
& \left. + (22-s)^{-1} \sum_{k=s}^{22} \log(VXvol_{t-k+s}) \right) + 1/2\hat{\sigma}_{\varepsilon}^{2} \Bigg)
\end{aligned} \tag{14}
$$

$$
\begin{aligned}
VXinv_{t+s|t} = \exp\Bigg(& \widehat{w}_0^{(t)} + \widehat{w}_1^{(t)} \log\left(VXinv_{t+s-1|t}\right) \\
& + \widehat{w}_2 \left(s^{-1} \sum_{k=1}^{s-1} \log\left(VXinv_{(t-k+s)|t}\right) \right. \\
& \left. + (5-s)^{-1} \sum_{k=s}^{5} \log\left(VXinv_{(t-k+s)}\right) \right) \\
& + \widehat{w}_3 \left(s^{-1} \sum_{k=1}^{s-1} \log\left(VXinv_{t-k+s|t}\right) \right. \\
& \left. + (22-s)^{-1} \sum_{k=s}^{22} \log(VXinv_{t-k+s}) \right) + 1/2\hat{\sigma}_{\varepsilon}^{2} \Bigg)
\end{aligned} \tag{15}
$$

Computing the returns

We generate the returns used in this study as described below. For each case we trade the product in question based on forecasts of the index and based on forecasts of the product itself. In the case of trading VXinv (based on VIX forecasts), based on one day ahead forecasts, if the VIX price forecast is greater than the VIX closing price then the return is calculated by subtracting the product's previous day closing price from the price it has today. If this is not true the return is calculated by subtracting the product's today price from the price it had one day ago. In the case of trading VXinv (based on the product's forecasts) the same principle is followed. If the product's price forecast is greater than the product's closing price then the return is calculated by subtracting the product's previous day closing price from the price it has today. If this is not true the return is calculated by subtracting the product's today price from the price it had one day ago.

5. Results

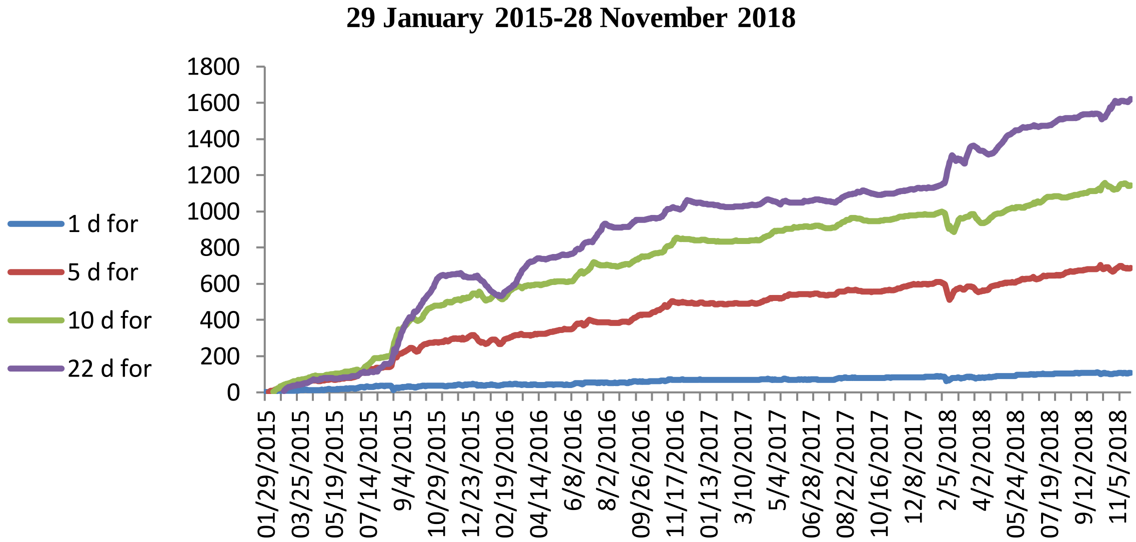
As it has already been mentioned we use three different time-series in order to reach our results. Two of the time-series are made from data provided by the CBOE official webpage while the third is based on data from the Investing.com official webpage. However, two different criteria have been used in order to construct said time-series. In more detail, the first time-series is based on CBOE data and the criterion used is that of the nearest month of expiration. The second time-series is based on data from CBOE as well but the criterion used is that of the highest trading volume. Finally, the third time- series is based on data from Investing.com and on the nearest month of expiration criterion. By running the models and conducting forecasts both on the index and its futures we gain various outputs. More specifically said outputs are: the returns of trading VIX based on VIX forecasts, the returns of trading the VIX futures based on VIX forecasts and the returns of trading the VIX futures based on the forecasts of these futures. It should be noted that the forecasts were conducted over different time horizons, namely over one, five, ten and twenty-two days ahead. In order to see in which case it is more beneficial to invest we compare the abovementioned outputs.

case 1: Trading the VIX

Table 2: Returns from trading the VIX based on VIX forecasts for all time horizons

Days of forecasting	1	5	10	22
Returns in dollars	108	138	114	74
Returns in dollars	108	689	1143	1622

Figure3: Figure of the returns of trading VIX based on VIX forecasts for all time horizons



In this case, we see that the returns are high, higher than those of the other cases. However we cannot use said returns in reality. According to the official CBOE webpage, the VIX index is not directly tradable[6]. According to Degiannakis (2008), in reality we cannot create a position by buying or short-selling the index itself, since the index is a volatility forecast not an asset. Therefore, said returns will have no impact on the rest of the analysis.

[6] The VIX index is not directly tradable, however we can gain exposure to the index by trading futures and options based on a portfolio of SPX options.

case 2: trading VIX futures(CBOE chain) based on the month of expiration criterion

Table 3: Returns from trading VXmon based on VIX forecasts

Days of forecasting	1	5	10	22
Returns in dollars	46	72	64	94
Returns in dollars	46	360	644	2,076

Table 4: Returns from trading VXmon based on VXmon forecasts

Days of forecasting	1	5	10	22
Returns in dollars	69	64	47	41
Returns in dollars	69	318	470	894

In this case we notice that we trade the futures based on the index's forecasts and based on forecasts of said futures. Due to the nature of VIX futures the contract multiplier for each contract is 1000$. That means that each number on the aggregated tables above is expressed in thousands of dollars. For example 46 means 46000$, 72 means 72000$, so on and so forth.

Figure 4: Figure of the returns of trading VXmon based on VIX forecasts for all time horizons

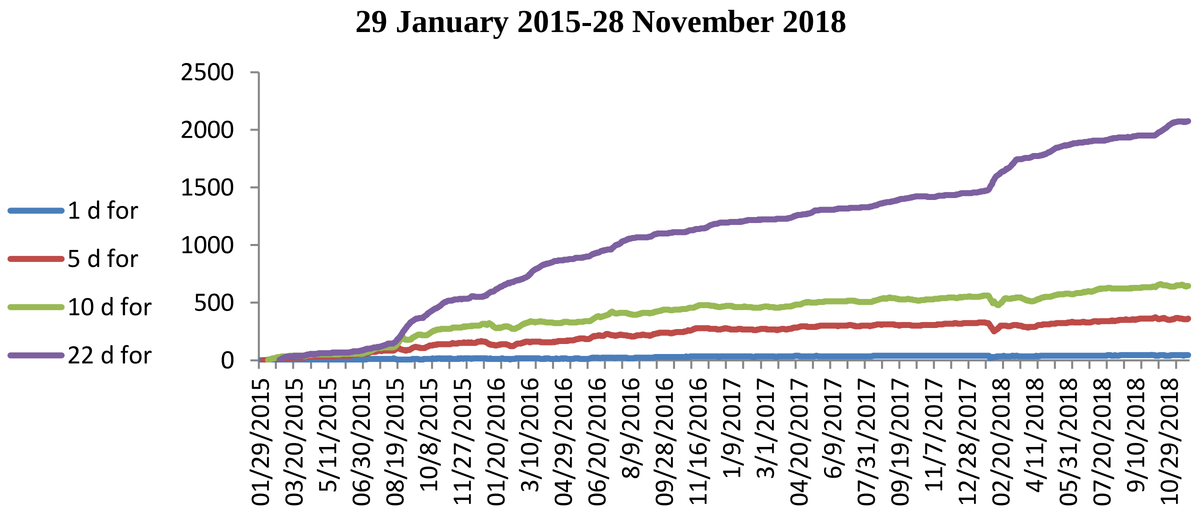
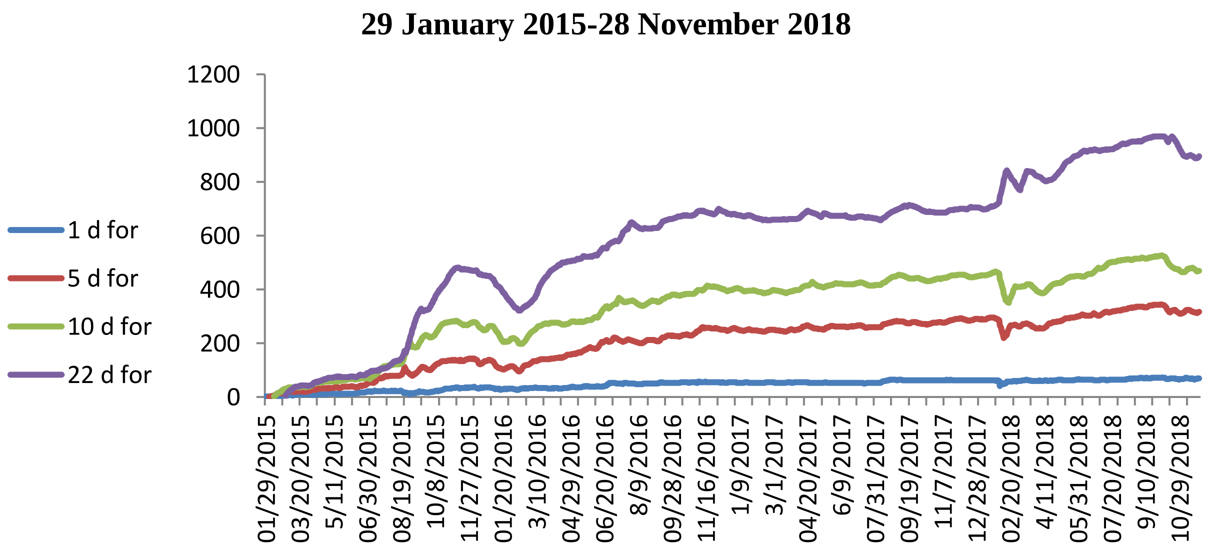


Figure 5: Figure of the returns of trading VXmon based on VXmon forecasts for all time horizons



It should be mentioned that the returns we got as outputs are the numbers on the second row of the aggregated tables. In more detail, 46$ is the return we would get if we traded the product for all the dates in our time-series, that is from 01/29/2015 until 11/28/2018. However, these numbers are not comparable with each other since they are extracted from forecasting on different time horizons. In order to make them comparable we divide the return number we got from trading the product for all the dates used in our time-series by the days of forecasting . For example, 360 divided by 5 equals 72, so on and so forth, thus the first row of the aggregated tables is created.

For the first aggregated table(**table3**) the results are read as follows:

By trading VXmon we could have 46000$ worth of profit based on our forecasts of VIX for one day ahead forecasts.

By trading VXmon we could have 72000$ worth of profit based on our forecasts of VIX for five day ahead forecasts or 360000$ worth of profit for every five days of trading the futures based on our forecasts of VIX for five days ahead.

By trading VXmon we could have 64000$ worth of profit based on our forecasts of VIX for ten day ahead forecasts or 640000$ worth of profit for every ten days of trading the futures based on our forecasts of VIX for ten days ahead.

By trading VXmon we could have 94000$ worth of profit based on our forecasts of VIX for twenty-two day ahead forecasts or 2076000$ worth of profit for every twenty-two days of trading the futures based on our forecasts of VIX for twenty-two days ahead.

For the second aggregated table (**table4**) the results are read as follows:

By trading VXmon we could have 69000$ worth of profit based on VXmon one day ahead forecasts.

By trading VXmon we could have 64000$ worth of profit based on VXmon five day ahead forecasts or 318000$ worth of profit for every five days of trading the futures based on VXmon five day ahead forecasts.

By trading VXmon we could have 47000$ worth of profit based on VXmon ten day ahead forecasts or 470000$ worth of profit for every ten days of trading the futures based on VXmon ten day ahead forecasts.

By trading VXmon we could have 41000$ worth of profit based on VXmon twenty-two day ahead forecasts or 894000$ worth of profit for every twenty-two days of trading the futures based on VXmon twenty-two day ahead forecasts.

case 3: Trading VIX futures(Investing.com chain) based on the month of expiration criterion

Table 5: Returns from trading VXinv based on VIX forecasts

Days of forecasting	1	5	10	22
Returns in dollars	52	71	64	41
Returns in dollars	52	353	645	894

Table 6: Returns from trading VXinv based on VXinv forecasts

Days of forecasting	1	5	10	22
Returns in dollars	72	64	48	43
Returns in dollars	72	318	480	942

Figure 6: Figure of the returns of trading VXinv based on VIX forecasts for all time horizons

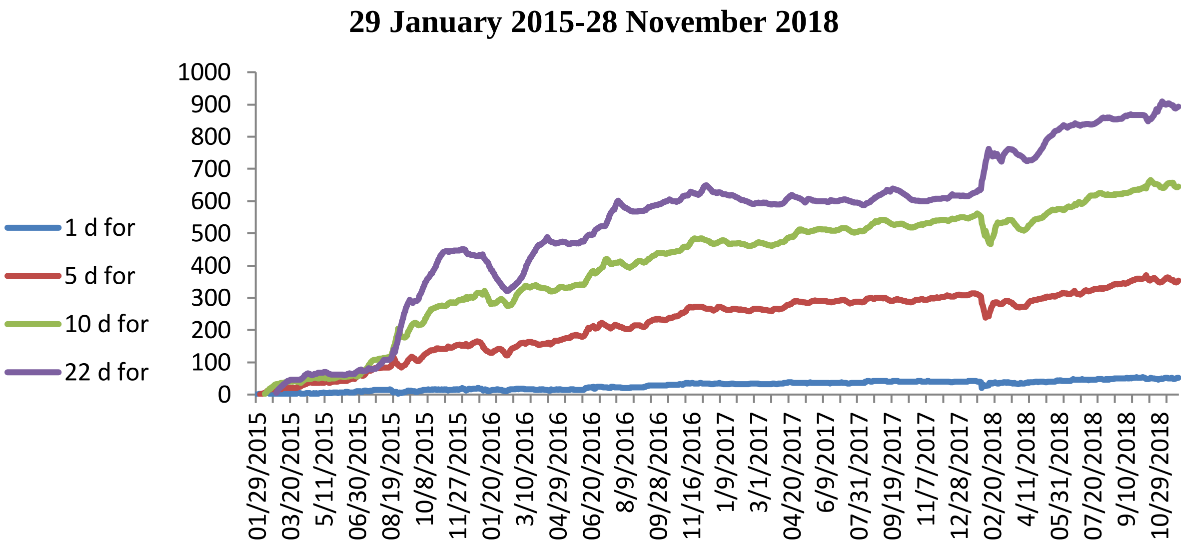
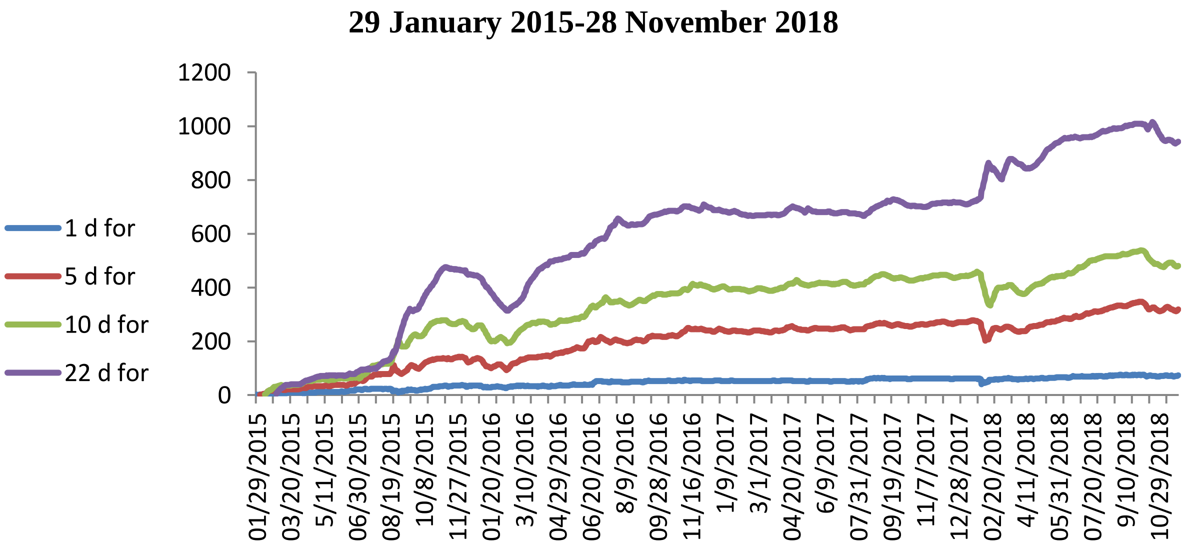


Figure 7: Figure of the returns of trading VXinv based on VXinv forecasts for all time horizons



In this case as well, we notice that we trade the futures based on the index's forecasts and based on forecasts of said futures. As before the values on the aggregated tables above are expressed in thousands of dollars.

For the first aggregated table(**table5**) the results are read as follows:

By trading VXinv we could have 52000$ worth of profit based on our forecasts of VIX for one day ahead forecasts.

By trading VXinv we could have 71000$ worth of profit based on our forecasts of VIX for five day ahead forecasts or 353000$ worth of profit for every five days of trading the futures based on our forecasts of VIX for five days ahead.

By trading VXinv we could have 64000$ worth of profit based on our forecasts of VIX for ten day ahead forecasts or 645000$ worth of profit for every ten days of trading the futures based on our forecasts of VIX for ten days ahead.

By trading VXinv we could have 41000$ worth of profit based on our forecasts of VIX for twenty-two day ahead forecasts or 894000$ worth of profit for every twenty-two of trading the futures based on our forecasts of VIX for twenty-two days ahead.

For the second aggregated table (**table6**) the results are read as follows:

By trading VXinv we could have 72000$ worth of profit based on VXinv one day ahead forecasts.

By trading VXinv we could have 64000$ worth of profit based on VXinv five day ahead forecasts or 318000$ worth of profit for every five days of trading the futures based on VXinv five day ahead forecasts.

By trading VXinv we could have 48000$ worth of profit based on VXinv ten day ahead forecasts or 480000$ worth of profit for every ten days of trading the futures based on VXinv ten day ahead forecasts.

By trading VXinv we could have 43000$ worth of profit based on VXinv twenty-two day ahead forecasts or 942000$ worth of profit for every twenty-two days of trading the futures based on VXinv twenty-two day ahead forecasts.

case 4: Trading VIX futures (CBOE chain) based on the highest trading volume criterion

Table7: Returns from trading VXvol based on VIX forecasts

Days of forecasting	1	5	10	22
Returns in dollars	39	64	59	42
Returns in dollars	39	319	588	917

Table8: Returns from trading VXvol based on VXvol forecasts

Days of forecasting	1	5	10	22
Returns in dollars	64	56	40	29
Returns in dollars	64	280	401	643

Figure 8: Figure of the returns of trading VXvol based on VIX forecasts for all time horizons

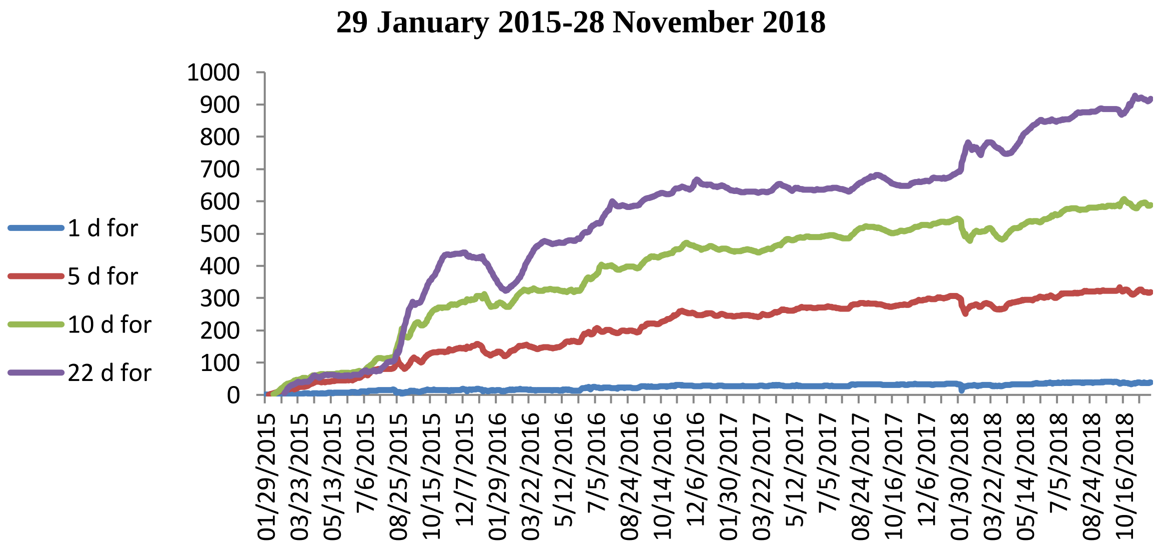
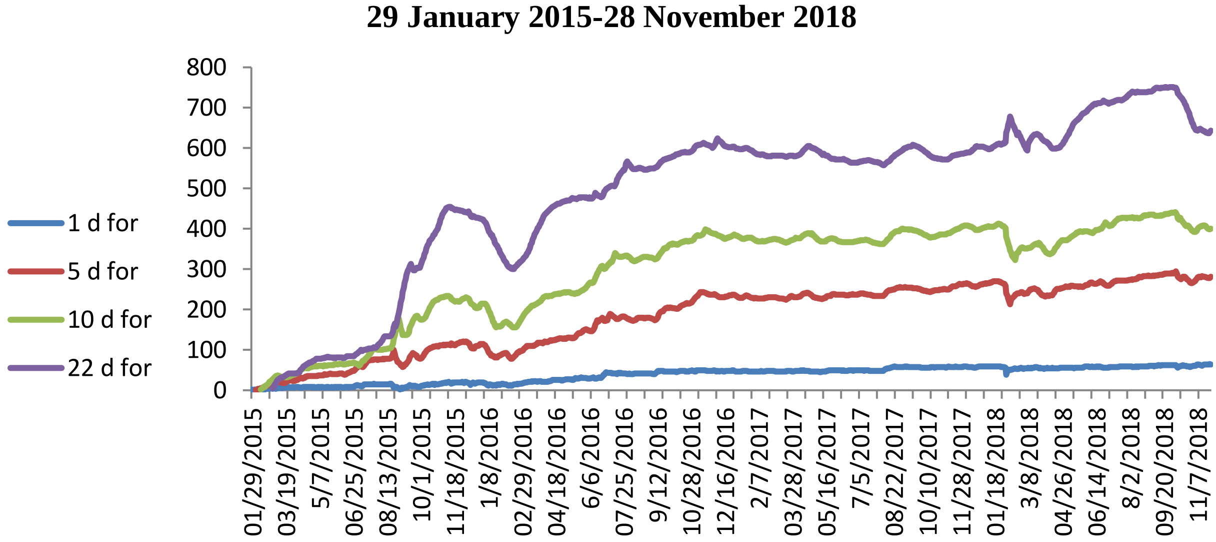


Figure 9: Figure of the returns of trading VXvol based on VXvol forecasts for all time horizons



Again, we notice that we trade the futures based on the index's forecasts and based on forecasts of said futures. As before the values on the aggregated tables above are expressed in thousands of dollars.

<u>For the first aggregated table (**table7**) the results are read as follows:</u>

By trading VXvol we could have 39000$ worth of profit based on our forecasts of VIX for one day ahead forecasts.

By trading VXvol we could have 64000$ worth of profit based on our forecasts of VIX for five day ahead forecasts or 319000$ worth of profit for every five days of trading the futures based on our forecasts of VIX for five days ahead.

By trading VXvol we could have 59000$ worth of profit based on our forecasts of VIX for ten day ahead forecasts or 588000$ worth of profit for every ten days of trading the futures based on our forecasts of VIX for ten days ahead.

By trading VXvol we could have 42000$ worth of profit based on our forecasts of VIX for twenty-two day ahead forecasts or 917000$ worth of profit for every twenty-two days of trading the futures based on our forecasts of VIX for twenty-two days ahead.

For the second aggregated table (**table8**) the results are read as follows:

By trading VXvol we could have 64000$ worth of profit based on VXvol one day ahead forecasts.

By trading VXvol we could have 56000$ worth of profit based on VXvol five day ahead forecasts or 280000$ worth of profit for every five days of trading the futures based on VXvol five day ahead forecasts.

By trading VXvol we could have 40000$ worth of profit based on VXvol ten day ahead forecasts or 401000$ worth of profit for every ten days of trading the futures based on VXvol ten day ahead forecasts.

By trading VXvol we could have 29000$ worth of profit based on VXvol twenty-two day ahead forecasts or 643000$ worth of profit for every twenty-two days of trading the futures based on VXvol twenty-two day ahead forecasts.

At this point it is important to mention that only returns that are viable and therefore worth investing on are the ones that are derived from the one day ahead forecasts. In the case of one day ahead forecasts the trader needs to maintain their position open for only a day and therefore pay for it only once where else they must pay for it multiple times. In this paper the longest- term time horizon is twenty-two days ahead. This practically means that the trader would have to pay twenty-two monetary units if they were to trade based on twenty-two day ahead forecasts in order to maintain their position but only one-twentieth of said monetary units if they were to trade based on one day ahead forecasts.

Therefore, we construct an aggregated table depicting the returns of the three cases that are of economic value (cases 2,3,4).

Table 9: Returns of economic value

	Case 2	Case 3	Case 4
Returns based on VIX forecasts	46	52	39
Returns based on forecasts of the respective futures	69	72	64

From the table above we see that the returns for each case, are higher when we trade the futures based on forecasts of the respective futures than when we trade the futures based on forecasts of the index. It is also evident, that the highest return is obtained in case 3.We can also see that returns are relatively close to each other particularly the ones of cases 2 and 3 which were constructed with the same criterion (the nearest month of expiration criterion).

In conclusion, the trader can achieve the highest profit(72000$) if they trade VXinv based on VXinv one day ahead forecasts.

6. Conclusion

This paper studies trading of the implied volatility index of the S&P 500, also known as VIX, and its futures. We use three different time series which are based on two different criteria. We also forecast over different time horizons and compare various outputs in order to take the optimal position. We conclude that the optimal decision in order to maximize profits would be to invest in the VIX futures chain based on Investing.com data and trade them using one day ahead forecasts.

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