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Abstract

This dissertation has been held as part of the MSc. Program of Applied Economics and Management, at the Department of Economic and Regional Development of Panteion University in Athens. The MSc thesis took place from July of 2015 to January of 2016. The thesis is dealing with the forecast of Value-at-Risk measure, based on a dataset consisting of Stock indices ($S\&P_{500}$, $EurostoXX_{50}$ and $FTSE_{100}$), Commodities (Copper, Silver and Gold COMEX) and Foreign Exchange Rates of Dollar (Euro, Canadian Dollar and British Pound FOREX). The forecasts of this empirical analysis have been done not only at one-day-ahead, as usual, but also at multi-steps-ahead for 95% and 99% confidence level, modeling both inter-day and intra-day data.

The economic uncertainty and volatility of today's business environment highlight the importance of incorporating risk assessment tools into forecasting processes. Consequently, Value-at-Risk is a field of financial econometrics, which fulfills the investors' prerequisites, henceforth it has been studied thoroughly. One main reason of this extensive research is the recent financial crisis, which intrigues the interest of risk managers and financial institutions. In order to provide more reliable Value-at-Risk (VaR) and Expected Shortfall (ES) forecasts, they attempt to investigate which models provide accurate and efficient predictions. Although there is a plethora of forecasting models and applications of forecasting volatility in the literature, this thesis has introduced a new adaptation at VaR estimate, as it presents the performance of inter-day volatility by estimating the AR(1)-GARCH(1,1)-skT model and intra-day volatility by estimating the AR(1)-HAR-RV-skT model. Moreover, this dissertation was held based on the recommendations of the Basel Committee of Banking Supervision.

Regarding for the results, the AR(1)-HAR-RV-skT model in an attempt to forecast volatility does not appear to improve the accuracy of the VaR forecasts for the 10-step-ahead and 20-step-ahead, both for 95% and 99% significance levels. Furthermore, the HAR model is not as much appropriate as expected to be for each of the different asset classes; Stocks, COMEX and FOREX. On the contrary regarding the one-step-ahead forecasts, the HAR specification overcomes the GARCH. In all the other cases, the GARCH specification is the superior model for forecasting the VaR measure.

Keywords: Value-at-Risk, VaR, Expected Shortfall, ES, volatility forecasting, inter-day data, intra-day data, multi-period-ahead, GARCH, HAR-RV, stocks, commodities, exchange rates, forecasting accuracy.

Περίληψη

Η διπλωματική αυτή πραγματοποιήθηκε ως μέρος του Μεταπτυχιακού Προγράμματος Εφαρμοσμένα Οικονομικά και Διοίκηση, του τμήματος Οικονομικής και Περιφερειακής Ανάπτυξης, Παντείου Πανεπιστημίου Αθήνας. Εκπονήθηκε κατά τη χρονική περίοδο Ιούλιος 2015 - Ιανουάριος 2016. Το θέμα της πλαισιώνεται γύρω από την πρόβλεψη και εκτίμηση του Κινδύνου (Value-at-Risk, VaR) χαρτοφυλακίων και συγκεκριμένα μετοχών εισηγμένων στο χρηματιστήριο ($S\&P_{500}$, $EurostoXX_{50}$, $FTSE_{100}$), εμπορεύσιμων υλικών (χαλκού, αργύρου και χρυσού), καθώς και συναλλαγματικών ισοτιμιών (Ευρώ, Καναδικού Δολαρίου και Βρετανικής Λίρας). Οι προβλέψεις της εμπειρικής ανάλυσης δεν έχουν πραγματοποιηθεί μόνο για μία μέρα μπροστά, ως είναι το σύνηθες σε άλλες τέτοιες έρευνες, αλλά περιλαμβάνει μία πολύ-περιοδική πρόβλεψη αρκετές ημέρες μπροστά. Συγκεκριμένα περιλαμβάνει επιπλέον 10 ημέρες μπροστά πρόβλεψη του κινδύνου (διάστημα μίας εβδομάδας σε χρηματιστηριακές βάσεις) και 20 ημέρες μπροστά (διάστημα ενός μήνα σε χρηματιστηριακές βάσεις). Το διάστημα εμπιστοσύνης που εξετάζεται στη παρούσα διπλωματική είναι το 95% και 99%, λαμβάνοντας δεδομένα σε ημερήσια (inter-day) και σε ενδοημερήσια (intra-day) βάση.

Η οικονομική αβεβαιότητα και μεταβλητότητα του οικονομικού περιβάλλοντος υπογραμμίζει τη σπουδαιότητα της ενσωμάτωσης νέων εργαλείων εκτίμησης και αξιολόγησης του κινδύνου. Κατά συνέπεια, η μοντελοποίηση του Value-at-Risk είναι ένας τομέας της οικονομετρίας, ο οποίος πληροί τις προϋποθέσεις των επενδυτών, και για το λόγο αυτό έχει μελετηθεί διεξοδικά. Ένας κύριος λόγος της εκτεταμένης αυτής μελέτης είναι η πρόσφατη οικονομική κρίση, η οποία πυροδότησε το ενδιαφέρον πολλών αναλυτών, καθώς και πολλών επιχειρήσεων ούτως ώστε να προβλέψουν με μεγαλύτερη ακρίβεια τον Κίνδυνο (VaR) που αναλογεί σε κάθε χαρτοφυλάκιο, αλλά και το ακριβές ποσό της αναμενόμενης απώλειας (ES, Expected Shortfall) του χαρτοφυλακίου, στη προσπάθεια τους να εντοπίσουν το μοντέλο εκείνο που αποδίδει μεγαλύτερη ακρίβεια και αποδοτικότητα. Παρά το γεγονός ότι υπάρχει πληθώρα προβλεπτικών μοντέλων, παραδειγμάτων και εφαρμογών που χρησιμοποιούνται με σκοπό να προβλέψουν την μεταβλητότητα σύμφωνα με την υπάρχουσα βιβλιογραφία, η παρούσα διπλωματική εισαγάγει μία νέα προσαρμογή εκτίμησης του Κινδύνου VaR, δεδομένου ότι παρουσιάζεται η μεταβλητότητα των ημερήσιων δεδομένων του μοντέλου AR(1)-GARCH(1,1)-skT και η μεταβλητότητα των ενδοημερήσιων δεδομένων του μοντέλου AR(1)-HAR-RV-skT. Επιπλέον, η παρούσα διατριβή πραγματοποιήθηκε με βάση τις συστάσεις της Επιτροπής της Βασιλείας για την Τραπεζική Εποπτεία.

Αναφορικά με τα αποτελέσματα της ανάλυσης, το μοντέλο AR(1)-HAR-RV-skT, στη προσπάθεια του να κάνει πρόβλεψη της μεταβλητότητας, δεν φαίνεται να βελτιώνει την ακρίβεια των προβλέψεων του VaR για τις προβλέψεις σε 10-ημέρες μπροστά και σε 20-ημέρες μπροστά, στο 95% και 99% διάστημα εμπιστοσύνης. Επίσης, το HAR μοντέλο αποδεικνύεται ότι δεν είναι τόσο κατάλληλο τελικά, όσο αναμένονταν να είναι, για κάθε μία από τις κατηγορίες, μετοχών, εμπορευμάτων και συναλλαγματικών ισοτιμιών. Αντίθετα, το HAR είναι καλύτερο μοντέλο έναντι του GARCH, για την πρόβλεψη μόνο κατά μία ημέρα μπροστά. Σε όλες τις άλλες περιπτώσεις το GARCH είναι ανώτερο του HAR.

Λέξεις-κλειδιά: Value-at-Risk, VaR, ES, πρόβλεψης μεταβλητότητας, ημερήσια δεδομένα, ενδοημερήσια δεδομένα, πολύ-περιοδική πρόβλεψη, GARCH, HAR-RV, μετοχές, εμπορεύματα, συναλλαγματικές ισοτιμίες, ακρίβεια προβλεπτικής ικανότητας.

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Symbols and Operators

- **t :** time index.
- $\mathbf{y_t}$ **:** the continuously compounded return series.
  Let $\{\boldsymbol{y_t}\}_{t=0}^{T} = \{\mathbf{log}(\boldsymbol{p_t}/\boldsymbol{p_{t-1}})\}_{t=1}^{T}$, where p_t is the closing price of the trading day t.
- $\boldsymbol{\mu_t}$ **:** denotes the conditional mean, $\mu_t = c_0(1-c_1) + c_1 y_{t-1}$.
- $\boldsymbol{\varepsilon_t}$ **:** the innovation process.
- $\mathbf{z_t}$ **:** the standardized residuals, vector process with zero mean, unit variance and zero covariance.
- $\boldsymbol{F(z_t;\nu,0,I)}$ **:** Multivariate density function of $\boldsymbol{z_t}$ and ν is the vector of parameters of density function $F(.;.)$.
- $\mathbf{I_{t-1\backslash t}}$ **:** the information set available at t=1.
- $\boldsymbol{\sigma^2}$ **:** the Unconditional Variance, $V(\varepsilon_t) = \sigma^2$.
- $\boldsymbol{\sigma_{i,j,t}}$ **:** the dynamic covariance between $c_{i,t}$ and $c_{j,t}$.
- $\boldsymbol{\sigma_t}^2$ **:** the Conditional Variance, $V(\varepsilon_t|I_{t-1}) \equiv \sigma_t^2(\theta)$.
- ***f*(.) :** the density function of $\{z_t\}_{t=0}^{T}$, $g(.)$, linear or non-linear.
- $\boldsymbol{\theta}$ **:** the vector of the unknown parameters.
- $\boldsymbol{F(a;\theta^t)}$ **:** the α^{th} quantile loss of the assumed distribution, given the estimated parameters θ at time t.
- $\boldsymbol{\mu_{t+1|t}}$ ***and*** $\boldsymbol{\sigma_{t+1|t}}$ **:** the one-step-ahead conditional forecasts of the mean and for the standard deviation.
- $\boldsymbol{VaR_{t+1|t}^{1-p}}$ **:** the VaR number of the next trading day (one-step-ahead VaR), given the information set at day t.
- $\boldsymbol{ES_{t+1|t}^{1-p}}$ **:** the ES number of the next trading day (one-step-ahead ES), given the information set at day t.
- $\boldsymbol{VaR_{t+\tau|t}^{1-p}}$ **:** the VaR number of long trading positions, for the next τ trading days (multi-period-ahead forecasts).
- $\boldsymbol{ES_{t+\tau|t}^{(1-p)}}$ **:** the ES number of long trading positions, for the next τ trading days (multi-period-ahead forecasts).
- $\boldsymbol{N\sim B(\tilde{T},p)}$ **:** a binomial distribution, with $\tilde{T}$ the out-of-sample observations.
- $\boldsymbol{\tilde{T}}$ **:** the out-of-sample observations.
- **T :** rolling sample of 1000 observations.
- $\boldsymbol{\hat{T}}$ **:** the total number of the log returns.
- $\boldsymbol{\psi_{t+1}}$ **:** the Loss Function of Lopez (equation 23, p.13, dissertation).
- $\boldsymbol{h_{t+1}^2}$ **:** the realized volatility[1] used as the measure of the true, but unobservable variance at the day t+1. (used in MSE at equation 26, p.14, dissertation).
- $\boldsymbol{X_{t,l}^{(i,i^*)}}$ **:** the DM-statistic, Diebold & Mariano ($X_{t,l}^{(i,i^*)} = L_{t,l}^{(i)} - L_{t,l}^{(i^*)}$).
- $\boldsymbol{i}$ **:** the benchmark model of DM-statistic.
- $\boldsymbol{i^*}$ **:** the competitive model of the DM-statistic.
- $\boldsymbol{X_{t,l}^{(i,i^*)}}$ **:** the SPA test, ($X_{t,l}^{(i,i^*)} = L_{t,l}^{(i)} - L_{t,l}^{(i^*)}$).

[1] The Realized Volatility is computed by the following equation:
$h_t^2 = \frac{\hat{\sigma}_{OC}^2+\hat{\sigma}_{CO}^2}{\hat{\sigma}_{OC}^2}\sum_{j=1}^{m-1}\left(100\left(\log\left(P_{(j+1/m),t}\right)-\log\left(P_{(j/m,t)}\right)\right)\right)^2.$

- $\boldsymbol{i}$ **:** the benchmark model of SPA test.
- $\boldsymbol{i}^*$ **:** the competitive model**s** of the SPA test.
- $\boldsymbol{\sigma_t^{2(IV)}}$ **:** the Integrated Volatility – IV, a variable which is not observable (equation 34, p.17, dissertation).
- $\boldsymbol{\sigma_t^{2(RV)}}$ **:** the Realized Volatility (RV), which is defined as the sum of squared returns observed over very small time intervals (equation 35, p.17, dissertation).
- $\boldsymbol{\sigma_t^{2(RV)}}$ **:** the volatility of inter-day and intra-day trading strategies of the Heterogeneous Autoregressive Realized Volatility, HAR-RV.
- $\left(\boldsymbol{\sigma^{(RV)}}\right)_{\boldsymbol{t-5:t-1}}$ **:** the volatility of the HAR-RV model for the medium term trading during the period of one week.
- $\left(\boldsymbol{\sigma^{(RV)}}\right)_{\boldsymbol{t-22:t-1}}$ **:** the volatility of the HAR-RV model enclose investment strategies during the period of one month or even longer time horizons.
- $\mathbf{w_0, w_1, w_2, w_3}$ **:** the coefficients on the intra-day squared returns of HAR-RV model, during the previous day.
- $\boldsymbol{u_t \sim i.i.d.N(0,1)}$ **:** the residuals of the HAR-RV model.
- $\boldsymbol{z_t \sim skT\,(0,1;v,g)}$ **:** the standardized residuals of the HAR-RV model, with skewed student-t distribution, zero mean and standardized volatility over the parameters; ν vector explains kurtosis and g vector explains asymmetry.
- $\boldsymbol{log\sqrt{252\sigma_t^{(RV)}}}$ **:** the annualized realized volatility of the HAR-RV model.
- $\boldsymbol{\left(log\sqrt{252\sigma_t^{(RV)}}\right)^2/252}$ **:** the daily log-returns of the Realized Volatility, as a dependent variable by the HAR-RV model.
- $\boldsymbol{\{\breve{z}_{i,1}\}_{i=1}^{MC}}$**:** random numbers from the skewed Student-t distribution.
- **MC:** denotes the number of draws.

Introduction

Value-at-risk (VaR) and expected shortfall (ES) have become two popular measures of market risk associated with an asset or portfolio of assets, during the last decade. In particular, the VaR has been chosen by the Basel Committee on Banking Supervision as the benchmark of risk measurement for capital requirements. Both the VaR and the ES have been used by financial institutions as asset and for minimizing risk, and have been rapidly developed as analytic tools to assess riskiness of trading activities.

One of the most important issues in finance is the choice of one benchmark volatility model to forecast the risk that an investor faces. After Engle R. F. (1982) seminal paper, many other researchers have tried to find the most appropriate risk model that predicts future variability of asset returns by employing various specifications, based on ARCH specifications. Hence, their results are confusing and conflicting, because there is no model that is deemed as adequate for all financial datasets, distributions, sample frequencies and applications. A good starting point to judge competitive models is the out-of-sample forecasting performance. On the one hand, many researchers have tried to find the best performing method for different financial markets and time horizons by using versions of the ARCH model, but there is not a clear agreement in the literature on the most adequate volatility specification. On the other hand, the availability of high frequency datasets rekindled the interest of academics to forecast risk.

Most of the studies have considered volatility as an unobservable variable and therefore used a fully specified conditional mean and conditional variance model to estimate and analyze that latent volatility. Modeling the unobserved conditional variance was one of the most prolific topics in the financial literature which led to many ARCH-GARCH developments and stochastic volatility models. An alternative approach is to construct an observable proxy for the latent volatility by using intra-day high frequency data. At this thesis, the intra-day data are modeled with the application of AR(1)-HAR-RV and the inter-day data represented with the AR(1)-GARCH(1,1), followed by the skewed Student-t distributional assumption both of them.

It is well-known that most of the empirical works are based on daily returns. Despite the majority of the studies in the literature, some of the most quintessential are; Giot & Laurent (2003) who proposed the asymmetric power of ARCH with skewed Student-t distributed innovations, APARCH-skT model, while Degiannakis (2004) suggested the fractionally integrated APARCH (FIAPARCH) model and stated that the FIAPARCH with skewed Student-t distributed innovations produces the most accurate VaR predictions among three stock indices (CAC_{40}, DAX_{30} and $FTSE_{100}$). Additionally, other researchers, such as Angelidis, Benos & Degiannakis (2004); *et.al.*, propose different volatility structures to estimate the daily VaR, but yet again without reaching a consensus and a common conclusion. They argued that the choice of the best performing model depends on the equity index.

However, by using high frequency data, researchers explore ways to extract more information that maybe it will enable them to forecast VaR accurately. To be more precise, Giot & Laurent (2004) compared the APARCH-skT model with an ARFIMAX specification, in their attempt to capture VaR for stock indices and exchange rates, as well. They conclude that the use of intra-day dataset did not improve the performance of the inter-day VaR model, fact that it is analyzed in more details at this dissertation, exploring not only stocks, as usual, but also with a dataset of Stocks, Commodities and Exchange Rates, respectively. Another important study that strengthens the results of this thesis according to the literature, has to do with Giot P. (2005) who estimated VaR at intra-day time horizons of 15 and 30 minutes and proposed that the GARCH model with skewed Student-t distributed innovations had the best overall performance, and that there were no significant differences between daily and intra-day VaR models once the intra-day seasonality in the volatility was taken into account.

All the above findings presented in the previous paragraph enhance the outcomes of this present dissertation. In the next chapters, we will try to answer the question if there is an adequate intra-day model for volatility forecasting in a variety of assets, not only for stocks, that gives accurate estimation. The innovative process of this paper concerns the time horizon of the forecasts. I have chosen to forecast AR(1)-GARCH(1,1)-skT and AR(1)-HAR-RV-skT models; the former represented the inter-day dataset and the latter represented the intra-day dataset, for one-step-ahead, 10-step-ahead and 20-step-ahead VaR, at 95% and 99% of confidence level. The most other studies in the literature have already applied empirical examples for one-step-ahead and ten-step-ahead. As a consequence, the long memory volatility of 20-days-ahead has not investigated in huge extent, until now with the outcomes of this research.

To summarize, although there are indications that the extended models produce the most accurate VaR forecasts, in some cases, a simpler model is preferable and especially, as the time horizon increases. For 10 and 20-day-ahead VaR forecasts, GARCH model is superior, instead of the one-day-ahead forecasts that AR(1)-HAR-RV-skT seems to be preferable, but with little differences from the other model. It was also found that the use of the intra-day datasets does not add to the forecasting power of the models.

The structure of this dissertation is as follows: Chapter 1 describes the literature reviews with examples and applications with a variety of different models, while Chapter 2 presents the scope of the research. Chapter 3 present at the first part data description, secondly describes the methodology of the two models (GARCH and HAR), according to Monte Carlo Simulation of the multi-step-ahead VaR and ES forecasts, and finally presents the empirical analysis and the results from these two models, after the backtesting procedure of Kupiec and Christoffersen tests. Chapter 4 concludes the dissertation and provides the final outcomes of this research.

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1.1) ARCH Volatility Models

This chapter encompasses issues, concerning the main literature of the theory of Risk Management and forecasting, by using ARCH volatility specifications. In econometrics, autoregressive conditional heteroskedasticity (ARCH) models are used to characterize and model observed time series. The primary purpose is to display a conceptual framework of the most attractive models commonly used nowadays in many financial applications. A wide variety of proposed ARCH specifications are observed in some of the following surveys; Engle R. F. (1982); Bollerslev (1986); Nelson (1991); Bollerslev & Mikkelsen (1996); Degiannakis (2004); *et.al*.

As an introduction of the ARCH volatility Models that will be followed, it would be necessary to be presented the notation of financial time series. Let $\{y_t\}_{t=0}^{T} = \{\log(p_t/p_{t-1})\}_{t=1}^{T}$ refer to the continuously compounded return series, where p_t is the closing price of the trading day t. The return series follows the stochastic process:

$$\begin{aligned} y_t &= \mu_t + \varepsilon_t \\ \mu_t &= c_0(1-c_1) + c_1 y_{t-1} \\ \varepsilon_t &= z_t \sigma_t \\ z_t &\sim iid\, f[E(z_t) = 0, V(z_t) = 1; \theta] \\ \sigma_t^2 &= g(I_{t-1}), \end{aligned} \tag{1}$$

where $E(y_t|I_{t-1}) \equiv \mu_t(0)$ denotes the conditional mean, given the information available at t=1, I_{t-1}, $\{\varepsilon_t\}_{t=0}^{T}$ is the innovation process with unconditional variance $V(\varepsilon_t) = \sigma^2$ and conditional variance $V(\varepsilon_t|I_{t-1}) \equiv \sigma_t^2(\theta)$, $f(.)$ is the density function of $\{z_t\}_{t=0}^{T}$, $g(.)$ in any of the functional forms presented in the model and θ is the vector of the unknown parameters.

Engle (1982) introduced the Autoregressive Conditional Heteroskedasticity Model, ARCH (q) and expressed the conditional variance as a linear function of the past q squared innovations.

$$\sigma_t^2 = a_0 + \sum_{i=1}^{q} a_i \varepsilon_{t-i}^2 \tag{2}$$

The parameters should satisfy the following prerequisites:

$$a_0 > 0 \; and \; a_i \geq 0 \; for \; i = 1, \ldots, q.$$

The reason that Engle was led to the innovation of ARCH Model was his attempt to investigate a model that had the inflation unpredictability as a first priority. He argued that the level of inflation was not a drawback, but the uncertainty about future cost and prices was. The uncertainty can be measured if it was changing over time, what econometricians called Heteroskedasticity; Engle R. F. (2003). Engle earned at 2002 a Nobel Prize for his innovation, shared with Clive Granger who had developed a test for bilinear time series models.

Tim Bollerslev (1986) proposed a generalization of ARCH Model, called Generalized ARCH or GARCH (p,q). The purpose of this new model was to generalize the simple Autoregressive Heteroskedasticity Model to an Autoregressive Moving Average Model. The GARCH(p,q) forecasted variance is a weighted average of three different variance forecasts; firstly, the constant variance which corresponds to the long run average, secondly, the forecast that was made in previous period and finally, the information set that was not available when the previous forecast was made.

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} b_j \sigma_{t-j}^2, \quad (3)$$

where $\alpha_0 > 0$ $for\ i = 1, \dots, q\ and$ $b_j \geq 0$ for $j = 1, \dots, p$.

If $\sum_{i=1}^{q} \alpha_i + \sum_{j=1}^{p} b_j < 1$ then the unconditional variance is equal to:

$$\sigma^2 = \frac{\alpha_0}{(1 - \sum_{i=1}^{q} a_i - \sum_{j=1}^{p} b_j)} .$$

The GARCH(p,q) model has been already used in many econometric analyses in order to forecast the risk, generating accurate forecasts. Hansen and Lunde (2005) proposed that there is none else model provide better volatility forecasts than the GARCH(1,1), comparing among 330 alternative models.

However, the use of GARCH is not always suggested in every occasion. For that reason, Taylor (1986) and Schwert (1989) introduced the Absolute GARCH Model or AGARCH(p,q), in which they argued that the conditional standard deviation is a linear function of its past values, as well as the past absolute innovations.

$$\sigma_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i |\varepsilon_{t-i}| + \sum_{j=1}^{p} b_j \sigma_{t-j} . \quad (4)$$

In this attempt the large shocks should have a smaller effect on the conditional variance of the AGARCH model than that of GARCH, respectively.

An alternative in the family of GARCH Models is IGARCH(p,q). The Integrated GARCH focused on the assumption that $\sum_{i=1}^{q} \alpha_i + \sum_{j=1}^{p} b_j \approx 1$, with the following equation:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^{p} b_j \sigma_{t-j}^2, \quad (5)$$

$$\text{where } \sum_{i=1}^{q} \alpha_i + \sum_{j=1}^{p} b_j = 1.$$

IGARCH models are unit-root GARCH Models. One significant characteristic of IGARCH makes the difference between the simple GARCH, concerns the unconditional variance, which is infinite. As a consequence, the above sentence indicates that the conditional variance remains important for all conditional volatility forecasts. Moreover there is a special form of IGARCH, the Exponentially Weighted Moving Average (EWMA), which is used by Risk Metrics[2]. The volatility forecast is computed as $\sigma_t^2 = \lambda\sigma_{t-1}^2 + (1-\lambda)\varepsilon_{t-1}^2$.The basic RiskMetrics model is equivalent to a normal IGARCH model where the autoregressive parameter is set at a prespecified value λ [(3)] and the coefficient of ε_{t-1}^2 is equal to 1-λ. However, Risk Metrics TM methodology, used in many studies, has been proved that underestimates the total risk.

Another important generalization of GARCH is the Exponential GARCH or EGARCH(p,q), introduced by Dan Nelson at 1991, in order to overcome some weaknesses of the GARCH model. He proposed that volatility could respond asymmetrically to past forecast errors; Nelson (1991). EGARCH models are appropriate when positive and negative shocks of equal magnitude might not contribute equally to volatility. The equation of EGARCH is following:

[2] The Risk Metrics variance model was first established in 1989, when Mr. Dennis Weatherstone, the new chairman of J.P. Morgan, asked for a daily report measuring and explaining the risks of his firm. Nearly four years later in 1992, J.P. Morgan launched the Risk Metrics TM methodology to the marketplace.

[3] λ=0,94.

$$\log \sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \left(\alpha_i \left| \frac{\varepsilon_{t-i}}{\sigma_{t-i}} \right| + \gamma_i \frac{\varepsilon_{t-i}}{\sigma_{t-i}} \right) + \sum_{i=1}^{p} b_j \log \sigma_{t-j}^2. \qquad (6)$$

In the equation (5), the logarithmic transformation ensures that the forecasts of the variance are always positive and the parameter $\gamma_i \neq 0$ depicts the asymmetric effect. If $\gamma_i = 0$ then a positive surprise $(\varepsilon_t > 0)$will have the same effect on volatility as a negative surprise$(\varepsilon_t < 0)$. This is well known as the leverage effect. While when $\gamma_i < 0$, it means that positive shocks generate less volatility than the negative ones.

The Threshold GARCH or TARCH(p,d,q) is one another model out of the most widely used. This specification allows a response of volatility to news with different coefficients for good and bad news.

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \gamma_1\, \varepsilon_{t-1}^2 d_{t-1} + \sum_{i=1}^{p} b_i\, \sigma_{t-i}^2\,. \qquad (7)$$

In this case, the dummy variable $d_t = 1$ if $\varepsilon_t < 0$ and $d_t = 0$ if $\varepsilon_t > 0$.

For the AGARCH specification or Asymmetric GARCH, a negative value of γ_i means that positive returns increase volatility less than negative returns.

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} (\alpha_i \varepsilon_{t-i}^2 + \gamma_1 \varepsilon_{t-1}) + \sum_{i=1}^{p} b_i \sigma_{t-i}^2\,. \qquad (8)$$

The Asymmetric Power ARCH or APARCH (p,q) model comprises most of the presented models. It was introduced by Ding, *et al.* (1993), without assuming that conditional variance should be a linear function of the lagged squared returns.

$$\sigma_t^\delta = \alpha_0 + \sum_{i=1}^{q} \alpha_i\, (|\varepsilon_{t-i}| - \gamma_i \varepsilon_{t-i})^\delta + \sum_{i=1}^{p} b_i \sigma_{t-i}^\delta\,, \qquad (9)$$

$$\text{where } \alpha_0 > 0,\ \alpha_i \geq 0,\ |\gamma_i| < 1,\ b_i \geq 0 \text{ and } \delta > 0.$$

Because the distribution of returns is often not symmetric, parametric VaR models faced difficulties in modeling correctly the tails of the distribution of returns. As a result, Giot and Laurent (2003) introduced the APARCH model, based on a different distribution than that of Normal[4]. They followed the skewed Student-t Distribution, so as to take into account the fat tails of the returns. This innovation enabled to measure the short and long trading positions more easily rather than that of the previous modeling processes, comparing to another similar empirical analysis; that of Mittnik and Paolella (2000), in which they used APARCH focused on long VaR only. By forecasting four daily stock indices; the French CAC40, the German DAX, the US NASDAQ and the Japanese NIKKEI, Giot and Laurent (2003) brought about considerable improvements at one-day-ahead VaR both for long and short trading positions.

Bailie (1996) tried to model long memory property in volatility, using a new model, which was the extended IGARCH(p,q). This model was called FIGARCH(p,d,q) and its primary purpose was to develop a more flexible class of processes, depending on the conditional variance, which gave the opportunity to explain in a better and more simple way the observed temporal dependencies in financial market volatility. As a consequence, Bailie introduced the FIGARCH (p,d,q) process in 1996, by replacing the first difference operator[5] from the IGARCH model; (1-L), with the differencing operator $(1\text{-L})^d$; Baillie, Bollerslev & Mikkelsen (1996).

$$\Theta(L)(1-L)^d \varepsilon_t^2 = a_0 + \big(1-\beta(L)\big)(\varepsilon_t^2 - \sigma_t^2), \qquad (10)$$

$$\text{which } 0 < d < 1$$

---

[4] See Appendix A': Types of Distributions and the Density Functions of them (p.65).

[5] $\Theta(L)(1-L)\varepsilon_t^2 = \alpha_0 + \big(1-\beta(L)\big)(\varepsilon_t^2 - \sigma_t^2)$

By all accounts, the FIGARCH combines many of the features of fractionally integrated process for the mean, when $d = 1$. Concurrently, FIGARCH has also a lot of similarities with the GARCH process for the conditional variance, when $d = 0$. Finally, it has been proved that FIGARCH model added flexibility when modeling long run volatility characteristics, as well as it seems to be more realistic from economic perspective, dominated by a hyperbolic rate of decay.

In the same year, another innovative model was displayed by Bollerslev and Mikkelsen (1996); that of Fractionally Integrated EGARCH / FIEGARCH (p,d,q). This new model was an extension of the previous EGARCH; Bollerslev, Mikkelsen, *et.al.* (1996).

$$\log(\sigma_t^2) = a_0\big(1 - B(L)\big) + (1 - L)^{-d}\,\big(1 + \Phi(L)\big)\Big(\gamma_1\Big(\Big|\frac{\varepsilon_{t-1}}{\sigma_{t-1}}\Big| - E\varepsilon t{-}1\sigma t{-}1 + \quad \gamma 2\varepsilon t{-}1\sigma t{-}1 + BL \log\sigma t2.$$

(11)

Tse (1998) was the person who developed the Fractionally Integrated Asymmetric Power ARCH (FIAPARCH) model, which allows for long memory and asymmetries in volatility.

$$\sigma_t^\delta = a_0 + \big(1 - B(L) - \big(1 - \Phi(L)\big)(1 - L)^{-d}\big)(|\varepsilon_t| - \gamma\varepsilon_t)^\delta + B(L)\log\big(\sigma_t^\delta\big), \qquad (12)$$

where $-1 < \gamma < 1$ and $\delta > 0$. When $\gamma < 0$, negative shocks give rise to higher volatility than positive ones. The opposite happens when $\gamma > 0$. However the FIAPARCH process reduces the importance of the FIGARCH process when $\gamma = 0 \; and \; d = 2$.

1.2) Value at Risk and Expected Shortfall

Risk Management is a standard prerequisite for all financial institutions nowadays. Numerous methods have been proposed to minimize the forecast error. Value-at-Risk (VaR) is the main risk management tool, used to compute accurately the risk of each financial asset. Particularly, VaR refers to a portfolio's worst outcome that is likely to occur at a given confidence level, over a specified period and is focused on the market risk; Angelidis & Degiannakis (2007). Market risk is defined as the risk that arises from unforeseen movements in market places. There are three methods of calculating VaR; the first category refers to the major representatives of parametric family, which are the Autoregressive Conditional Heteroskedasticity (ARCH) models. The second category, the non-parametric modeling relies on actual prices without assuming any specific distribution and the main representative of this category is the Historical Simulation. The last category is the semi-parametric family that combines the two above frameworks. Filtered Historical Simulation (FHS) and Extreme Value Theory (EVT) are the representative methods of the third category. As far as the appropriate methods of model evaluation, there are mainly two; the evaluation of the statistical properties of VaR forecasts and the construction of a loss function that measures the distance between the predicted VaR and the actual portfolio's outcome. In the first stage, the statistical accuracy of the models is examined. In the second stage, a loss function is applied in order to investigate whether the differences among the models, that pass the first stage, are statistically significant. It is also essential that VaR has been adopted by bank regulators. Specifically, according to the Basle Committee proposal (1995a, 1995b), the VaR methodology can be used by a variety of financial institutions to calculate capital charges with accordance to their financial risk, let alone banks could determine their daily capital charge by following the three below proposals; Angelidis & Degiannakis (2007):

1. The 99% confidence level must be used.
2. The holding period must be set to 10 trading days, in the attempt investors to be able to liquidate their positions due to price changes.
3. Banks could calculate VaR by implementing internal models.

Proposing a historical review of Value-at-Risk, the first approach of inserting the use of VaR into practical examples was made by New York Stock Exchange (NYSE) on 1922, by imposing to the members of the firm to hold capital equal to 10% of their assets. Moreover, many researchers had played an important role on this issue; Leavens (1945) presented the first quantitative example of VaR, Markowitz and Roy (1952) suggested VaR measures individually, which were based on the covariances of risk factors and finally, Baumol (1963) presented a measure focused on standard deviation adjusted to a confidence level, in which the reflection of user's attitude to risk was obvious; Angelidis & Degiannakis (2007). Not to mention of course, a widespread method of calculating VaR; the Risk Metrics system applied on the internet and introduced by JP Morgan in 1994.

In this paragraph will be made an attempt the formulation of VaR to be described by using the following equation:

$$p = Pr\left(y_t \leq VaR_t^{(1-p)}\right) = \int_{-\infty}^{VaR_t^{(1-p)}} \frac{1}{\sqrt{2\pi}} exp\left(-\frac{1}{2}y_t^2\right) dy_t. \tag{13}$$

Where P_t be the observed value of the portfolio at time t and the profit (or loss) (P/L) for period t-1 to t, equals to $y_t = \ln(P_t) - \ln(P_{t-1})$. The next figure depicts accurately this relation, under the assumption that $y_t \sim N(0,1)$ and the probability of the loss will be less than $VaR_t^{(1-p)} = -1{,}645$, as well as the confidence level[6] is 95% (See Figure 1). Having estimated the parameters of the models, the VaR number for the next trading day (calculating the one-step-ahead VaR), given the information set at day t, is computed as:

$$VaR_{t+1|t}^{1-p} = \mu_{t+1|t} + F(a;\theta^t)\sigma_{t+1|t}, \tag{14}$$

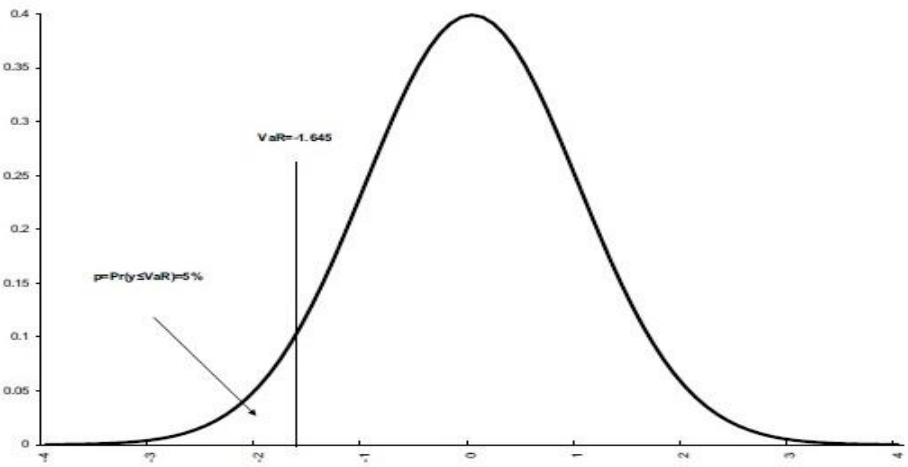
where $F(a;\theta^t)$ is the α^{th} quantile loss of the assumed distribution, given the estimated parameters θ at time t, $\mu_{t+1|t}$ *and* $\sigma_{t+1|t}$ are the conditional forecasts of the mean and for the standard deviation, respectively.

However, there arose a variety of criticisms about the risk management tool; VaR. Value-at-Risk (VaR) has become a standard risk measure for financial risk management due to its conceptual simplicity, ease of computation, and ready applicability. Nevertheless, VaR has been charged as having several conceptual problems. First of all, Taleb (1997) and Hoppe (1998) argued that the underlying statistical assumptions of VaR modeling have been violated, while Beder (1995) claimed that different risk management techniques generate different VaR forecasts and as a consequence, the risk estimations should probably be imprecise. Another criticism concerns the sub-additivity procedure; many econometricians claimed that VaR is not necessarily sub-additive. This leads to the inference that if risks are not sub-additive, the sum of them might underestimate the total risk. To be more precise, the VaR of a portfolio may be greater than the sum of individual VaRs. For that

[6] For example, if confidence level is 95% means that for a capital of 100.000€, VaR equals to 1.645€.

reason, there arose a need of introducing another risk measure in order to remedy these shortcomings; Angelidis & Degiannakis (2007).

Figure 1: For $p = Pr\left(y_t \leq VaR_t^{(1-p)}\right) = 5\%$ and the $VaR_t^{(1-p)} = -1{,}645$ under the assumption that $y_t \sim N(0,1)$.



Source: Angelidis & Degiannakis (2007), Econometric Modeling of Value-at-Risk, Nova Science Publishers, p.5/1-53.

Finally, VaR does not give any indication about the size of the potential loss, given the fact that loss exceeds VaR. For instance, if a VaR violation occurs, a risk manager expects to lose more than the VaR prediction. In other words, VaR gives important information about the potential loss, but does not indicate information about the expected loss. For all these reasons, Artzner, *et.al.* (1997); (1998) and Delbaen (2002) introduced the Expected Shortfall (ES) risk measure.

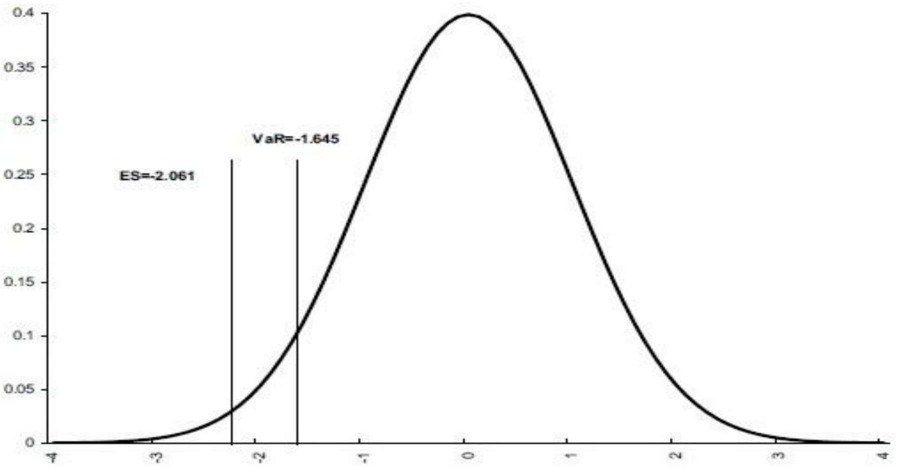
Expected Shortfall (ES) is equal to the expected value of loss, given that a VaR violation is occurred. Another definition depicts Expected shortfall as the conditional expectation of loss that takes into account losses beyond the VaR level (See Figure 2). Expected Shortfall is a reliable measure computing loss, especially during market turmoil, because VaR seems to be unreliable under market stress and VaR may underestimate risk, not to mention the fact that is the most attractive coherent risk measure, which satisfies the following four prerequisites; Artzner, *et.al.* (1997):

1. Sub-additivity,
2. Homogeneity,
3. Monotonicity,
4. Risk-free condition.

The one-step-ahead Expected Shortfall (ES) forecast for long trading positions is the one-day-ahead expected value of the loss, given that the returns t+1 fall below the corresponding value of the VaR forecast. The above sentence is well described at the following equation:

$$ES_{t+1|t}^{(1-p)} = E\left(y_{t+1} \middle| \left(y_{t+1} \leq VaR_{t+1|t}^{(1-p)}\right)\right). \quad (15)$$

Figure 2: $For\ y_t \sim N(0,1)\ and\ p = Pr\left(y_t \leq VaR_t^{(1-p)}\right) = 5\%$, the $VaR_t^{(1-p)}\ and\ the\ ES_t^{(1-p)}$.



Source: Angelidis & Degiannakis (2007), Econometric Modeling of Value-at-Risk, Nova Science Publishers, p.7/1-53.

In order to compare the VaR and ES, a lot of authors have studied about these two risk measures and proposed different opinions; one of them was that of Yamai's and Yoshiba's (2004). Particularly, they implied that VaR is not such a reliable measure during market turmoil. They claimed that there were several conceptual problems with VaR. Among these problems, an important one was that VaR disregards any loss beyond the VaR level, what we call "tail risk". And as a consequence, they showed that the expected shortfall requires a larger sample size than VaR to provide the same accurate results; Yamai & Yoshiba (2004).

To conclude, both Value-at-Risk and Expected Shortfall are two necessary measures utilized to minimize the forecast error. However, the Expected Shortfall (ES) is a little better risk measure, because firstly it informs the risk manager what to expect whether a VaR violation is occurred; Artzner, *et.al*. (1997; 1998), secondly ES could not mislead investors, contrary to VaR; Yamai & Yoshiba (2004) and finally, ES estimates might be more accurate than the VaR ones; Mausser, Rosen & *et.al.* (2000).

1.3) Monte Carlo Simulation: The multi period VaR and ES forecasts

The multi period VaR could be estimated utilizing a variety of different techniques; among others are the parametric approaches, the non-parametric and the semi-parametric ones, as they were explained in more details in the previous 1.2 subsection. Despite all the above well-known forms applying multi period VaR, a new one distinguishing approach was that of Monte Carlo Simulation. Monte Carlo Simulation depicts the reliability of quasi maximum likelihood estimation methods and by all accounts has a competitive advantage, because new empirical evidences have already shown that the apparent long run dependence, for example of stock index volatility, will be better described by a reverting mean of fractional integrated process. This is illustrated by the fact that the future conditional variance of the optimal forecast will be dissipated at a slow hyperbolic rate, and as a result this means more accurate forecasts. Consequently, the key innovation of using Monte Carlo Simulation is the estimation of multiple-step-ahead VaR and ES for the FIGARCH-skT specification, using a number of steps arising from a new algorithm; Christoffersen P. F. (2003); Xekalaki & Degiannakis (2010).

To generate the τ-step-ahead VaR and ES forecasts for the AR(1)-FIGARCH(1,d,1)-skT model, Monte Carlo Simulation technique is employed. At the first step required to be produced leptokurtic and asymmetrically conditionally distributed log-returns. The second and third steps are used to obtain estimates for multi period VaR and ES based on the fractional integrated operator. Furthermore, the out-of-sample observations at those steps are divided into overlapping intervals. The use of overlapping intervals is quite important to avoid the shortcoming of autocorrelation in the forecast errors. For more details about the steps of Monte Carlo Approach, check the following paper: Degiannakis, Dent & Floros (2012). The τ-day-ahead VaR and ES forecast, respectively, for long trading positions is defined as:

$$VaR_{t+\tau|t}^{1-p} = \mu_{t+\tau|t} + F(a;\theta^t)\sigma_{t+\tau|t} \tag{16}$$

and

$$ES_{t+\tau|t}^{(1-p)} = E\left(y_{t+\tau}\middle|\left(y_{t+\tau} \leq VaR_{t+\tau|t}^{(1-p)}\right)\right). \tag{17}$$

Innovative was the analysis that Dionne, Duchesne & Pacurar (2009) presented in their paper, using the Monte Carlo Simulation technique for a different frequency data set at this time; the intraday data, which will be analyzed in further detail in a following subsection. Dionne, *et.al.* (2009) tried to estimate intraday VaR using tick-by-tick data. The model that was used on that analysis was a log ACD-ARMA-EGARCH model[7], which finally the approach produced reliable estimates of IVaR (Intraday Value-at-Risk). The strong advantage of this innovative approach has to do with the greater information content and the greater flexibility of the intraday time horizon. They proposed an extension of GARCH models for tick-by-tick data; the ultra-high-frequency (UHF) GARCH model introduced by Engle (2000), to specify the joint density of the high-frequency returns. The advantage of this model is that it explicitly accounts for the irregular time-spacing of the data by considering durations when modeling returns; Dionne, Duchesne & Pacurar (2009); Degiannakis, Dent & Pacurar (2012).

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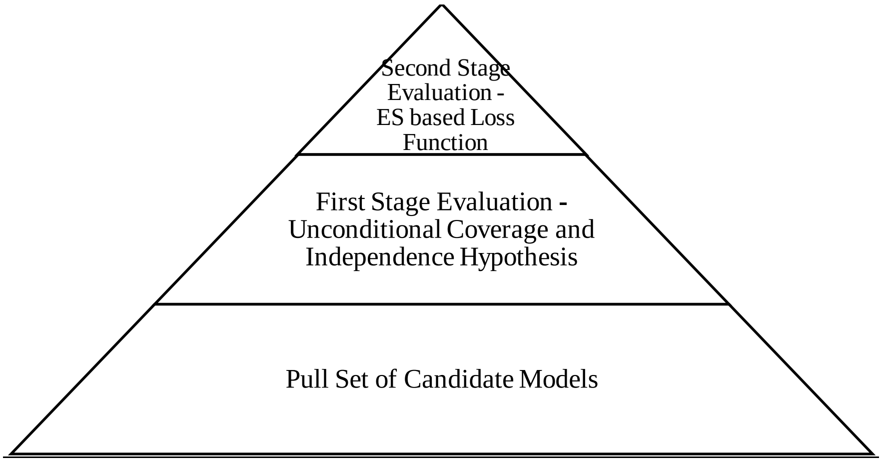
[7] ACD-ARMA-EGARCH model is an Autoregressive Conditional Duration- Autoregressive Moving Average EGARCH (exponential) model. The ACD model was introduced by Engle and Russell (1998) to taking into account the irregular spacing of such data.

1.4) Backtesting Value-at-Risk: The statistical properties of VaR forecasts (First Stage Evaluation)

It is well-known that Value-at-Risk must neither overestimate nor underestimate the expected VaR number, because it is obvious that in both cases, the financial institution allocates the wrong amount of capital. To be more precise, in the former case of overestimation, risk managers of the firm charge a higher amount of capital than really needed. Finally, in the latter case of underestimation VaR, managers charge a lower rate of capital than that of really needed and as a result, their firm remains uncovered toward the risk; the regulatory capital may not be enough to cover the market risk, as unfortunately they do not manage to forecast accurately the increased losses. For all these reasons, there arose the need of using a new risk management tool. The simplest method to evaluate the accuracy of the risk models is to record the total number of violations in order to determine the factor k. The smaller k is, the better the model predicts VaR, but this formula can only be applied at 99% significant interval and only when the holding period is up to 10 trading days. Otherwise, there are some alternative statistical techniques of evaluating VaR models, among others the quintessential one is Kupiec and Christoffersen's method, called Backtesting Procedure; Angelidis & Degiannakis (2007).

Having presented a wide variety of different risk management techniques, now it is time to discuss the statistical evaluation of these forms, and especially the statistical properties of VaR forecasts. Taken into consideration that VaR is never observed, not even after a violation is occurred, the first step is to calculate the VaR values as a number and then classify the risk models through examining the statistical properties of the forecasts. This approach divided into two stages (See Figure 3). In the first stage, a model is only considered as adequate under the assumption of no rejection by both the unconditional and independence hypothesis. The first hypothesis examines if the average number of violations is statistically equal to the excepted one and the second hypothesis if these violations are independent. At this subsection, we focused on the first stage of statistical evaluation, given more information about the Backtesting Criterion of Kupiec (1995) and Christoffersen (1998); (2003). As far as the second stage is concerned, it will be analyzed in more details at the following (1.5) subsection.

Figure 3: The statistical properties of VaR forecasts: The two stages.



Source: *Angelidis & Degiannakis (2007), Backtesting VaR models: A two-stage procedure, Journal of Risk Model Validation, p.10/1-22.*

1.4.1) Unconditional Coverage of Kupiec

The most widespread used test developed by Kupiec (1995), examines the hypothesis if the exception rate is statistically equal to the expected rate. The null hypothesis denotes that the model is adequate if the appropriate likelihood ratio statistic is:

$$LR_{UC} = 2\log\left(\left(1-\frac{N}{\tilde{T}}\right)^{\tilde{T}-N}\left(\frac{N}{\tilde{T}}\right)^{N}\right) - 2\log\left((1-\rho)^{\tilde{T}-N}\rho^{N}\right) \sim X_1^2 \quad (18)$$

where $N = \sum_{t=1}^{\tilde{T}} \tilde{I}_t$ is the number of days over a period $\tilde{T}$ that a violation occurred and as a result the portfolio loss was larger than the VaR estimate[8], and ρ is the expected ratio of violations. Otherwise, the risk model will be rejected if it generates too many or too few violations; hence, the risk manager accepts a model that generates dependent exceptions.

$$\tilde{I}_{t+1} = \begin{cases} 1, & if \ \ y_{t+1} < VaR_{t+1|t}^{(1-p)} \\ 0, & if \ \ y_{t+1} \geq VaR_{t+1|t}^{(1-p)} \end{cases} \quad (19)$$

According to (Kupiec, 1995), the number of violations follows a binominal distribution $N \sim B(\tilde{T}, p)$. The null hypothesis and the opposite are:

$$\begin{aligned} &H_0: N/\tilde{T} = p \\ &H_1: N/\tilde{T} \neq p. \end{aligned} \quad (20)$$

The Unconditional Coverage of Kupiec is X^2 (Chi square) distribution with one degree of freedom. As we can see in the Table 1 the "no rejection regions" of N differ into various sample sizes and confidence levels, as well.

Table 1: Unconditional Coverage 'no rejection' regions for 95% significance level.

Confidence level	**Evaluation sample size**			
	250	**500**	**750**	**1000**
5%	7 ≤ N ≤ 19	17 ≤ N ≤ 35	27 ≤ N ≤ 49	38 ≤ N ≤ 64
1%	1 ≤ N ≤ 6	2 ≤ N ≤ 9	3 ≤ N ≤ 13	5 ≤ N ≤ 16
0,5%	0 ≤ N ≤ 4	1 ≤ N ≤ 6	1 ≤ N ≤ 8	2 ≤ N ≤ 9
0,10%	0 ≤ N ≤ 1	0 ≤ N ≤ 2	0 ≤ N ≤ 3	0 ≤ N ≤ 3
0,01%	0 ≤ N ≤ 0	0 ≤ N ≤ 0	0 ≤ N ≤ 1	0 ≤ N ≤ 1

Source: Angelidis & Degiannakis (2007), Econometric Modeling of Value-at-Risk, Nova Science Publishers, p.28/1-53.

Lastly, the Unconditional Coverage Test, part of the first stage of statistical evaluation procedure, has the right to reject a model for both high and low failures. Hence, Kupiec's distribution stated as poor enough and as a consequence, this shortcoming comes to be overcome by the advent of an auxiliary criterion; the Conditional Coverage Test; Angelidis & Degiannakis (2007; 2007).

[8] We evaluate the accuracy of risk models for long trading positions. Alternatively, for short trading positions $\tilde{I}_{t+1} = 0$ if $y_{t+1} \geq VaR_{t+1|t}^{(1-p)}$ and 1 otherwise.

1.4.2) Conditional Coverage of Christoffersen

Christoffersen (1998) developed a more elaborate criterion that of conditional coverage test, in which combined the Kupiec's former criterion. Practically, Christoffersen examined concurrently, the total number of failures by checking if is equal to the expected number and the VaR failure process if it is independently distributed or not. The hypotheses presented on the second backtesting criterion are defined as:

$$H_0: N/\tilde{T} = p \ \ and \ \ \pi_{01} = \pi_{11} = \rho \tag{21}$$ [9]

$$H_0: N/\tilde{T} \neq p \ \ and \ \ \pi_{01} \neq \pi_{11} \neq \rho. \tag{22}$$

The null hypothesis expresses in the first part that the total number of violations is equal to the expected p and the second part expresses that the failure process is independent. The likelihood ratio statistics of the Conditional Test of Christoffersen (1998) are described in the following two equations; one for the independence and the other for the conditionality:

$$LR_{IN} = 2(\log((1-\pi_{01})^{n00}\pi_{01}^{n01}(1-\pi_{11})^{n10}\pi_{11}^{n11}) - \log((1-\pi_0)^{n00+n10}\pi_0^{n01+n11})) \sim X_1^2 \tag{23}$$

$$LR_{CC} = 2\log((1-\rho)^{T-N}\rho^N) + 2\log((1-\pi_{01})^{n00}\pi_{01}^{n01}(1-\pi_{11})^{n10}\pi_{11}^{n11}) \sim X_1^2. \tag{24}$$

Where N is the number of days that a violation is occurred over a period T and ρ is the desired coverage rate. Under this framework, a risk model is rejected if it generates either too many or too few violations; Christoffersen (1998); (2003).

Taking all the above into account, the main advantage of using the above two backtesting tests is the fact that the managers could easily reject a VaR model that generates too many or too few clustered violations. However, their drawback is that these two backtesting procedures cannot classify the models based only on the p-values of these tests.

1.5) Loss Functions: The statistical significance of VaR forecasts (Second Stage Evaluation)

As mentioned at the previous subsection, the statistical accuracy of the VaR forecasts are proved by the two backtesting tests with the unconditional and conditional coverage and consequently, if a model is not rejected means that forecasts VaR accurately. However, it is a common phenomenon more than one model to be characterized as adequate and as a result, the risk managers will not be able to choose the most appropriate technique. The weakness of backtesting test to attribute accurate results at scale of 100%, leads to the excessive need of the second stage of VaR evaluation. Lopez (1999) proposed a forecast evaluation framework which is focused on a loss function. Under this new evaluation framework, risk managers are capable to classify the models and find a utility function that releases their concerns. Loss functions measure the accuracy of the VaR forecasts on the basis of the distance between the observed returns and the forecasted VaR values, given that a violation is occurred; Angelidis & Degiannakis (2007). In other words, the adequacy of the models is investigated by the construction of a loss function that measures the squared

[9] where $\pi_{ij} = n_{ij}/\sum_j n_{ij}$ are the corresponding probabilities. And i, j=1 denotes that a violation has occurred, whereas i, j=0 indicates exactly the opposite.

distance between actually daily returns and the one-day-ahead VaR forecasts; or the multi-period VaR forecasts, respectively.

Through the Lopez (1999) approach, a VaR model is penalized when an exception takes place. So, one model is preferred over another if it yields a lower total loss value. Particularly, Lopez suggested the following loss function, which accounts for the magnitude of the tail losses $\left(\left(VaR_{t+1|t} - y_{t+1}\right)^2\right)$:

$$\psi_{t+1} = \begin{cases} 1 + \left(VaR_{t+1|t} - y_{t+1}\right)^2 & if\ a\ violation\ is\ occured \\ 0 & otherwise \end{cases} \quad (25)$$

The loss function of Lopez (Equation 23) adds a score of one whenever a violation occurs. The preferable one is the model that minimizes the total loss, $\Psi = \sum_{t=1}^{T} \Psi_t$; Degiannakis (2004); Angelidis & Degiannakis (2007); Lopez (1999).

Despite the useful part of that innovation, Lopez's (1999) approach faces two drawbacks that have to be taken into consideration. The first disadvantage argues that if a model is not checked through the backtesting tests, maybe this model does not generate any exception and it will be deemed as adequate and superior over all the other, as $\Psi_{t+1} = 0$, although something like that would be totally wrong. In order to remedy this shortcoming, Sarma, Thomas & Shah (2003) suggested a two-stage backtesting procedure. In the first stage, they tested the statistical accuracy of the models through the well-known conditional and unconditional coverage tests. As a second step, they proposed the Firm's Loss Function (FLF) by penalizing failures but also imposing a penalty reflecting the cost of capital suffered on other days:

$$\psi_{t+1} = \begin{cases} \left(y_{t+1} - VaR_{t+1|t}^{(1-p)}\right)^2, & if\ y_{t+1} < VaR_{t+1|t}^{(1-p)}, \\ -a_c VaR_{t+1|t}^{(1-p)}, & if\ \ y_{t+1} \geq VaR_{t+1|t}^{(1-p)} \end{cases} \quad (26)$$

where a_c is a measure of cost of capital opportunity; Sarma, Thomas & Shah (2003). By this new technique of Sarma, *et.al* (2003) is ensured that the models that have not been rejected in the first stage of evaluation, forecast VaR accurately.

The second drawback of Lopez's approach (1999) is that the return y_{t+1} should be better compared with Expected Shortfall rather than VaR, because VaR does not give any indication about the size of the expected loss, given a violation occurs for long trading positions. For that reason, Angelidis and Degiannakis at their paper (2007) proposed a new method to overcome the second shortcoming of Lopez's (1999) loss function. They constructed the extended loss function of Lopez with ES and not with the VaR, which was defined as:

$$\psi_{t+1} = \begin{cases} \left(y_{t+1} - ES_{t+1|t}^{(1-p)}\right)^2, & if\ y_{t+1} < VaR_{t+1|t}^{(1-p)}, \\ 0 & , if\ \ y_{t+1} \geq VaR_{t+1|t}^{(1-p)} \end{cases} \quad (27)$$

Now it arise the query of how the adequate models can be evaluated in the second stage. All the information to answer this question is in the next subsection 1.5.1 and 1.5.2.

1.5.1) Statistical Accuracy (MSE, HASE, LE)

In order to judge the models and acquire adequate forecasts in the second stage, we can use three different loss functions to compute the statistical accuracy. The first one is the Mean Squared Error (MSE), which depicts the squared distance between observed and predicted values. MSE is one of the most popular measures in evaluating forecasting accuracy. Therefore, it is not always such a reliable method, and especially when volatility is the variable under study, since symmetric loss functions may be responsible for the high non-linear environment. When something like that happens, there are other two methods that can be used; the HASE and LE, which take into consideration the heteroskedastic framework; Angelidis & Degiannakis (2008).

If we want to measure the statistical accuracy of the models with the loss function of MSE, we use the following equation:

$$MSE = T^{-1} \sum_{t=1}^{T} \left(h_{t+1}^2 - \sigma_{t+1|t}^2\right)^2 \tag{28}$$

Where the h_{t+1}^2 is the realized volatility[10] used as the measure of the true, but unobservable variance at the day t+1. The one-day-ahead variance is $\sigma_{t+1|t}^2$ and T is the number of the forecasts.

The other two loss functions; Heteroskedasticity-Adjusted Squared Error (HASE) and Logarithmic Error (LE) are more elaborate loss functions, which they take into account the heteroskedasticity framework. As a result, HASE and LE are based on asymmetric loss functions and they presented in the following equations:

$$HASE = T^{-1} \sum_{t=1}^{T} \left(1 - \frac{h_{t+1}^2}{\sigma_{t+1|t}^2}\right)^2, \tag{29}$$

and

$$LE = T^{-1} \sum_{t=1}^{T} \log\left(\frac{h_{t+1}^2}{\sigma_{t+1|t}^2}\right)^2. \tag{30}$$

Not to mention that HASE was introduced by Bollerslev and Ghysels (1996), as well as, LE was introduced by Pagan and Schwert (1990).

1.5.2) Statistical Significance (DM, SPA, MCS)

The statistical significance of the volatility forecasts was investigated by:

1. the Diebold and Mariano (DM) Statistic (1995),
2. the Hansen's Superior Predictive Ability (SPA) Test (2005), and
3. the Model Confidence Set Statistic (MCS) of Hansen, *et.al* (2005).

The above three significance statistics are the most frequently used tests in a large range of studies, with the MCS statistic be one of the most recent methods.

The DM statistic is the t-statistic derived from the regression of $X_{t,l}^{(i,i^*)} = L_{t,l}^{(i)} - L_{t,l}^{(i^*)}$ in connection with heteroskedastic and consistent (HAC) standard errors. Let *i* be the benchmark model with the lowest loss function value, the $L_{t,l}^{(i)}$ is the value of the loss function *l* at the time *t* of the benchmark model *i*. The null hypothesis designates that the benchmark model *i* has equal predictive ability with the model i^*, for $i^*=1, \ldots, M$. The alternative hypothesis states that the benchmark model has superior predictive ability over the competitive model i*; Diebold & Mariano (1995).

[10] The Realized Volatility is computed by the following equation:
$$h_t^2 = \frac{\hat{\sigma}_{OC}^2 + \hat{\sigma}_{CO}^2}{\hat{\sigma}_{OC}^2} \sum_{j=1}^{m-1} \left(100\left(\log\left(P_{(j+1/m),t}\right) - \log\left(P_{(j/m,t)}\right)\right)\right)^2.$$

Hansen (2005) introduced the Superior Predictive Ability (SPA) test that is used to compare the forecasting ability of the one benchmark model against its M competitor models. The advantage of SPA test is that the comparison of the models become simultaneously for all the models between the benchmark and all the others respectively, contrary to the DM statistic that the comparison carries out at one-by-one models. The hypothesis of the SPA test is:

$$H_0: E\left(X_t^{i,1} \ldots X_t^{i,M}\right)' \leq 0$$
$$H_1: E\left(X_t^{i,1} \ldots X_t^{i,M}\right)' > 0 \tag{31}$$

where $X_{t,l}^{(i,i^*)} = L_{t,l}^{(i)} - L_{t,l}^{(i^*)}$, the best performing one model is i and all the other competitive models denoted as i^*, for $i^*=1, \ldots, M$. The null hypothesis that the benchmark model i is not outperformed by the other competitive models is tested with the following statistic, T^{SPA}:

$$T^{SPA} = max \frac{M^{1/2}\overline{X_{\iota^*}}}{\sqrt{Var\left(M^{1/2}\overline{X_{\iota^*}}\right)}}, \tag{32}$$

for $i^*=1, \ldots, M$, where $\overline{X_{\iota^*}} = \frac{1}{T}\sum_{t=1}^{T} X_t^{(i,i^*)}$. The $Var\left(M^{1/2}\overline{X_{\iota^*}}\right)$ is calculating according to the stationary bootstrap of Politis and Romano (1994) methodology, not to mention that White (2000b) is the source of many of the ideas that underlies the bootstrap implementation; Hansen P. R. (2005); Angelidis & Degiannakis (2007).

The Model Confidence Set (MCS), introduced by Hansen, *et.al* (2005), is an innovative process, due to the fact that the MCS acknowledges the limitations of the data. Especially, when the set of competing models is quite large then many applications may not yield a single model that significantly dominates all competitors, because the data is not sufficiently informative to give an adequate answer to the question of “which is the best forecasting model”. As a consequence, through the MCS statistic, it is now possible to reduce the set of models to a smaller set; a model confidence set that is guaranteed to accommodate the best forecasting model, under a pre-specified level of confidence. So the best model is unlikely to be replicated for all criteria. The objective of the model confidence set (MCS) procedure is to determine the M^*, that consists of the benchmark models; more than one on this occasion from a collection of models M_0. It is proposed a bootstrap implementation of the MCS procedure that is very convenient when the number of models is large. The bootstrap implementation is simple to use in practice and avoids the need to estimate a high-dimensional covariance matrix (White, 2000b). The MCS procedure is based on an equivalence test, the δ_M, and elimination rule, e_M. The equivalence test is applied to the set of objects $M = M_0$. If δ_M is rejected, there is evidence that the models M is not so good, and as a result the e_M is used to eliminate the object with poor sample performance from M. Finally, the MCS procedure yields p-values for each of their models. A model with a small MCS p-value makes it unlikely that model i is one of the best models (is a member of M^*); Hansen, Lunde, *et.al.* (2005). The hypotheses that are being tested have the following form:

$$H_0: E\left(d_{ij,t}\right) = 0 \quad for\ all\ i,j \in M$$
$$H_1: E\left(d_{ij,t}\right) \neq 0 \quad for\ all\ i,j \in M. \tag{33}$$

1.6) Realized Volatility and Intra-Day Data

Up until now, everything concerns volatility prediction has been analyzed with much detail. One of the most significant issues in financial environment is the choice of the appropriate volatility model in order a risk manager to forecast the risk that his clients face. A lot of researchers have spent many years of study, focused on the inter-day volatility forecasts. Inter-day trading, or more commonly known as End-of-Day Trading would be when a position is held overnight or for multiple days. Now it's time to present another technique of VaR forecasting procedure, by using ultra-high-frequency data. This alternative technique is known as the Intra-day Realized Volatility Models. Intra-day denotes a situation of buying and selling indexes within the same market day. The availability of high frequency data rekindled the interest of many researchers to forecast risk, because the volatility estimates based on intra-day returns which are more accurate than those of daily ones. This is illustrated by the fact that the squared daily returns are unbiased, but noisy estimator of volatility; Angelidis & Degiannakis (2008).

There seems to be many proponents to forecast Value-at-Risk with this new alternative way. Many researchers use high frequency dataset in their analyses, due to explore ways to extract more information to enable them to forecast VaR accurately. The origin of high and ultra high frequency data concept was not such a contemporary process; Merton (1980) already mentioned it, provided data sampled at a high frequency level, let alone that the sum of squared realizations can be used to estimate the variance of an *i.i.d*[11]. Andersen and Bollerslev (1998a) at their paper showed that daily realized volatility may be constructed simply by summing up intra-day squared returns. Assuming that a day can be divided in N equal periods and if $r_{i,t}$ denotes the intra-daily return of the i^{th} interval of day t, then the daily volatility for day t can be written as:

$$\left(\sum_{i=1}^{N} r_{i,t}\right)^2 = \sum_{l=1}^{N} r_{i,t}^2 + 2\sum_{i=1}^{N}\sum_{j=i+1}^{N} r_{j,t} r_{j-i,t} \ . \quad (34)$$

If the returns have zero mean and are uncorrelated, then $\left(\sum_{i=1}^{N} r_{i,t}\right)^2$ is a consistent and unbiased estimator of the daily variance, σ_t^2; Andersen and Bollerslev (1998a). Because all squared returns on the right side of the (32) equation are observed when intra-day data are available, then the $\left(\sum_{i=1}^{N} r_{i,t}\right)^2$ is called Realized Volatility at daily returns.

Let $\log\left(P(t)\right)$ be considered as the instantaneous logarithmic price of a financial asset follows:

$$d\ln\left(P(t)\right) = \sigma(t)dW(t) \ , \quad (35)$$

where $\sigma(t)$ is the volatility of the instantaneous returns and $W(t)$ is the standard Wiener[12] process. The Integrated Volatility,$\sigma_t^{2(IV)}$, over the time interval $(l-1,l)$ is equal to:

[11] *i.i.d:* Independent and identically distributed random variables.

[12] A standard Wiener process (often called Brownian motion) on the interval $[0,T]$ is a random variable $\left(\mathrm{W}(t)\right)$ that depends continuously on $t \in [0,\mathrm{T}]$ and satisfies the following: $W(0) = 0$, $0 \le s < t \le T$.

$$W(t) - W(s) \sim \sqrt{t-s}N(0,1) \ ,$$

$$\sigma_t^{2(IV)} = \int_{t-1}^{t} \sigma^2(x)dx \,. \tag{36}$$

The Integrated Volatility is a variable which is not observable. As a consequence, the Integrated Volatility (IV) can be estimated by the Realized Volatility (RV),$\sigma_t^{2(RV)}$ which is defined as the sum of squared returns observed over very small time intervals.

$$\sigma_t^{2(RV)} = \sum_{j=1}^{m-1}\left(\log\left(P_{(j+1/m),t}\right) - \log\left(P_{(j/m),t}\right)\right)^2, \tag{37}$$

Where $P_{(m),t}$ consists of the financial asset prices during period t with sampling frequency m. Lastly, the Realized Volatility converge in probability to the Integrated Volatility:

$$p\lim_{m\to\infty}\left(\sum_{j=1}^{m-1}\left(\log\left(P_{(j+1/m),t}\right) - \log\left(P_{(j/m),t}\right)\right)^2\right) = \sigma_t^{2(IV)}. \tag{38}$$

Another approach was that of Marten's (2002), who proposed accounting the overnight returns without inserting the noisy effect of daily returns. The equation of Martens' approach defined as:

$$\sigma_t^{2(RV)} = \frac{\sigma_{OC}^2 + \sigma_{CO}^2}{\sigma_{OC}^2} \sum_{j=1}^{m-1}\left(100\left(\log P_{(j+1/m),t}\right) - \log\left(P_{(j/m),t}\right)\right)^2, \tag{39}$$

where σ_{OC}^2 is the open-to-close sample variance. σ_{CO}^2 is the close-to-open sample variance. Moreover, Engle and Sun (2005) suggested another important addendum to the Realized Volatility Method; they proposed an econometric model for the joint distribution of tick-by-tick return and duration, taking into account the market microstructure effects; Engle & Sun (2005); Angelidis & Degiannakis (2007).

Additionally, the contribution of Corsi's research is depicted as one of the top quintessential processes. He introduced in his paper; Corsi (2004), the Heterogeneous Autoregressive for Realized Volatility (HAR-RV) model, which has the following form:

$$\sigma_t^{2(RV)} = w_0 + w_1\sigma_{t-1}^{(RV)} + w_2\left(\sigma^{(RV)}\right)_{t-5:t-1} + w_3\left(\sigma^{(RV)}\right)_{t-22:t-1} + \varepsilon_t \,, \tag{40}$$

The HAR-RV model is an autoregressive structure of the realized volatilities over different interval sizes. As far as the $\sigma_{t-1}^{(RV)}$ is concerned, it accounts for the volatility of inter-day and intra-day trading strategies, thus the $\left(\sigma^{(RV)}\right)_{t-5:t-1}$ accounts for medium term trading, let alone the $\left(\sigma^{(RV)}\right)_{t-22:t-1}$ enclose investment strategies during the period of one month or even longer time horizons, as Corsi F. (2004) applied. The heterogeneity is the reason of the volatility creation, through the different time spaces. Last but not least, Corsi *et.al* tried to extend his further model by implementing a new specification model, that of HAR-GARCH(p,q) model. For further details, there is an analytical research into the paper of Corsi, Mittnik, *et.al* (2005).

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where $N(0,1)$ is a normal distribution with zero mean and unit variance. Because the normal distribution is used, the process is often referred to as Gaussian. For use on a computer, we discredited the Wiener process with a time-step *dt* as:$dW \sim \sqrt{dt}N(0,1)$.
(*Source:* https://me.ucsb.edu/~moehlis/APC591/tutorials/tutorial7/node2.html (University of California, Santa Barbara)).

Finally, it is of great importance to describe the model used at Realized Volatility process of intra-day trading strategies. This model is the ARFIMAX(k,l) Model or else, the Fractionally Integrated ARMAX. The purpose of this model is to modeling the long memory property of the realized volatility, which accounts for recent developments in the ultra-high frequency financial modeling:

$$y_t = c_0' + \left(1 - c'(L)\right)^{-1} z_t \sigma' \tilde{h}_{t|t-1} \quad (41)$$

$$z_{t \sim i.i.d}\ skT\ (0{,}1; \upsilon', g) \quad (42)$$

$$\tilde{h}_{t|t-1}^2 = exp\left(\log \tilde{h}_{t|t-1}^2 + \frac{1}{2}\sigma_u^2\right) \quad (43)$$

$$\left(1 - a'(L)\right)(1 - L)^{d'}(\log h_t^2 - w_0' - w_1' y_{t-1} - \gamma' d_{t-1}' y_{t-1}) = \left(1 + b'(L)\right) u_t \quad (44)$$

$$u_{t \sim i.i.d}\ N(0, \sigma_u^2) \quad (45)$$

Generally, the AR(k)-ARFIMAX(p,q) specification accounts for:

1. Non-synchronous trading positions,
2. Fractional Integration of the Intra-day Volatility,
3. Asymmetric and leptokurtic conditional and unconditional distribution of returns.

Andersen & Bollerslev (1998a), Giot & Laurent (2004) and Angelidis & Degiannakis (2007).

However, there are others who are opposed to the importance of that alternative volatility measure. A quintessential example was that of Giot and Laurent (Giot & Laurent, 2004), in which they compared the APARCH-skT model with an ARFIMAX specification in an attempt to compute VaR for stock indexes and exchange rates as well. They supported that the use of intra-day data did not improve the performance of the inter-day VaR model. Therefore, Giot (2005) continued estimate the intra-day VaR of 15 and 30 minutes, despite his different point of view. For another time, he came to a consensus that there were no significant differences between daily and intra-day VaR models. As a result, he claimed that the use of the intra-day data does not add something further to the forecasting power of the models. To summarize, the meaning of this paragraph is to describe that although, there are indications that the extended models produce the most accurate and valid VaR forecasts, in some cases, a simpler one may be preferable.

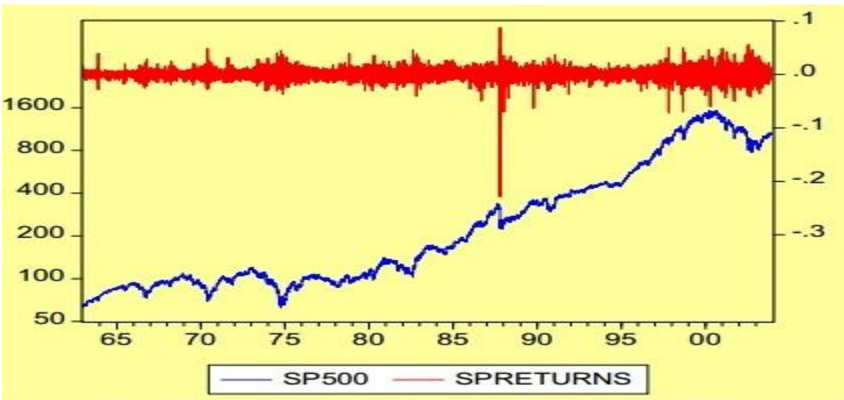
The recent literature on realized volatility and the huge literature on daily volatility models seem to indicate that a researcher faces a twofold dilemma of what method to choose when daily volatility is to be modeled. It is not such a simple choice as it seems. There are weaknesses with either realized volatility or the daily volatility models. For instance, if someone decides to model daily volatility using daily realized volatility, then intra-day dataset is needed so that corresponding intra-day returns can be computed. Furthermore, even today intra-day data remain extremely costly and are not readily available for all assets. On the contrary, working with daily data is relatively simple and the data are broadly available. However, if all the above shortcomings of Realized Volatility method are overcome in some way, then undoubtedly, the intraday level will be a much better model.

1.7) Empirical Analyses of Value-at-Risk Theory, using ARCH Models

Beside all these elaborate information about all the commonly used models in order to forecasting VaR accurately, there are many empirical analyses, utilizing a variety of portfolios and market stock indices. Researchers in their attempt to calculate losses made some innovations and as a consequence, they finally extended the ideas of their predecessors. The following paragraphs will be presented some of them. Long memory[13] in volatility has been documented across a range of equity indices; the $S\&P_{500}$; Engle R. F. (2003); Bollerslev & Mikkelsen (1996); Angelidis & Benos and Degiannakis (2004); *et.al.*, the $Nikkei_{225}$; Ding & Granger (1996); Giot & Laurent (2003); Angelidis, Benos & Degiannakis (2004); *et.al.*, the DAX_{30}; Angelidis, Benos; *et.al.*(2004); Angelidis & Degiannakis (2008); Giot & Laurent (2003); *et.al.*, the $FTSE_{100}$; Angelidis, Benos & Degiannakis (2004); *et.al.*, the CAC_{40}; Giot & Laurent (2003); *et.al.*, not to mention the exchange rates of Deutschemark-U.S.$, Baillie, Bollerslev & Mikkelsen (1996); Ding & Granger (1996); *et.al.* and finally, the US$/UK£ exchange rate; Angelidis & Degiannakis (2007); *et.al.* All these above are the most commonly used indices and exchange rates, which are inferred in the literature. There are several studies that investigate the parametric ARCH procedures. By all accounts, all parametric ARCH models share the same goal; modeling the conditional variance as a function of past squared returns; Engle R. F. (2003), *et.al.*

To begin with the first empirical analysis, in his Nobel lecture, Engle (2003) illustrated the use of ARCH models for financial applications. He proposed an extended analysis of $S\&P_{500}$ index at daily levels from 1963 to the end of 2003 (See Figure 4).

Figure 4: $S\&P_{500}$ daily returns and returns from January 1963 to November 2003.



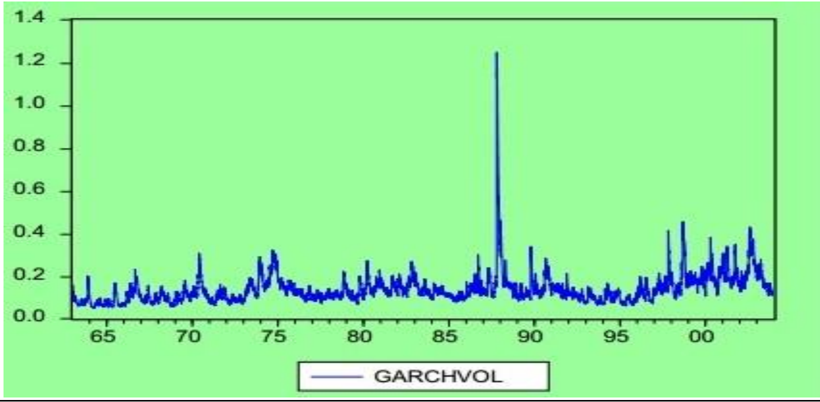
Source: Robert Engle (2003), Risk and Volatility: Econometric Models and Financial Practice, Nobel Lecture, p.332/326-349.

This analysis provides an information set about how ARCH models are used for risk management and option pricing. Engle used GARCH(1,1), which gave weights to the

[13] The slow decline of the autocorrelations in the volatility series suggests a long memory process, as Baillie proposed in 1996. The first contribution in this regard was Taylor (1986), who noticed that the absolute values of stock returns tended to have very slow decaying autocorrelations. Ding, Granger and Engle (1993) noted the same fact, concerning the daily returns; Baillie R. (1996).

unconditional variance and the previous day forecasts. The results of GARCH(1,1) were not enough satisfactory (See Figure 5), since it appeared that the long run variance had a tiny effect and might not be significant, which is not correct.

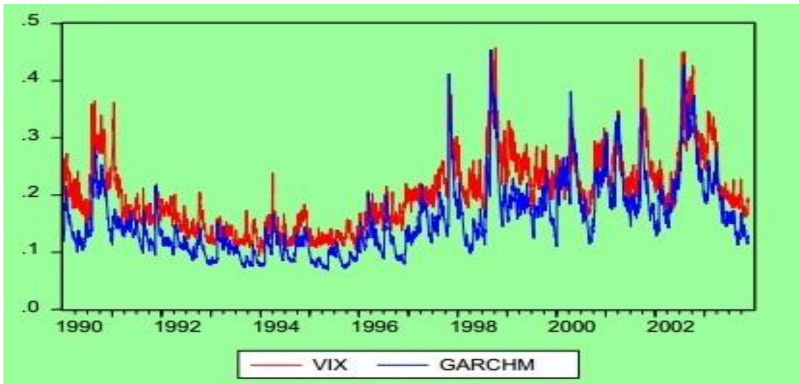
Figure 5: GARCH(1,1) volatilities.



Source: Robert Engle (2003), Risk and Volatility: Econometric Models and Financial Practice, Nobel Lecture, p.339/326-349.

For that reason, he utilized an asymmetric volatility model; the TARCH model, considering the CBOE Volatility Index (VIX)[14]. Finally, the TARCH volatilities forecasted out to one month, due to VIX method. All in all, the outcome was quite similar although the TARCH was little lower than the VIX. (See Figure 6)

Figure 6: Implied Volatilities and GARCH volatilities.



Source: Robert Engle (2003), Risk and Volatility: Econometric Models and Financial Practice, Nobel Lecture, p.341/326-349.

Another empirical analysis in Angelidis, Benos and Degiannakis article (2004) implemented three volatility models; GARCH, TARCH and EGARCH, under three different distributional assumptions; Normal, Student-t and GED[15]. This study shows that the more flexible a GARCH model is, the more adequate is in the

[14] The CBOE Volatility Index (VIX) is a key measure of market expectations of near-term volatility conveyed by $S\&P_{500}$ stock index option prices in one-month returns. Since its introduction in 1993, VIX has been considered by many to be the world's premier barometer of investor sentiment and market volatility; Chungi, Tsai, Wang & Weng (2011), (Source: www.cboe.com/micro/vix/vixintro.aspx)

[15] GED: Generalized Error Distribution.

volatility forecasting. Moreover, the above three models that have been chosen are able to capture the most considerable characteristics of financial markets. They used four historical sample sizes with 500, 1000, 1500 and 2000 observations, in order to estimate the 95% and 99% one-day-ahead VaR. A restricted sample size could generate more accurate one-step-ahead VaR forecasts, since it incorporates changes in trading behavior more effectively. The equity indices portfolios used in this analysis were the $S\&P_{500}$, the $NIKKEI_{225}$, the $FTSE_{100}$, the CAC_{40} and the DAX_{30}. The overall conclusion of this research was that the VaR estimate was less often rejected at the 95% confidence level. In the assumption of normally distributed returns, the results were weak, due to the fact that the vast majority of models underestimated the risk at high confidence level, yielding sufficient p-values for the 95%, but extremely low ones for the 99%, respectively. As far as the Student-t distribution is concerned, the GARCH and EGARCH models generate better forecasts rather than TARCH. Hence, by increasing the confidence level, there arose some more complicated results, because both symmetric and asymmetric models had been selected as statistical adequate, which is incorrect. In this occasion, the choice of the sample size has turned to be one of the most crucial factors. Last but not least, the GED had similar effects as the Normal distribution for 95% confidence level, but yielded better results for 99%. To conclude, the combination of leptokurtic distribution and a simple asymmetric volatility model, such as the EGARCH in this occasion, attributed the best combination, concerning the five indices of this example.

Ding, Granger and Engle (1993) investigated a long memory property of the stock market returns series. They found not only that there was substantially higher correlation between absolute returns than returns themselves, but the transformation of the absolute return $|r_t|^d$ also has quite high autocorrelation for long lags. Additionally, in another empirical analysis of the same authors; Ding and Granger (1996) found that absolute returns and their power transformations were highly correlated; Ding & Granger (1996). A systematic study of this can also be found in Taylor's analysis (1986). In this research, they investigated the autocorrelation structure of $|r_t|^d$, where $|r_t|$ is the daily $S\&P_{500}$ stock market return and (d) is a positive number.

On the other hand, Angelidis and Degiannakis (2007), in their paper proposed a two-step backtesting procedure, where in the first step all the rejected models are discarded by the univariate VaR backtesting procedure and in the second step; a multivariate superior predictive test is occurred, chosen one model as the benchmark. Following this procedure of superior predictive test (SPA), the statistical significance of the volatility forecasts is investigated. Particularly, in the empirical analysis, Angelidis and Degiannakis (2007) used three financial markets; US stock - the $S\&P_{500}$, the commodity market of Gold and the exchange rate of US (\$)/UK (£), under four distributional assumptions; that of Normal, Student-t, GED and skewed Student. The sample size of this analysis was from April 4th, 1988 through April 5th, 2005. The purpose of this study was to find the best model predicting accurately VaR for those three financial markets. To be more precise for each financial market, firstly for the $S\&P_{500}$ index, the FIEGARCH-GED model (for 95% confidence level and for long trading position) was the most accurate one. Secondly, for Gold commodity range, five models generated accurate predictions for both confidence levels and both trading positions; GARCH-GED, IGARCH-GED, FIAGARCH-GED, FIAGARCHC-GED and FIAPARCHC-GED. Furthermore, for the US (\$)/UK (£) exchange rate, the choice is not such straightforward. For long (short) trading positions at 99% confidence level, the best overall distribution seems to be the GED (Normal), whereas for the other cases the results are mixed. Hence, the model which appears to have the best overall performance was EGARCH-N. After the SPA test, the results were displayed in the following table. (See Table 2)

Table 2: The proposed models that forecast accurately VaR and ES for each dataset, after the SPA test.

Market	**Model**
S&P500	FIEGARCH-N
Gold bullion US$ per Troy ounce	GARCH-GED/IGARCH-GED
US$/UK£	EGARCH-N

Source: Angelidis and Degiannakis (2007), Backtesting VaR models: A two-stage procedure, Journal of Risk Model Validation p.19/1-22.

In an attempt to illustrate the innovative approach of Monte Carlo Simulation, Degiannakis S., Dent P. and Floros C. (2012) presented in their paper an empirical application of forecasting one-step-ahead, 10-step and 20-step-ahead VaR and ES, modeling volatility for 10 of the most worldwide known stock indices. The data period was from January, 12 of 1989 until February, 12 of 2009. VaR and ES are calculated for 95% and 99% confidence level, by considering long memory within the conditional variance process and skewed Student-t distributed innovations. In this paper not only be analyzed the leptokurtosis, but also the asymmetry of the portfolio returns was investigated. The main purpose and contribution of this paper was to propose a new adaptation of the Monte Carlo Simulation technique of Christoffersen (2003) in order to forecasting multiple-step-ahead VaR and ES, respectively. Particularly, the models used by the authors were FIGARCH-skT and GARCH-skT. According to the conditional coverage test, the results are turned over to the FIGARCH-skT model, as it produced an adequate forecasting performance for the most out of the 10 indices tested. Additionally, the MSE results of the ES figures have shown that the FIGARCH-skT model is generally lower than those of GARCH-skT, especially when the forecasting margin increases. The SPA test led to the inference that the null hypothesis of the superiority of the optimal model (FIGARCH-skT) was not rejected. To conclude, the fractional integrated model seems to outperform the simple GARCH both for 95% and 99% confidence level, for the 10-day-ahead and for the 20-day-ahead time horizon. As far as the one-day-ahead time horizon is concerned, the long memory structured model did not perform better results rather than those of short memory.

According to the empirical analysis of another paper of Degiannakis (2004), the ability of volatility models, under the ARCH framework, was investigated to produce accurate forecasts of one-day-ahead realized intra-day volatility and one-day-ahead VaR, using five-minute linearly interpolated prices. In order to investigate the predictability of the models, firstly, he used two statistical criteria to measure the distance between the predicted and realized intra-day volatility and secondly, he computed the VaR and investigated which model can predict the next-day's financial loss in the most accurate way. To evaluate the ability of the models in forecasting one-step-ahead intra-day volatility, he used the Heteroskedasticity-Adjusted Squared Error (HASE) and the Logarithmic Error (LE) loss functions. As we can see in the Table 3, in which presents the values of HASE and LE loss functions, the FIAPARCH(1,1)-skT model either yields the lowest value of the loss functions or produces volatility forecasts whose predictive accuracy is not statistically significant to the forecasts of the model with the lowest value of the loss function. Only, in the case of the $FTSE_{100}$ and the LE loss function, the FIAPARCH(1,1)-skT model is statistically significant to the FIAPARCH(1,1)-N model, which yields the lowest value of the LE loss function.

Table 3: Presents the HASE and LE loss functions and the relative Diebold & Mariano Statistics.

MODEL	CAD		DAX		FTSE	
	HASE LOSS FUNCTION	DM STATISTIC	HASE LOSS FUNCTION	DM STATISTIC	HASE LOSS FUNCTION	DM STATISTIC
GARCH(1,1)-N	9,045655	**-2,52774**	0,485761	-1,94042	0,488443	-1,58756
IGARCH(1,1)-N	7,970780	**-2,21427**	0,447328	-	0,479234	-1,14582
APARCH(1,1)-N	7,349019	**-2,47113**	0,474691	-0,57499	0,454476	-0,64122
FIAPARCH(1,1)-N	6,341504	-1,51987	0,464788	-0,49778	0,445565	-
FIAPARCH(1,1)-skT	6,252786	-	0,452104	-0,11375	0,453901	-1,44746
MODEL	HASE LOSS FUNCTION	DM STATISTIC	HASE LOSS FUNCTION	DM STATISTIC	HASE LOSS FUNCTION	DM STATISTIC
GARCH(1,1)-N	0,762832	***-5,67353***	1,473186	***-7,59009***	1,207131	***-5,60130***
IGARCH(1,1)-N	0,891378	***-8,71770***	1,570528	***-10,1103***	1,213684	***-5,53343***
APARCH(1,1)-N	0,704857	-	1,292694	***-6,26598***	1,139915	***-3,81473***
FIAPARCH(1,1)-N	0,724752	**-2,02689**	1,456512	***-7,11328***	1,062456	-
FIAPARCH(1,1)-skT	0,719542	-1,26751	1,136389	-	1,079832	***-2,71098***

*Bold Font: Statistically significant at 5%
*Bold Italics Font: Statistically significant at 1%

Source: Degiannakis (2004), Volatility Forecasting: Evidence from a FIAPARCH-skT Model, p.17/1-24.

The statistical significance of the volatility forecasts was investigated using the Diebold & Mariano statistic; Diebold & Mariano (1995); Degiannakis (2004). The stock indexes[16] used on this analysis are three; CAC_{40}, DAX_{30} and $FTSE_{100}$ and the forecasting period started from July 10th, 1989 to June 30th, 2003. Table 4 presents the adequacy of the models, by the construction of a loss function that measure the squared distance between actual daily returns and one-step-ahead VaR.

[16] See Appendix B: Figure 23 that presents the CAC40, DAX30 and FTSE100 stock index daily returns in the period from July 10th, 1987 to June 30th, 2003 (p.66). And also see Appendix C: Figure 24 that presents the CAC40, DAX30 and FTSE100 stock index for the realized intra-day volatility and the relative one-day-ahead forecasts of FIAPARCH(1,1)-skT (p.67).

Table 4: Presents the LE loss functions measuring the squared distance between the actual daily returns and the one-day ahead VaR forecasts and the relative DM Statistics, with a=% and a=.% confidence interval, respectively.

α=5%						
Long Positions						
MODEL	**CAD40**		**DAX30**		**FTSE100**	
	HASE LOSS FUNCTION	DM STATISTIC	HASE LOSS FUNCTION	DM STATISTIC	HASE LOSS FUNCTION	DM STATISTIC
GARCH(1,1)-N	0,065551	***-3,89179***	0,087170	***-4,007482***	0,041990	***-3,816774***
IGARCH(1,1)-N	0,055388	***-2,47118***	0,066870	**-2,338562**	0,037960	***-3,222050***
APARCH(1,1)-N	0,065596	***-5,40206***	0,087871	***-4,897153***	0,037681	***-4,473745***
FIAPARCH(1,1)-N	0,063253	***-5,97315***	0,086078	***-4,510446***	0,037282	***-5,822649***
FIAPARCH(1,1)-skT	0,042675	-	0,053523	-	0,023751	-
Short Positions						
MODEL	**CAD40**		**DAX30**		**FTSE100**	
	HASE LOSS FUNCTION	DM STATISTIC	HASE LOSS FUNCTION	DM STATISTIC	HASE LOSS FUNCTION	DM STATISTIC
GARCH(1,1)-N	0,044903	***-3,85406***	0,038727	***-3,288739***	0,017074	***-2,740793***
IGARCH(1,1)-N	0,035263	-1,74338	0,029910	-0,837396	0,014234	-1,132025
APARCH(1,1)-N	0,041037	***-4,98423***	0,037286	***-4,913541***	0,014402	***-2,031883***
FIAPARCH(1,1)-N	0,040013	***-5,04504***	0,037238	***-3,385501***	0,015622	-4,657467
FIAPARCH(1,1)-skT	0,029958	-	0,027336	-	0,012470	-
α=1%						
Long Positions						
MODEL	**CAD40**		**DAX30**		**FTSE100**	
	HASE LOSS FUNCTION	DM STATISTIC	HASE LOSS FUNCTION	DM STATISTIC	HASE LOSS FUNCTION	DM STATISTIC
GARCH(1,1)-N	0,018611	**-2,07918**	0,026076	-2,071137	0,011895	**-2,148599**
IGARCH(1,1)-N	0,016305	-1,81311	0,017439	-1,813110	0,010593	-1,930740
APARCH(1,1)-N	0,017799	***-2,70251***	0,027427	**-2,480588**	0,009448	**-2,334256**
FIAPARCH(1,1)-N	0,015945	***-2,72616***	0,025671	**-2,344866**	0,008977	***-2,815065***
FIAPARCH(1,1)-skT	0,007119	-	0,009478	-	0,003468	-
Short Positions						
MODEL	**CAD40**		**DAX30**		**FTSE100**	
	HASE LOSS FUNCTION	DM STATISTIC	HASE LOSS FUNCTION	DM STATISTIC	HASE LOSS FUNCTION	DM STATISTIC
GARCH(1,1)-N	0,006970	***-2,42590***	0,007536	**-2,356176**	0,002387	-1,785638
IGARCH(1,1)-N	0,003923	-1,68471	0,005083	-1,723925	0,001786	-1,142707
APARCH(1,1)-N	0,006059	***-2,63054***	0,007156	***-2,655640***	0,001846	-1,735150
FIAPARCH(1,1)-N	0,005716	***-2,69535***	0,009186	**-2,425682**	0,002126	***-2,891783***
FIAPARCH(1,1)-skT	0,002142	-	0,003344	-	0,001103	-
*Bold Font: Statistically significant at 5%						
*Bold Italics Font: Statistically significant at 1%						

Source: Degiannakis S. (2004), Volatility Forecasting: Evidence from a FIAPARCH-skT Model, p.18/1-24.

Under a more careful study of table 4, the accuracy of the FIAPARCH(1,1)-skT model's VaR predictions is statistically superior in the majority of the cases. To reach a conclusion, the empirical analysis of Degiannakis (2004) have shown that the extended ARCH model; FIAPARCH(1,1)-skT, generates the most accurate volatility forecasts in the majority of the cases. As a result, this study led to the inference that flexible models produce accurate volatility forecasts; Giot & Laurent (2003).

To take all the above empirical analyses into consideration and in an attempt to collect as much information as it is possible about the ARCH models and their performance, we lead to the inference that there are some weaknesses through the variety of empirical analyses focused on. First of all, the ARCH models assume that positive and negative shocks have the same effects on volatility because they depend on the square of the previous shocks. In practice, it is an undeniable fact that the price of a financial asset responds differently to positive and negative shocks, respectively. As a result the above thesis is not much reliable. Another drawback concerns the source of variations of financial time series, in which the ARCH models do not provide any current insight in order to understanding these sources. Finally, ARCH models are likely to overpredict the volatility due to the fact that they respond slowly to large isolated shocks to the return series; Tsay (2005) .

Indisputably, the Value-at-Risk is a field of financial econometrics that has been studied thoroughly. One main reason of this extensive research is the resent financial crisis, which intrigue the interest of risk managers, let alone of financial institutions, in order to provide more reliable Value-at-Risk (VaR) and Expected Shortfall (ES) forecasts. All they want to do is to minimize the losses amount of their capitals. They utilize a huge number of models, as there is a majority of existing models in the literature, in their attempt to find the benchmark one having accuracy and efficiency. Although there is a plethora of forecasting models, the financial institutions have to abide by the recommendations of the Basel Committee of Banking Supervision[17].

An enormous variety of VaR models have been tested in the literature including both parametric and non-parametric models. The results have not been entirely consistent, often suggesting that the optimum choice of model, as well as the distributional assumptions, may depend upon a number of factors including the market for which the model is being estimated, the length and the frequency of the data series, and whether or not the VaR relates to short or long trading positions; Angelidis, *et.al.* (2004); Shao, *et.al.* (2009). For all these reasons, this dissertation has been done to clarify any doubt concerning the appropriateness and accuracy of a model to be chosen, among stock indices, commodities and exchange rates.

The method that my dissertation follows is the use of AR(1)-GARCH(1,1) model, representing the short memory trading positions, compared with that of intra-day high frequency data using the Heterogeneous Autoregressive Realized Volatility, AR(1)-HAR-RV model. Surprisingly the fact that it does not formally belong to the class of long memory models; the HAR-RV model is able to reproduce the same memory persistence observed in volatility. The distribution of the GARCH(1,1) is the skewed Student-t (skT) and respectively, the AR(1)-HAR-RV has been estimated under the skewed Student-t distribution as well. Concerning the frequency of these forecasts, I have used one-day-ahead, 10-day-ahead and 20-day-ahead VaR and ES forecasts for the GARCH(1,1) model. In addition to the previous parameterization, AR(1)-HAR-RV has been forecasted into daily basis estimation, after its logarithmic modification from annualized realized volatility to daily realized volatility. Afterward this transformation, the frequency of the data used for HAR-RV model is again one-step-ahead, 10-step-ahead and 20-step-ahead forecasts estimates.

To support the choice of GARCH(1,1) and in accordance to the literature, the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model has been shown to produce reasonable low and high frequency VaR forecasts across a variety of markets and under different distributional assumptions. Some of the studies concluded that the use of a skewed instead of a symmetrical distribution for the standardized residuals produces superior VaR forecasts. On the other hand, Angelidis & Degiannakis (2007) conclude that the student-t and the skewed Student-t overestimate the true VaR and as a consequence, they implied that other distributions such as the Normal may be more appropriate for the standardized residuals.

Last but not least, it is an undeniable fact that a single return only offers a weak signal about the current level of volatility. To be more precise, the GARCH models are poorly suited for situations where volatility changes rapidly to a new level,

[17] Basel II VaR quantitative requirements include: a) daily-basis estimation, b) confidence level set of 99%, c) one year minimum sample extension with quarterly or more frequent updates, d) no specific models prescribed, for instance, banks are free to adopt their own schemes, e) regular backtesting testing programme for validation purposes. (Basel Committee on Banking Supervision, 2009)

due to a GARCH model is slow at ‘catching up’ and it will take many periods for the conditional variance to reach its new level, as discussed in Andersen, Bollerslev, *et.al.* (2003). For this reason, high-frequency financial data are now widely available by introducing a number of realized measures of volatility; Hansen, Huang, *et.al.* (2011). As a consequence, interested enough is the analysis of the Heterogeneous Autoregressive Realized Volatility, HAR-RV model.

The Heterogeneous Autoregressive Realized Volatility, AR(1)-HAR-RV model encompasses many advantages. First of all, the model retains a structure that enables to the realized volatility estimates to be aggregated at different scales in order to have realized volatility measures of the integrated volatility over different periods: daily, weekly and monthly. This is a strong advantage and the reason is so simple to explain. Typically a financial market is composed by participants having a large spectrum of dealing frequency. On the one side of the dealing spectrum, there are dealers, market makers and intraday speculator, with very high intraday frequency. On the other side, there are central banks, commercial organization and, for example, pension fund investors with their currency hedging. Each such participant has different reaction times to news, related to his time horizon and characteristic dealing frequency. The basic idea is that agents with different time horizons perceive, react and cause different types of volatility components. Simplifying a bit, the model of HAR-RV can easily identify three primary volatility components: the short-term with daily or higher dealing frequency, the medium-term typically made of portfolio manager who rebalance their positions weekly, and the long-term with a characteristic time of one or more months (Corsi F. , 2002). Finally, it is a surprise that although the HAR-RV model does not formally belong to the class of long memory models, it is able to reproduce the same memory persistence observed in volatility as well as many of the other main stylized facts of financial data. For all these reasons, I chose the Heterogeneous Autoregressive Realized Volatility, AR(1)-HAR-RV model, as the second more attractive in order to lead to a conclusion after the comparison with the GARCH(1,1).

To conclude, the aim of this analysis that will be followed in the below chapter is to provide in further detail empirical evidence favoring or not the Realized Volatility of HAR-RV model within high frequency data. In other words, the purpose of this research is to be determined whether the short memory GARCH model is outperformed for forecasting not only at daily basis estimation, but also at multi-period VaR for longer time horizons, such as 10-day and 20-day ahead forecasts, let alone to be investigated the superiority one of these two models.

Chapter 3: Empirical Analysis and Results

3.1) Data Description

In the empirical analysis of this dissertation, I used three types of financial asset classes; these three types are stock indices, commodities and foreign exchange rates. In order to examine the robustness of the forecasts of the selected volatility models, the VaR and ES forecasts are generated using daily logarithmic returns data from 3 stock indices, 3 commodities and finally, 3 exchange rates. The 3 stock indices are the Standard and Poors 500 from USA ($S\&P_{500}$) with 3901 observations, the Europe Stock 50 ($EurostoXX_{50}$) with 3949 observations and the Financial Times Stock Exchange 100 from London stock market ($FTSE_{100}$) with 3912 observations. The 3 commodities are the Copper Commodity of High Quality (HG) with 3897 observations, the Silver Commodity (SV) with 3897 observations, as well as the Gold Commodity (GC) with again 3897 observations. The 3 foreign exchange rates are the Euro Exchange Rate (EC) based on USA Dollar (EUR/USD) with 3898 observations, the British Pound Exchange Rate (BC) based on USA Dollar (GBP/USD) with 3899 observations and finally, the Canadian Dollar Exchange Rate (CD) based on USA Dollar (CAD/USD) with 3899 observations.

Concerning the stock indices, the sample used for this dissertation considers data from major world stock market indices with the longest continuous history. For example, the Standard and Poor's 500, $S\&P_{500}$, is an American stock market index based on the market capitalizations of the 500 largest companies. The $S\&P_{500}$ index components and their weightings are determined by the S&P Dow Jones Indices. It is one of the most commonly followed equity indices, and many consider it one of the best representations of the U.S. stock market, and a bellwether for the U.S. The Financial Times Stock Exchange Index, also called the FTSE 100 Index is a share index of the 100 largest companies listed on the London Stock Exchange with the highest market capitalization. It is seen as a gauge of prosperity for businesses regulated by UK company law. The $EurostoXX_{50}$ is a stock index of Eurozone stocks designed by STOXX, an index provider owned by Deutsche Börse Group and SIX Group. It is made up of fifty of the largest and most liquid stocks. The index futures and options on the $EurostoXX_{50}$ are among the most liquid products in Europe and the world. Moreover, the Copper, Gold and Silver are the most publicly quoted metal commodities. Particularly, gold commodity also tends to act as a safe-haven investment in times of volatility and uncertainty[18].

The data from the nine asset prices cover a range of fifteen years, during the period from 3rd of January, 2000 to 5th of August, 2015 and were conditioned to remove any non-trading days. To avoid outliers that would result from half trading days and diminish the problem of seasonality, I removed days that stock markets were not active for more than six and a half hours between 9:30 a.m. and 4:00 p.m. Furthermore, inactive trading days were excluded when stock markets were closed the whole day, such as weekends and public or local holidays; for instance the day after Thanksgiving and days around Christmas.

Albeit the number of the total log-returns ($\widehat{T}$) of each of the 9 stock indices, commodities and exchange rates of dollar, I utilized in my research the formulation of

[18] See Appendix D: The closing values of indicators; S&P500 and Gold Commodity through the years (p.69).

an out-of-sample log-returns ($\tilde{T}$), based on a rolling sample[19] (T) of 1000 observations. A total of $\tilde{T} = \hat{T} - T$ out-of-sample forecasts were produced for each model, with the parameters of the models re-estimated each trading day. Moreover, the approach used in order to divide the out-of-sample estimation period was the non-overlapping intervals. The out-of-sample observations for each index, commodity or exchange rate, $\tilde{T}$, are divided into $\tilde{T}/\tau$ non-overlapping intervals of observations, with τ observations in each interval. By using different sample periods, we were able to investigate whether the risk management techniques are robust across various time periods and specifically, select a model that is not affected by the chosen sample period. Furthermore, this procedure ensures that the observations of each sample would not repeat and consequently, is necessary to avoid autocorrelation in the forecast errors. Due to the use of non-overlapping intervals, as the forecasting time horizon increases, the number of VaR and ES forecasts produced decreases by a factor equal to the length of the forecast period. As a result, particularly for the 20-step-ahead time horizon, the results of the Kupiec and Christoffersen tests are highly sensitive to the number of VaR violations such that a very small number of additional violations can be pivotal in determining whether or not the forecasting performance of the model is deemed to be adequate; Degiannakis, *et.al.* (2013).

Descriptive statistics for the daily log returns for the selected indices, commodities and exchange rates are presented in the Table 5. The mean is not significantly different from zero and would not make any difference to the outcome. From the elements of the table, all of the returns distributions are leptokurtic, due to the fact that the Kurtosis is a large positive value for all the nine asset prices. This high peak and corresponding fat tails means the distribution is more clustered around the mean than in a mesokurtic or platykurtic distribution, and will have a relatively smaller standard deviation (See Figure 7).

Table 5: Descriptive Statistics for the daily log returns.

Table 5: Descriptive Statistics								
Index	**Obs.**	**Mean**	**Median**	**Std. dev**	**Skewness**	**Kurtosis**	**Jarque-Bera**	**Probability**
Stock Indices								
S&P500	3901	0,021517	0,078196	1,238977	-0,049957	17,79049	26443,65	0,000000
EurostocXX50	3949	0,010520	0,065985	1,537287	-0,094972	10,26722	6493,768	0,000000
FTSE100	3912	0,013624	0,059112	1,299864	-0,125363	16,04128	20643,38	0,000000
Commodities								
HG (Copper COMEX)	3897	0,025732	0,04955	1,958289	-0,191981	6,414826	1425,380	0,000000
SV (Silver COMEX)	3897	0,028808	0,151172	2,221797	-1,041196	9,544504	5693,436	0,000000
GC (Gold COMEX)	3897	0,033317	0,045465	1,237483	-0,359605	8,295233	3447,038	0,000000
Foreign Exchange Rates								
EUR/USD (EC)	3898	-0,005012	0,007898	0,643137	-0,022133	4,655992	331,3706	0,000000
GBP/USD (BP)	3899	-0,005398	0,000000	0,60121	-0,594169	7,621704	2750,703	0,000000
CAD/USD (CD)	3899	-0,000644	0,010132	0,633168	-0,146147	5,666509	869,1815	0,000000

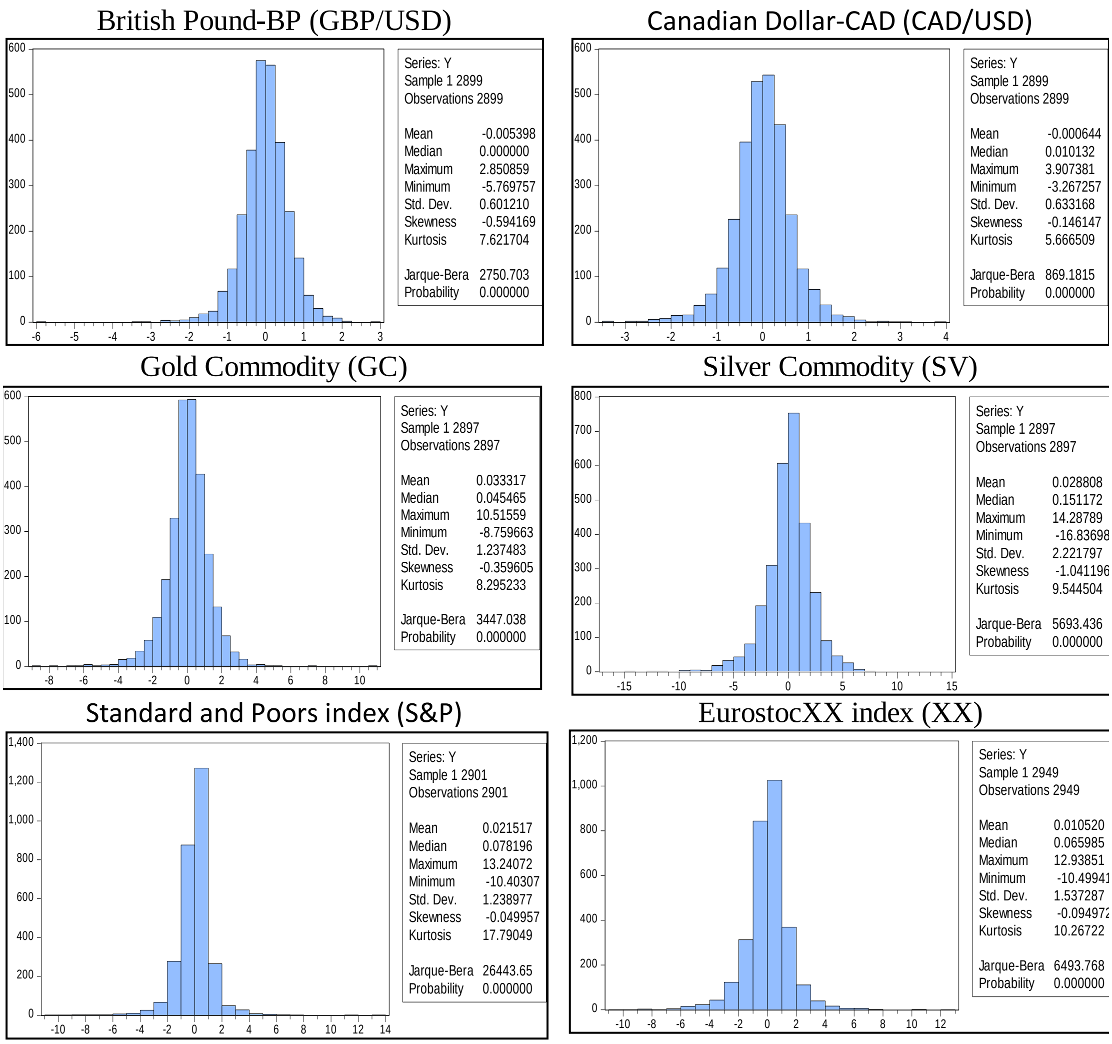
*The last column of Table 5 presents the p-values of the Jarque-Bera test which has as its null hypothesis that the returns series follow a Gaussian distribution.

Moreover, all the indices, commodities and foreign exchange rates of dollar are negatively skewed, as we can see in the Table 5 at the Skewness column. Negative

[19] A rolling forecasts is an add/drop process for predicting the future over a set period of time. It is well-known as FIFO (First In – First Out) method of forecasting. Rolling forecasts are often used in long-term weather predictions, project managements, supply chain management and financial planning. If for example an organization needs to anticipate operating expenses a year in advance, the rolling period would be 12 months. After the 1st month had passed, that month would be dropped from the beginning of the forecast and another month would be added to the end of the forecast.

skewness means that the data points are skewed to the left of the data average. The Jarque-Bera results indicate that none of the log-returns follow a Gaussian distribution, as it is shown in both Table 5 and Figure 7, in which Figure 7 concerns a random representation of the histograms for six out of the nine assets of this analysis. This is illustrated by the fact that the p-values are all zero and as a consequence, this means that the null hypotheses in which the return series follow a Gaussian distribution are rejected for all the asset prices. As far as the autocorrelation is concerned, examining the correlograms for the indices, commodities and exchange rates of dollar, I led to the inference that there is first degree autocorrelation and particularly, negative autocorrelation. The existence of correlation indicates a relationship between two variables in which one variable increases as the other decreases, and vice versa. Hence, the negative correlation means that the relationship that appears to exist between two variables is negative 100% of the time and it is sited at the left axis. Correlograms for the absolute log returns of the 9 asset prices are available upon request from the author.

Figure 7: Histograms of random 6 out of 9 asset prices, indicate the leptokurtic distribution.



In Figure 7, I used indicatively six out of 9 asset prices, in order to present the leptokurtic distribution.

The forecasts for the GARCH(1,1)-skT and AR(1)-HAR-RV-skT have been estimated into the 95% confidence level (α=5%) and as the Basel Committee indicates, into the 99% confidence level (α=1%), as well. All the available data analyzed in the previous paragraphs are presented to the following figures. These figures introduce the daily log returns for each one of the 9 stocks, commodities and exchange rates.

Figure 8: Daily log returns of the S&P$_{500}$

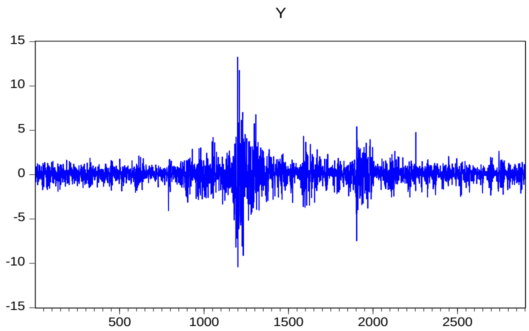


Figure 9: Daily log returns of the EurostoXX$_{50}$

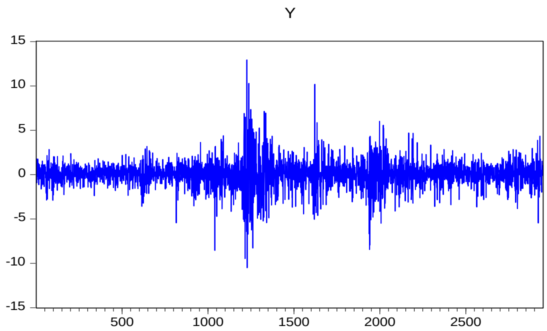


Figure 10: Daily log returns of the $FTSE_{100}$

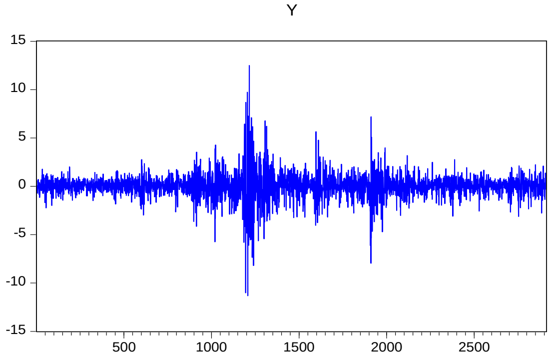


Figure 11: Daily log returns of HG Commodity

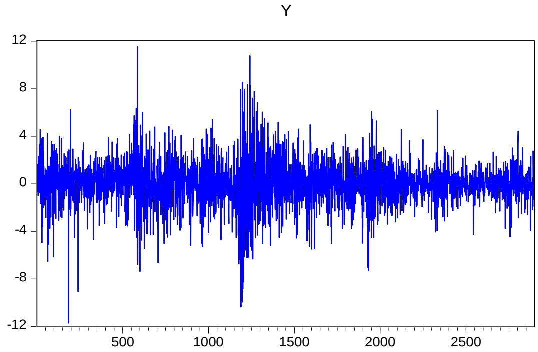


Figure 12: Daily log returns of the GC-Gold

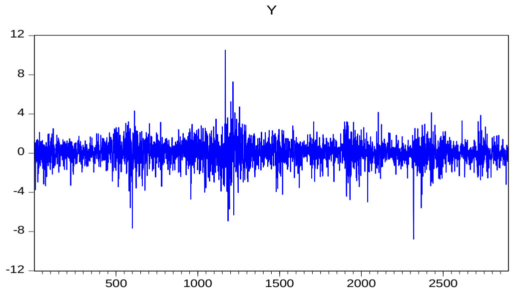


Figure 13: Daily log returns of the SV-Silver

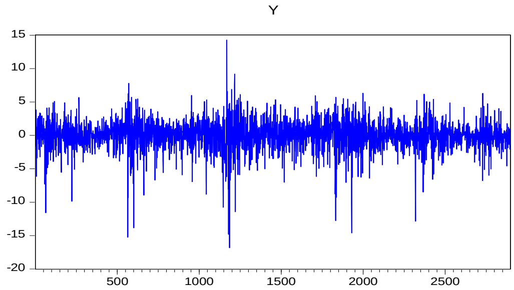


Figure 14: Daily log returns of the EUR/USD

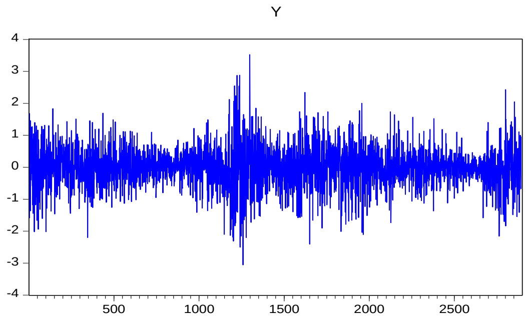


Figure 15: Daily log returns of the GBP/USD

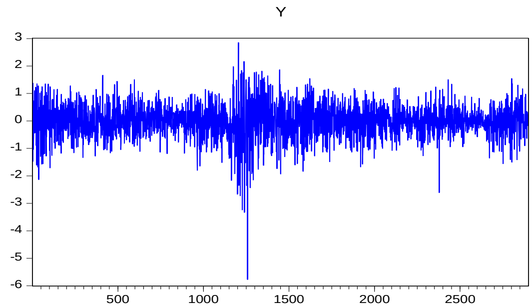
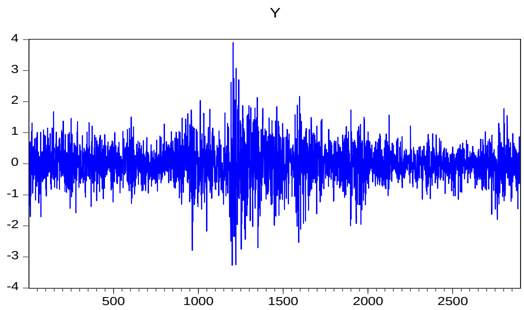


Figure 16: Daily log returns of the CAD/USD



The above figures; from Figure 8 to 16, show the graphs of the series for each one of the nine asset prices. It is clear that in almost all the graphs, there are the same periods of intense volatility clustering. The first cluster of volatility encompasses the observations approximately from 1000 to 1500 observations' period. These observations in accordance to the data set of the analysis fluctuate during the year of 2004. However, another essential but less intense cluster of volatility was that of the years around 2001 to 2002, in which terrorists attacked the World Trade Center in New York and as a result, several financial markets in the United States remained closed at least for a week. Last but not least, another intrigued cluster of volatility concerns the period around 2008 and 2009. These years reflect to the known credit crunch of 2008 and are the observations between 1900 and 2100, at the Figures 8 to 16. Despite all the above, thus the graphs are oscillating around zero, indicating that the series have a constant mean.

3.2) Methodology of AR(1)-GARCH(1,1)-skT model

The empirical success of the Generalized Autoregressive Heteroskedasticity (GARCH) framework, by Engle R. F. (1982) and Bollerslev T. (1986), has been widely spotlighted by many researchers in order to model high-frequency volatility and calculate VaR and ES to select the optimal GARCH specification. Literature provides evidence that among the simple models, the GARCH(1,1) is the most adequate one. As a result, in this section will be described all the methodology used in order to build the model of GARCH(1,1), followed by the skewed Student-t distribution, not only forecasting the one-day-ahead 95% and 99% of Value-at-Risk (VaR) and Expected Shortfall (ES), but also forecasting the ten-day-ahead and 20-day-ahead VaR and ES, as the Basel Committee mandatorily suggests[20]. All the estimations have been done in this dissertation took part for 9 major worldwide assets; 3 stock indices, 3 commodities and 3 exchange rates of dollar, as they analytically described in the previous subsection; that of data description.

The equations used to calculate the one-day-ahead VaR and ES were presented in the first chapter of this dissertation (see eq.14 and eq.15 at pages 10-11) and respectively, the equations used for the multi-period forecasts of 10-step-ahead and 20-step ahead were presented at eq.16 for VaR and at eq.17 for ES (see at page 13). It is also important to illustrate that in order to build the multi-period GARCH(1,1) model, I utilized the new adaptation of the Monte Carlo simulation technique, that firstly introduced by Christoffersen P. F. (2003).

Consequently, in order to calculate multiple VaR and ES for AR(1)-GARCH(1,1)-skT model, I have used a number of steps arising from a new algorithm, as Christoffersen P. F. (1998) had done in his paper and as well as Xekalaki & Degiannakis (2010), presented the Monte Carlo simulation. Furthermore, it is essential to add, at this point, the information that I utilized in my analysis and the density function[21] proposed by Fernandez and Steel (1998)[22]:

$$y_t = \mu_t + \varepsilon_t = c_0(1 - c_1) + c_1 y_{t-1} + \varepsilon_t \tag{46}$$
$$\varepsilon_t = \sigma_t z_t$$
$$\sigma_t^2 = a_0 + a_1 \varepsilon_{t-1}^2 + b_1 \sigma_{t-1}^2$$
$$z_t \sim iid\ skT(0,1;\nu,g)$$

[20] Financial institutions are required by the Basel Committee to calculate the VaR of their positions for at least a 10-day holding period so as to calculate their minimum capital risk requirements (Basel Committee on Banking Supervision, 2009).

[21] See Appendix A, eq. 107 (for the skewed Student-t distribution), p.65.

[22] Note that AR(1) is presented as $y_t = c_0 + e_t,\ e_t = c_1 e_{t-1} + \varepsilon_t,\ thus\ (y_t - c_0) = c_1(y_{t-1} - c_0) + \varepsilon_t$.

$$f(skT)(z_t; g, \nu) = \begin{cases} \frac{2s}{g+g^{-1}} f(g(sz_t + m); \nu) \;\; if \; z_t < -\frac{m}{s} \\ \frac{2s}{g+g^{-1}} f\left(\frac{sz_t+m}{g}; \nu\right) \;\; if \; z_t \geq -\frac{m}{s} \end{cases}$$

where g and ν are the asymmetry and tail parameters of the distribution, $m = \Gamma[(\nu-1)/2]\sqrt{(\nu-2)}\left[\Gamma(\nu/2)\sqrt{\pi}\right]^{-1}(g - g^{-1})$ and $s^2 = (g^2 + g^{-2} - 1) - m^2$ and as a result, $s = \sqrt{g^2 + g^{-2} - m^2 - 1}$.

Based on Xekalaki & Degiannakis (2010) and Christoffersen P. F. (2003), a Monte Carlo simulation algorithm for computing $VaR_{t+\tau}^{95\%}$ *and* $ES_{t+\tau}^{95\%}$ based on Generalized Autoregressive Heteroskedasticity (GARCH) model is presented. Consider the AR(1)-GARCH(1,1)-skT with the above framework (46) and finally $z_t \sim skT(0,1,g,\nu)$ *and* $y_t \sim skT(\mu_t, \sigma_t^2, g, \nu)$, the τ-day-ahead 95% VaR and ES are obtained as following:

One – day – ahead

Step 1: It is required to produce leptokurtic and asymmetrically conditionally distributed log-returns. As a result, at step 1, I define the scheme as follows, so as to create random draws from the skewed Student-t distribution based on Fernandez & Steel (1998) and Lambert, *et.al.* (2002).

- Step 1.1: Forecast the one-day-ahead conditional standard deviation based on the simulated $\breve{\varepsilon}_{t+1}$:

  $$\sigma_{t+1|t} = \sqrt{a_0^{(t)} + a_1^{(t)}\varepsilon_{t|t}^2 + b_1^{(t)}\sigma_{t|t}^2} \qquad (47)$$

- Step 1.2: Generate random numbers, $\{\breve{z}_{i,1}\}_{i=1}^{MC}$ from the skewed Student-t distribution, where MC=5000 denotes the number of draws. The pseudo-random numbers are used to compute the innovations for period t+1 onwards.
- Step 1.3: Create the hypothetical returns of time t+1, as:

  $$\breve{y}_{i,t+1} = \sigma_{t+1|t}\breve{z}_{i,1} + c_0^{(t)}\left(1 - c_1^{(t)}\right) + c_1^{(t)} y_t \;\; for \; i = 1, \dots, MC. \qquad (48)$$

  The return at time t+1 is generated in accordance to the AR(1) progress.
- Step 1.4: Compute the simulated error term

  $$\breve{\varepsilon}_{t+1} = \sigma_{t+1|t}\breve{z}_{i,1}. \qquad (49)$$

- Step 1.5: Calculate the 1-day-ahead 95% and 99% VaR and ES as:

  $$VaR_{t+1|t}^{95\%} = \mu_{t+1|t} + F\left(a; \theta^{(t)}\right)\sigma_{t+1|t} \qquad (50)$$

  $$ES_{t+1|t}^{95\%} = E\left(\breve{y}_{t+1} \middle| \left(\breve{y}_{t+1} \leq VaR_{t+1|t}^{95\%}\right)\right) \qquad (51)$$

  And in the same way, we calculate the 99% VaR and ES: $VaR_{t+1|t}^{99\%}$ *and* $ES_{t+1|t}^{99\%}$.

Two – day – ahead

Step 2: At this step 2, I define the scheme as follows, so as to create random draws from the skewed Student-t distribution.

- Step 2.1: Create the forecast standard deviation of time t+2 based on the simulated $\breve{\varepsilon}_{t+2}$:

  $$\breve{\sigma}_{t+2|t} = \sqrt{a_0^{(t)} + a_1^{(t)}\breve{\varepsilon}_{t+1|t}^2 + b_1^{(t)}\sigma_{t+1|t}^2} \qquad (52)$$

- Step 2.2: Generate random numbers, $\{\breve{z}_{i,2}\}_{i=1}^{MC}$ from the skewed Student-t distribution, where MC=5000 denotes the number of draws. The pseudo-random numbers are used to compute the innovations for period t+2 onwards.
- Step 2.3: Create the hypothetical returns of time t+2, as:
$$\breve{y}_{i,t+2} = \breve{\sigma}_{t+2|t}\breve{z}_{i,2} + c_0^{(t)}\left(1 - c_1^{(t)}\right) + c_1^{(t)}\breve{y}_{i,t+1} \ \ for\ i = 1, \dots, MC. \tag{53}$$
The return at time t+2 is generated in accordance to the AR(1) progress.
- Step 2.4: Compute the simulated error term
$$\breve{\varepsilon}_{t+2} = \breve{\sigma}_{t+2|t}\breve{z}_{i,2}. \tag{54}$$
- Step 2.5: Calculate the 2-days-ahead 95% and 99% VaR and ES as:
$$VaR_{t+2|t}^{95\%} = F_a\left(\{\breve{y}_{i,t+2}\}_{i=1}^{MC}\right) \tag{55}$$
$$ES_{t+2|t}^{95\%} = E\left(\breve{y}_{t+2}\middle|\left(\breve{y}_{t+2} \le VaR_{t+2|t}^{95\%}\right)\right) \tag{56}$$
And in the same way, calculate 99% VaR and ES: $VaR_{t+2|t}^{99\%}\ and\ ES_{t+2|t}^{99\%}$.

Three – day – ahead

Step 3: At this step 3, I define the scheme as follows, so as to create random draws from the skewed Student-t distribution.

- Step 3.1: Create the forecast standard deviation of time t+3 based on the simulated $\breve{\varepsilon}_{t+3}$:
$$\breve{\sigma}_{t+3|t} = \sqrt{a_0^{(t)} + a_1^{(t)}\breve{\varepsilon}_{t+2|t}^2 + b_1^{(t)}\sigma_{t+2|t}^2} \tag{57}$$
- Step 3.2: Generate random numbers, $\{\breve{z}_{i,3}\}_{i=1}^{MC}$ from the skewed Student-t distribution, where MC=5000 denotes the number of draws. The pseudo-random numbers are used to compute the innovations for period t+3 onwards.
- Step 3.3: Create the hypothetical returns of time t+3, as:
$$\breve{y}_{i,t+3} = \breve{\sigma}_{t+3|t}\breve{z}_{i,3} + c_0^{(t)}\left(1 - c_1^{(t)}\right) + c_1^{(t)}\breve{y}_{i,t+2} \ \ for\ i = 1, \dots, MC. \tag{58}$$
The return at time t+3 is generated in accordance to the AR(1) progress.
- Step 3.4: Compute the simulated error term
$$\breve{\varepsilon}_{t+3} = \breve{\sigma}_{t+3|t}\breve{z}_{i,3}. \tag{59}$$
- Step 3.5: Calculate the 3-days-ahead 95% and 99% VaR and ES as:
$$VaR_{t+3|t}^{95\%} = F_a\left(\{\breve{y}_{i,t+3}\}_{i=1}^{MC}\right) \tag{60}$$
$$ES_{t+3|t}^{95\%} = E\left(\breve{y}_{t+3}\middle|\left(\breve{y}_{t+3} \le VaR_{t+3|t}^{95\%}\right)\right) \tag{61}$$
And in the same way, calculate 99% VaR and ES: $VaR_{t+3|t}^{99\%}\ and\ ES_{t+3|t}^{99\%}$.

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Ten-day-ahead

Step 10: At this step 10, I define the scheme as follows, so as to create random draws from the skewed Student-t distribution.

- Step 10.1: Create the forecast standard deviation of time t+10 based on the simulated $\breve{\varepsilon}_{t+10}$:
$$\breve{\sigma}_{t+10|t} = \sqrt{a_0^{(t)} + a_1^{(t)}\breve{\varepsilon}_{t+9|t}^2 + b_1^{(t)}\sigma_{t+9|t}^2} \tag{62}$$
- Step 10.2: Generate random numbers, $\{\breve{z}_{i,10}\}_{i=1}^{MC}$ from the skewed Student-t distribution, where MC=5000 denotes the number of draws. The pseudo-

random numbers are used to compute the innovations for period t+10 onwards.

- Step 10.3: Create the hypothetical returns of time t+10, as:
$$\breve{y}_{i,t+10} = \breve{\sigma}_{t+10|t}\breve{z}_{i,10} + c_0^{(t)}\left(1 - c_1^{(t)}\right) + c_1^{(t)}\breve{y}_{i,t+9} \ for\ i = 1, \ldots, MC. \quad (63)$$
The return at time t+10 is generated in accordance to the AR(1) progress.
- Step 10.4: Compute the simulated error term
$$\breve{\varepsilon}_{t+10} = \breve{\sigma}_{t+10|t}\breve{z}_{i,10}. \quad (64)$$
- Step 10.5: Calculate the 10-days-ahead 95% and 99% VaR and ES as:
$$VaR_{t+10|t}^{95\%} = F_a\left(\{\breve{y}_{i,t+10}\}_{i=1}^{MC}\right) \quad (65)$$
$$ES_{t+10|t}^{95\%} = E\left(\breve{y}_{t+10}\middle|\left(\breve{y}_{t+10} \le VaR_{t+10|t}^{95\%}\right)\right) \quad (66)$$
And in the same way, calculate 99% VaR and ES: $VaR_{t+10|t}^{99\%}\ and\ ES_{t+10|t}^{99\%}$.

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Twenty-day-ahead

Step 20: At this step 20, I define the scheme as follows, so as to create random draws from the skewed Student-t distribution.

- Step 20.1: Create the forecast standard deviation of time t+20 based on the simulated $\breve{\varepsilon}_{t+20}$:
$$\breve{\sigma}_{t+20|t} = \sqrt{a_0^{(t)} + a_1^{(t)}\breve{\varepsilon}_{t+19|t}^2 + b_1^{(t)}\sigma_{t+19|t}^2} \quad (67)$$
- Step 20.2: Generate random numbers, $\{\breve{z}_{i,20}\}_{i=1}^{MC}$ from the skewed Student-t distribution, where MC=5000 denotes the number of draws. The pseudo-random numbers are used to compute the innovations for period t+20 onwards.
- Step 20.3: Create the hypothetical returns of time t+20, as:
$$\breve{y}_{i,t+20} = \breve{\sigma}_{t+20|t}\breve{z}_{i,20} + c_0^{(t)}\left(1 - c_1^{(t)}\right) + c_1^{(t)}\breve{y}_{i,t+19} \ for\ i = 1, \ldots, MC. \quad (68)$$
The return at time t+20 is generated in accordance to the AR(1) progress.
- Step 20.4: Compute the simulated error term
$$\breve{\varepsilon}_{t+20} = \breve{\sigma}_{t+20|t}\breve{z}_{i,20}. \quad (69)$$
- Step 20.5: Calculate the 20-days-ahead 95% and 99% VaR and ES as:
$$VaR_{t+20|t}^{95\%} = F_a\left(\{\breve{y}_{i,t+20}\}_{i=1}^{MC}\right) \quad (70)$$
$$ES_{t+20|t}^{95\%} = E\left(\breve{y}_{t+20}\middle|\left(\breve{y}_{t+20} \le VaR_{t+20|t}^{95\%}\right)\right) \quad (71)$$
And in the same way, calculate 99% VaR and ES: $VaR_{t+20|t}^{99\%}\ and\ ES_{t+20|t}^{99\%}$.

τ-day-ahead

(A General Approximation for multi-period forecasting)

Step τ: This step is used for obtaining estimates for multi-period VaR and ES, and especially for τ-days-ahead forecasts. We also create random draws from the skewed Student-t distribution based on Fernandez & Steel (1998); Lambert, *et.al.* (2002); Degiannakis, *et.al.* (2012); Christoffersen P. (1998).

- Step τ.1: Create the forecast standard deviation of time t+τ based on the simulated $\breve{\varepsilon}_{t+\tau}$:

$$\breve{\sigma}_{t+\tau|t} = \sqrt{a_0^{(t)} + a_1^{(t)} \breve{\varepsilon}_{t+\tau-1|t}^2 + b_1^{(t)} \sigma_{t+\tau-1|t}^2} \tag{72}$$

- Step τ.2: Generate random numbers, $\{\breve{z}_{i,\tau}\}_{i=1}^{MC}$ from the skewed Student-t distribution, where MC=5000 denotes the number of draws. The pseudo-random numbers are used to compute the innovations for period t+τ onwards.
- Step τ.3: Create the hypothetical returns of time t+τ, as:
  $$\breve{y}_{i,t+\tau} = \breve{\sigma}_{t+\tau|t} \breve{z}_{i,\tau} + c_0^{(t)}\left(1 - c_1^{(t)}\right) + c_1^{(t)} \breve{y}_{i,t+\tau-1} \ for\ i = 1, \dots, MC. \tag{73}$$
  The return at time t+τ is generated in accordance to the AR(1) progress.
- Step 20.4: Compute the simulated error term
  $$\breve{\varepsilon}_{t+\tau} = \breve{\sigma}_{t+\tau|t} \breve{z}_{i,\tau}. \tag{74}$$
- Step 20.5: Calculate the τ-days-ahead 95% and 99% VaR and ES as:
  $$VaR_{t+\tau|t}^{95\%} = F_a\left(\{\breve{y}_{i,t+\tau}\}_{i=1}^{MC}\right) \tag{75}$$
  $$ES_{t+\tau|t}^{95\%} = E\left(\breve{y}_{t+\tau} \middle| \left(\breve{y}_{t+\tau} \leq VaR_{t+\tau|t}^{95\%}\right)\right) \tag{76}$$
  And in the same way, calculate 99% VaR and ES: $VaR_{t+\tau|t}^{99\%}\ and\ ES_{t+\tau|t}^{99\%}$.

3.3) Methodology of AR(1)-HAR-RV-skT model

The present section introduces the realized volatility model and forecasting the time series behavior of volatility, which is able to reproduce the memory persistence observed in the data. The realized volatility generates an additive cascade of different volatility components, depending on the actions of different types of market participants. This additive volatility cascade leads to a simple AR-type model in the realized volatility with the feature of considering volatilities realized over different time horizons. The basic idea is that the market participants have a different perspective of their investment horizons. The name of this innovative model is Heterogeneous Autoregressive model of the Realized Volatility (HAR-RV), by Corsi F. (2004); Andersen, *et.al.* (2005). Surprisingly, in spite of the fact that it does not formally belong to the class of long memory models, the HAR-RV model is able to reproduce the same memory persistence observed in volatility as well as many of the other main stylized facts of financial data.

As a consequence, in this section will be described all the methodology used in order to build the model of AR(1)-HAR-RV, followed by the skewed Student-t distribution, and forecasting the intra-day data for 95% and 99% Value-at-Risk (VaR) and Expected Shortfall (ES), as the Basel Committee imposed for the last one. A strong advantage of the HAR-RV model is the fact that the realized volatility estimates are aggregated at different scales in order to have realized volatility measures of the integrated volatility over different periods: daily, weekly and monthly. Additionally, in the analysis of the second model, it has been used the logarithmic transformation of the annualized realized volatility, $\sqrt{252\sigma_t^{2(RV)}}$.

The HAR-RV model for the logarithmic transformation of the annualized realized volatility $\sqrt{252\sigma_t^{2(RV)}}$, is defined as:

$$\log\sqrt{252\sigma_t^{2(RV)}} = w_0 + w_1 \log\sqrt{252\sigma_{t-1}^{2(RV)}} +$$
$$w_2 \log\sqrt{252\sigma_{t-5:t-1}^{2(RV)}} + w_3 \log\sqrt{252\sigma_{t-22:t-1}^{2(RV)}} + u_t, \quad (77)$$
where $u_t \sim i.i.d\ N(0,1)$.

The AR(1)-HAR-RV-skT model is defined as an AR(1) process for the daily log-returns, $y_t = c_0^t(1 - c_1^t) + c_1^t y_{t-1} + \varepsilon_t$.

The unpredictable component ε_t, is designed to follow the skewed Student-*t* distribution (see also Appendix A, eq. 107, p.65) conditional on the most recently available information set, or $\varepsilon_t = \left(y_t - c_0^{(t)} c_1^{(t)} y_{t-1}\right)|I_{t-1} \sim skT(0,1; g, v)$. Moreover, the unpredictable component is decomposed as $\varepsilon_t = z_t \sigma_t^{(RV)}$.

The daily volatility is estimated by a HAR model for the $\log\sqrt{252\sigma_t^{2(RV)}}$, being the dependent variable:

$$y_t = c_0^t(1 - c_1^t) + c_1^t y_{t-1} + z_t \sigma_t^{(RV)} \quad (78)$$

The prediction of the RV volatility has been acquired using the following approximation:

$$\sigma_t^{2(RV)} = exp\left(\left(\widehat{w}_0 + \widehat{w}_1 \log\sqrt{252\sigma_{t-1}^{2(RV)}} + \widehat{w}_2 \log\sqrt{252\sigma_{t-5:t-1}^{2(RV)}} + \widehat{w}_3 \log\sqrt{252\sigma_{t-22:t-1}^{2(RV)}}\right)^2 / 252\right)$$

and $z_t \sim skT(0,1; g, v)$. (79)

Based on Xekalaki & Degiannakis (2010) and Christoffersen (2003), a Monte Carlo simulation algorithm for computing $VaR_{t+\tau}^{95\%}$ and $ES_{t+\tau}^{95\%}$ based on Heterogeneous Autoregressive model of the Realized Volatility (HAR-RV) model is illustrated. Consider the AR(1)-HAR-RV-skT with the above framework of equations 71-73, and finally $z_t \sim skT(0,1, g, v)$ and $y_t \sim skT(\mu_t, \sigma_t^2, g, v)$, the τ-day-ahead 95% VaR and ES are obtained as following:

One – day – ahead

Step 1: In this step, I computed the one-day-ahead HAR - Realized Volatility.

- Step 1.1: Compute the one-day-ahead realized volatility:

$$\sigma_{t+1|t}^{2(RV)} = exp\left(\left(\widehat{w}_0 + \widehat{w}_1 \log\sqrt{252\sigma_t^{2(RV)}} + \widehat{w}_2 \log\sqrt{252\sigma_{t-4:t}^{2(RV)}} + \widehat{w}_3 \log\sqrt{252\sigma_{t-21:t}^{2(RV)}}\right)^2 / 252\right) \quad (80)$$

  Note that $\log\sqrt{252\sigma_{t-4:t}^{2(RV)}}$ denotes the average of i) actual values for points in time prior to *t* and ii) predicted values for points in time subsequent time *t*. The same case holds for $\log\sqrt{252\sigma_{t-21:t}^{2(RV)}}$.

- Step 1.2: Generate MC=5000 random numbers, $\{\breve{z}_{i,1}\}_{i=1}^{MC}$, from the skewed Student-t distribution, to be used to simulate the innovations for period t+1 onwards.
- Step 1.3: Create the hypothetical returns of time t+1, as:
$$\breve{y}_{i,t+1} = \sigma_{t+1|t}^{(RV)} \breve{z}_{i,1} + c_0^{(t)}\left(1 - c_1^{(t)}\right) + c_1^{(t)} \breve{y}_{i,t} \ \ for\ i = 1, \dots, MC. \quad (81)$$
The return at time t+1 is generated in accordance to the AR(1) progress. The value of the unpredictable component is $\breve{\varepsilon}_{t+1} = \sigma_{t+1|t}^{(RV)}\, \breve{z}_{i,1}$.
- Step 1.4: Calculate the 1-day-ahead 95% and 99% VaR and ES as following:
$$VaR_{t+1|t}^{95\%} = \mu_{t+1|t} + F(a; \theta^t)\left(\{\breve{y}_{i,t+1}\}_{i=1}^{MC}\right) \quad (82)$$
$$ES_{t+1|t}^{95\%} = \tilde{k}^{-1} \sum_{i=1}^{\tilde{k}} \left(VaR_{t+1|t}^{\left(1-0.05+i0.05\left(\tilde{k}+1\right)^{-1}\right)} \right) \quad (83)$$
And in the same way, calculate 99% VaR and ES: $VaR_{t+1|t}^{99\%}\ and\ ES_{t+1|t}^{99\%}$.

Two – day – ahead

Step 2: These steps are used for obtaining estimates for multi-period VaR and ES, and especially for 2-day-ahead forecasts. We also create random draws from the skewed Student-t distribution based on Fernandez & Steel (1998); Lambert, *et.al.* (2002); Degiannakis *et.al.* (2012); Christoffersen P. (1998); Clements, *et.al.* (2006).

- Step 2.1: Compute the two-day-ahead realized volatility:
$$\sigma_{t+2|t}^{2(RV)} = exp\left(\left(\widehat{w}_0 + \widehat{w}_1 \log\sqrt{252\sigma_{t+1}^{2(RV)}} + \widehat{w}_2 \log\sqrt{252\sigma_{t-3:t+1}^{2(RV)}} + \widehat{w}_3 \log\sqrt{252\sigma_{t-20:t+1}^{2(RV)}} \right)^2 /252 \right) \quad (84)$$
- Step 2.2: Generate MC=5000 random numbers, $\{\breve{z}_{i,2}\}_{i=1}^{MC}$, from the skewed Student-t distribution, to be used to simulate the innovations for period t+2 onwards.
- Step 2.3: Create the hypothetical returns of time t+2, as:
$$\breve{y}_{i,t+2} = \sigma_{t+2|t}^{(RV)} \breve{z}_{i,2} + c_0^{(t)}\left(1 - c_1^{(t)}\right) + c_1^{(t)} \breve{y}_{i,t+1} \ \ for\ i = 1, \dots, MC \quad (85)$$
The return at time t+2 is generated in accordance to the AR(1) progress. The value of the unpredictable component is $\breve{\varepsilon}_{t+2} = \sigma_{t+2|t}^{2(RV)}\, \breve{z}_{i,2}$.
- Step 2.4: Calculate the 2-day-ahead 95% and 99% VaR and ES as following:
$$VaR_{t+2|t}^{95\%} = \mu_{t+2|t} + F(a; \theta^t)\left(\{\breve{y}_{i,t+2}\}_{i=1}^{MC}\right) \quad (86)$$
$$ES_{t+2|t}^{95\%} = \tilde{k}^{-1} \sum_{i=1}^{\tilde{k}} \left(VaR_{t+2|t}^{\left(1-0.05+i0.05\left(\tilde{k}+1\right)^{-1}\right)} \right) \quad (87)$$
And in the same way, calculate 99% VaR and ES: $VaR_{t+2|t}^{99\%}\ and\ ES_{t+2|t}^{99\%}$.

Three – day – ahead

Step .3: These steps are used for obtaining estimates for multi-period VaR and ES, and especially for 3-day-ahead forecasts. We also create random draws from the skewed Student-t distribution.

- Step 3.1: Compute the three-day-ahead realized volatility:

$$\sigma_{t+3|t}^{2(RV)} = exp\left(\left(\widehat{w}_0 + \widehat{w}_1 \log\sqrt{252\sigma_{t+2}^{2(RV)}} + \widehat{w}_2 \log\sqrt{252\sigma_{t-2:t+2}^{2(RV)}} + \widehat{w}_3 \log\sqrt{252\sigma_{t-19:t+2}^{2(RV)}}\right)^2 /252\right) \tag{88}$$

- Step 3.2: Generate MC=5000 random numbers, $\{\breve{z}_{i,3}\}_{i=1}^{MC}$, from the skewed Student-t distribution, to be used to simulate the innovations for period t+3 onwards.
- Step 3.3: Create the hypothetical returns of time t+3, as:
  $$\breve{y}_{i,t+3} = \sigma_{t+3|t}^{(RV)}\breve{z}_{i,3} + c_0^{(t)}\left(1 - c_1^{(t)}\right) + c_1^{(t)}\breve{y}_{i,t+2} \; for\; i = 1, \dots, MC. \tag{89}$$
  The return at time t+3 is generated in accordance to the AR(1) progress. The value of the unpredictable component is $\breve{\varepsilon}_{t+3} = \sigma_{t+3|t}^{(RV)}\,\breve{z}_{i,3}$.
- Step 3.4: Calculate the 3-day-ahead 95% and 99% VaR and ES as following:
  $$VaR_{t+3|t}^{95\%} = \mu_{t+3|t} + F(a;\theta^t)\left(\{\breve{y}_{i,t+3}\}_{i=1}^{MC}\right) \tag{90}$$
  $$ES_{t+3|t}^{95\%} = \tilde{k}^{-1}\sum_{i=1}^{\tilde{k}}\left(VaR_{t+3|t}^{\left(1-0.05+i0.05(\tilde{k}+1)^{-1}\right)}\right) \tag{91}$$
  And in the same way, calculate 99% VaR and ES: $VaR_{t+3|t}^{99\%}\; and\; ES_{t+3|t}^{99\%}$.

…

Ten – day – ahead

Step 10: This step is used for obtaining estimates for multi-period VaR and ES, and especially for 10-day-ahead forecasts. We also create random draws from the skewed Student-t distribution.

- Step 10.1: Compute the ten-day-ahead realized volatility:
  $$\sigma_{t+10|t}^{2(RV)} = exp\left(\left(\widehat{w}_0 + \widehat{w}_1 \log\sqrt{252\sigma_{t+9}^{2(RV)}} + \widehat{w}_2 \log\sqrt{252\sigma_{t+5:t+9}^{2(RV)}} + \widehat{w}_3 \log\sqrt{252\sigma_{t-12:t+9}^{2(RV)}}\right)^2 /252\right) \tag{92}$$
- Step 10.2: Generate MC=5000 random numbers, $\{\breve{z}_{i,10}\}_{i=1}^{MC}$, from the skewed Student-t distribution, to be used to simulate the innovations for period t+10 onwards.
- Step 10.3: Create the hypothetical returns of time t+10, as:
  $$\breve{y}_{i,t+10} = \sigma_{t+10|t}^{(RV)}\breve{z}_{i,10} + c_0^{(t)}\left(1 - c_1^{(t)}\right) + c_1^{(t)}\breve{y}_{i,t+9} \; for\; i = 1, \dots, MC. \tag{93}$$
  The return at time t+10 is generated in accordance to the AR(1) progress. The value of the unpredictable component is $\breve{\varepsilon}_{t+10} = \sigma_{t+10|t}^{(RV)}\,\breve{z}_{i,10}$.
- Step 10.4: Calculate the 10-day-ahead 95% and 99% VaR and ES as following:
  $$VaR_{t+10|t}^{95\%} = \mu_{t+10|t} + F(a;\theta^t)\left(\{\breve{y}_{i,t+10}\}_{i=1}^{MC}\right) \tag{94}$$
  $$ES_{t+10|t}^{95\%} = \tilde{k}^{-1}\sum_{i=1}^{\tilde{k}}\left(VaR_{t+10|t}^{\left(1-0.05+i0.05(\tilde{k}+1)^{-1}\right)}\right) \tag{95}$$
  And in the same way, calculate 99% VaR and ES: $VaR_{t+10|t}^{99\%}\; and\; ES_{t+10|t}^{99\%}$.

…

Twenty – day – ahead

Step 20: These steps are used for obtaining estimates for multi-period VaR and ES, and especially for 20-day-ahead forecasts. We also create random draws from the skewed Student-t distribution.

- Step 20.1: Compute the twenty-day-ahead realized volatility:
$$\sigma_{t+20|t}^{2(RV)} = exp\left(\left(\widehat{w}_0 + \widehat{w}_1 \log\sqrt{252\sigma_{t+19}^{2(RV)}} + \widehat{w}_2 \log\sqrt{252\sigma_{t+15:t+19}^{2(RV)}} + \widehat{w}_3 \log\sqrt{252\sigma_{t-2:t+19}^{2(RV)}}\right)^2 /252\right) \quad (96)$$
- Step 20.2: Generate MC=5000 random numbers, $\{\breve{z}_{i,20}\}_{i=1}^{MC}$, from the skewed Student-t distribution, to be used to simulate the innovations for period t+20 onwards.
- Step 20.3: Create the hypothetical returns of time t+20, as:
$$\breve{y}_{i,t+20} = \sigma_{t+20|t}^{(RV)} \breve{z}_{i,20} + c_0^{(t)}\left(1 - c_1^{(t)}\right) + c_1^{(t)} \breve{y}_{i,t+19} \; for\; i = 1, \dots, MC. \quad (97)$$
The return at time t+20 is generated in accordance to the AR(1) progress. The value of the unpredictable component is $\breve{\varepsilon}_{t+20} = \sigma_{t+20|t}^{(RV)} \breve{z}_{i,20}$.
- Step 20.4: Calculate the 20-day-ahead 95% and 99% VaR and ES as following:
$$VaR_{t+20|t}^{95\%} = \mu_{t+20|t} + F(a;\theta^t)\left(\{\breve{y}_{i,t+20}\}_{i=1}^{MC}\right) \quad (98)$$
$$ES_{t+20|t}^{95\%} = \tilde{k}^{-1} \sum_{i=1}^{\tilde{k}} \left(VaR_{t+20|t}^{\left(1-0.05+i0.05(\tilde{k}+1)^{-1}\right)}\right) \quad (99)$$
And in the same way, calculate 99% VaR and ES: $VaR_{t+20|t}^{99\%}\; and\; ES_{t+20|t}^{99\%}$

τ – day – ahead

(A General Approximation for multi-period forecasting)

Step τ: These steps are used for obtaining estimates for multi-period VaR and ES, and especially for *τ*-day-ahead forecasts. We also create random draws from the skewed Student-t distribution based on Fernandez & Steel (1998); Lambert, *et.al.* (2002); Degiannakis *et.al.* (2012); Christoffersen P. (1998); Clements, *et.al.* (2006).

- Step τ.1: Compute the τ-day-ahead realized volatility:
$$\sigma_{t+\tau|t}^{2(RV)} = exp\left(\left(\widehat{w}_0 + \widehat{w}_1 \log\sqrt{252\sigma_{t+\tau-1|t}^{2(RV)}} + \widehat{w}_2 \log\sqrt{252\sigma_{t+\tau-5:t+\tau-1}^{2(RV)}} + \widehat{w}_3 \log\sqrt{252\sigma_{t+\tau-22:t+\tau-1}^{2(RV)}}\right)^2 /252\right) \quad (100)$$
- Step τ.2: Generate MC=5000 random numbers, $\{\breve{z}_{i,\tau}\}_{i=1}^{MC}$, from the skewed Student-t distribution, to be used to simulate the innovations for period t+τ onwards.
- Step τ.3: Create the hypothetical returns of time t+τ, as:
$$\breve{y}_{i,t+\tau} = \sigma_{t+\tau|t}^{(RV)} \breve{z}_{i,\tau} + c_0^{(t)}\left(1 - c_1^{(t)}\right) + c_1^{(t)} \breve{y}_{i,t+\tau-1} \; for\; i = 1, \dots, MC. \quad (101)$$
The return at time t+τ is generated in accordance to the AR(1) progress. The value of the unpredictable component is $\breve{\varepsilon}_{t+\tau} = \sigma_{t+\tau|t}^{(RV)} \breve{z}_{i,\tau}$.
- Step τ.4: Calculate the τ-days-ahead 95% and 99% VaR and ES as following:

$$VaR^{95\%}_{t+\tau|t} = \mu_{t+\tau|t} + F(a;\theta^{t})\left(\{\breve{y}_{i,t+\tau}\}_{i=1}^{MC}\right) \quad (102)$$

$$ES^{95\%}_{t+\tau|t} = \tilde{k}^{-1}\sum_{i=1}^{\tilde{k}}\left(VaR_{t+\tau|t}^{\left(1-0.05+i0.05\left(\tilde{k}+1\right)^{-1}\right)}\right) \quad (103)$$

And in the same way, calculate 99% VaR and ES: $VaR^{99\%}_{t+\tau|t}$ and $ES^{99\%}_{t+\tau|t}$.

3.4) Empirical Results; VaR, ES and the accuracy of the two models

The results for one-step-ahead $VaR^{95\%}_{t+1}$ and ES forecasting are presented in Table 6, across the nine assets; 3 stock indices, 3 metal-COMEX (metal commodity exchange) and 3 FOREX of Dollar (foreign exchange rates). The results of one-step-ahead 95% do not only include the average values of VaR and ES, but also present the backtesting tests of Kupiec and Christoffersen and the Mean Squared Errors (MSE) for ES, as well. The models examined in Table 6 are the following two; the AR(1)-GARCH(1,1)-skT and the AR(1)-HAR-RV-skT models.

Overall, the Generalized Autoregressive model of conditional volatility seems not to improve at all the forecasting accuracy of VaR, across the nine assets for the one-step-ahead time horizon. To begin with, the first part of the analysis of Table 6 will be focused on the VaR analysis of AR(1)-GARCH(1,1)-skT model. The results appear to corroborate the findings of other researchers from the literature; Angelidis, *et.al.* (2004); McMillan & Kambouroudis (2009), and particularly, the results indicate that VaR models are not much robust across different markets. As a consequence, the optimal model varies from one index to another. For instance, at the category of Stock indices the optimal model differs from the optimal of FOREX category, the same as well as happens for the COMEX, respectively. Now according to the results of the (Kupiec, 1995) backtesting procedure, the observed violation rate is not statistically equal to the expected violation rate (5%) in more than the half cases, as indicate the red color of the Kupiec p-values column. In more details, this column of Table 6 presents the rejection or not of the null hypothesis; if the exception rate is statistically equal to the expected rate. The null hypothesis denotes that the model is adequate, but when the null hypothesis rejected, the observed violation rate will be smaller than the expected one. As a result, for the GARCH model, the most accurate category is FOREX (the foreign exchange rate) indices, which all p-values of EURO, British Pound and CAD, are all higher than the 5% confidence level (α=0,05). Contrary to the previous model (GARCH) of conditional volatility, the HAR model for 95% of one-day-ahead forecasting seems to be better, due to the p-values of Kupiec test which are higher than the expected 0.05 value, except for $EurostoXX_{50}$, $FTSE_{100}$ from stocks and Silver (SV) from COMEX market, where these values were rejected.

A second criterion to check the accuracy of one-step-ahead GARCH and HAR-RV model is the Christoffersen test, a more elaborate criterion in which Christoffersen (1998) combined the Kupiec's former criterion. Practically, conditional test examined concurrently the total number of failures by checking if is equal to the expected number and the VaR failure process if it is independently distributed or not. As we see again in Table 6, all the independence p-values of Christoffersen test are correct for the two models as well, since no one rejected for α=0.05. Furthermore, it is important to mention that the percentage of observed exception rate is lower than 5% for all the nine assets (Stocks, COMEX, and FOREX) at the GARCH model. This is a good point because it means that GARCH model does not underestimate the true VaR figure. Additionally, from the column of observed exception rate, we can easily determine that the most suitable category is that of FOREX market (foreign exchange rate), due to the fact that the percentage of violations are quite close to the 5% confidence level.

Table 6: One-step-ahead $VaR^{95\%}$ and ES modeling results.

Table 6: 1-Step-ahead VaR and ES Modeling Results (95%)							
Part A. GARCH-skT							
Index	**Number of 1-step-ahead VaR forecasts**	**Average VaR**	**Average ES**	**Average MSE**	**Observed Exception Rate**	**Kupiec p-value**	**Independence Christoffersen**
Stock Indices	**GARCH-skT**						
S&P500	2901	-2,079061	-2,916988	**0,018416**	4,03%	0,013542	0,894239
EurostocXX50	2949	-2,925082	-4,216117	0,027287	3,29%	0,000006	0,144379
FTSE100	2912	-2,087907	-2,852440	0,024185	4,02%	0,011929	0,729514
Commodities	**GARCH-skT**						
HG (Copper COMEX)	2897	-3,376016	-4,649356	0,069705	3,52%	0,000117	0,826045
SV (Silver COMEX)	2897	-4,699689	-6,947506	0,115868	2,59%	0,000000	0,181849
GC (Gold COMEX)	2897	-2,377837	-3,381763	**0,027238**	3,14%	0,000001	0,932133
Foreign Exchange Rates	**GARCH-skT**						
EUR/USD (EC)	2898	-1,093904	-1,436670	**0,003504**	**4,42%**	0,142009	0,172338
GBP/USD (BP)	2899	-1,011634	-1,322828	**0,006960**	**4,52%**	0,227191	0,684228
CAD/USD (CD)	2899	-1,019235	-1,323052	**0,003341**	**4,66%**	0,391234	0,490631
Part B. AR(1)-HAR-RV-skT							
Index	**Number of 1-step-ahead VaR forecasts**	**Average VaR**	**Average ES**	**Average MSE**	**Observed Exception Rate**	**Kupiec p-value**	**Independence Christoffersen**
Stock Indices	**AR(1)-HAR-RV-skT**						
S&P500	2901	-1,921296	-2,695686	**0,015017**	**4,45%**	0,163817	0,185135
EurostocXX50	2949	-2,646304	-3,809621	0,035701	3,93%	0,005852	0,268402
FTSE100	2912	-1,968041	-2,688660	0,023128	3,91%	0,005294	0,470672
Commodities	**AR(1)-HAR-RV**						
HG (Copper COMEX)	2897	-3,157860	-4,352708	0,090732	**4,76%**	0,553438	0,344276
SV (Silver COMEX)	2897	-4,138996	-6,118718	0,143401	3,21%	0,000002	0,110139
GC (Gold COMEX)	2897	-2,153282	-3,058488	**0,039716**	4,31%	0,083441	0,788992
Foreign Exchange Rates	**AR(1)-HAR-RV**						
EUR/USD (EC)	2898	-1,011098	-1,327306	**0,003826**	5,18%	0,665520	0,648122
GBP/USD (BP)	2899	-0,961006	-1,256177	**0,006487**	**5,17%**	0,668640	0,769083
CAD/USD (CD)	2899	-1,009675	-1,310138	**0,003412**	4,86%	0,735307	0,123485

*The red color indicates rejection of the null hypothesis that the backtesting criterion is accurate; taking into consideration that red value is smaller than 5% significance level.

*The bold fond of MSE column denotes the lowest value for 5% significance level.

*The bold fond of observed exception rate column denotes the most suitable value around 5% significance level.

The best among the three exchange rates seems to be the Canadian Dollar (CAD) with the smallest deviation of 0.34% units for the GARCH model, as comparing the total number of violations (4.66%) to the expected number of 5%. However, as we see at the graph of CAD (Figure 17), there is not total coverage of Value-at-Risk, using GARCH(1,1) because the red line of VaR does not catch all the negative peaks, representing the failures of $VaR^{95\%}$ estimation. On the other hand, the observed exception rates of HAR-RV model are quite satisfactory mainly for the FOREX category, since all the three exchange rates are quite similar to 5%, not to mention that EUR/USD (5,18%) and GBP/USD (5.17%), are the FOREX assets with the smallest deviation from the expected rate (Figure 18). Despite the fact that they are higher than 5% and as a result, this indicates a kind of underestimation of the true VaR figure, the failure of the forecasts limited just to 0.18 and 0.17 units, respectively, a really small difference. As a consequence, HAR model forecasts well enough for FOREX category, as well.

Figure 17: Graph of daily log-CAD/USD FOREX and one-step-ahead $VaR^{95\%}$ of GARCH model.

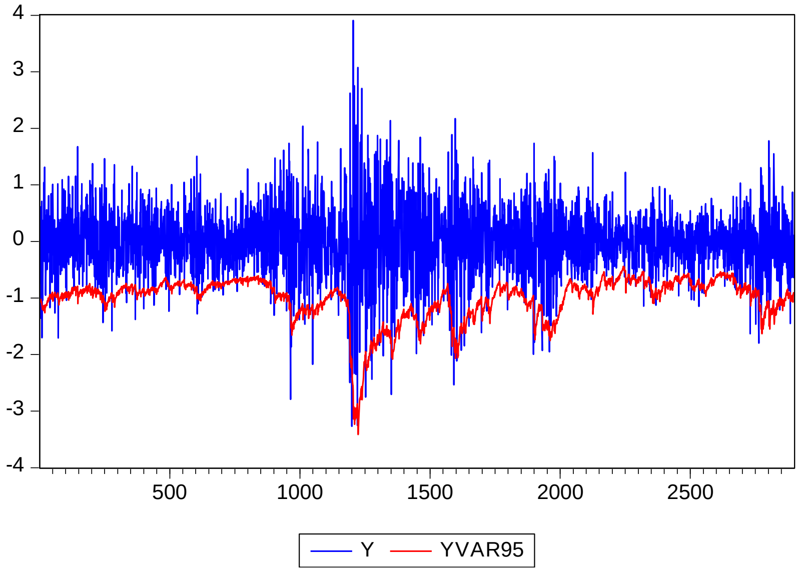
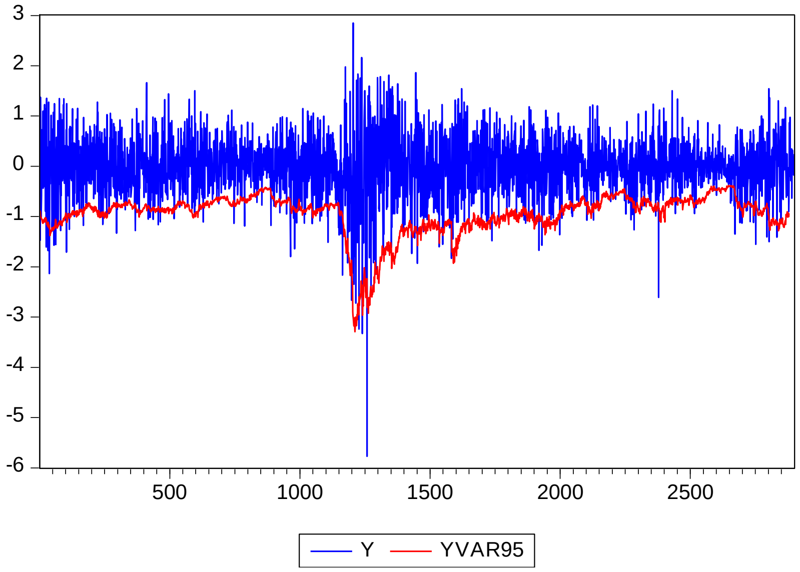


Figure 18: Graph of daily log-GBP/USD FOREX and one-step-ahead $VaR^{95\%}$ of HAR-RV model.



*The other graphs of daily log-returns are all available from the author on request.

The second part of the analysis concerns the ES measure that reports to the risk managers the expected loss of their investments, if a violation, or in other words, an extreme event occurs. At Table 6 there are the average ES of both GARCH and HAR-RV models. As we can see in Figure 17 an interesting period to focus on, which is characterized by high volatility, will be around at the 1200^{th} observation and in the same way in Figure 18, will be around at the 1250^{th} observation. Turning to the estimates for the quadratic loss function that measures the distance between actual returns and expected returns in the event of a VaR violation (MSE of ES), the AR(1)-GARCH(1,1)-skT model produces lower values of the MSE for the one-day-ahead and especially, for the exchange rates (Euro, GBP, CAD), when from the other two markets, the smallest MSE value were S&P$_{500}$ for stock indices and Gold from commodities. The same results applied for the second model; AR(1)-HAR-RV-skT just with little differences in the MSE values. The best market, concerning the lowest MSE was again the FOREX at one-step-ahead HAR-RV model of 95% confidence level. Finally, the HAR-RV specification seems to be preferable rather than the simple GARCH(1,1), as HAR satisfies the most of the prerequisites, concerning the evaluation and the accuracy of VaR.

Table 7: 10-step-ahead VaR$^{95\%}$ and ES modeling results.

Table 7: 10-Step-ahead VaR and ES Modeling Results (95%)								
Part A. GARCH-skT								
Index	**Number of 10-step-ahead VaR forecasts**	**Average VaR**	**Average ES**	**Average MSE**	**Observed Exception Rate**	**Kupiec p-value**	**Independence Christoffersen**	**Conditional Coverage Christoffersen**
Stock Indices	**GARCH-skT**							
S&P500	290	-1,787855	-2,361605	0,068746	5,17%	0,894479	0,199893	0,435886
EurostocXX50	294	-2,427851	-3,161031	0,209316	**5,10%**	0,945827	0,788741	0,962515
FTSE100	291	-1,846542	-2,440472	0,178061	5,84%	0,522282	0,145521	0,282526
Commodities	**GARCH-skT**							
HG (Copper COMEX)	289	-2,917354	-3,722252	0,130557	**4,50%**	0,682967	0,267495	0,497491
SV (Silver COMEX)	289	-3,594869	-4,574125	0,414053	6,57%	0,244433	0,506262	0,407256
GC (Gold COMEX)	289	-1,952748	-2,484206	0,097983	8,30%	0,018587	0,036584	0,007049
Foreign Exchange Rates	**GARCH-skT**							
EUR/USD (EC)	289	-1,017000	-1,285351	**0,004900**	4,50%	0,682967	0,119736	0,274230
GBP/USD (BP)	289	-0,937435	-1,185187	**0,007209**	4,50%	0,681997	0,267495	0,497222
CAD/USD (CD)	289	-0,967634	-1,230028	0,012442	**5,19%**	0,892335	0,801553	0,960074
Part B. AR(1)-HAR-RV-skT								
Index	**Number of 10-step-ahead VaR forecasts**	**Average VaR**	**Average ES**	**Average MSE**	**Observed Exception Rate**	**Kupiec p-value**	**Independence Christoffersen**	**Conditional Coverage Christoffersen**
Stock Indices	**AR(1)-HAR-RV-skT**							
S&P500	290	-1,512188	-1,905477	0,155025	7,24%	0,100002	0,246049	0,131921
EurostocXX50	294	-2,045223	-2,562995	0,261478	6,80%	0,181864	0,726299	0,385803
FTSE100	291	-1,640489	-2,063329	0,193879	7,22%	0,103331	0,630244	0,236325
Commodities	**AR(1)-HAR-RV**							
HG (Copper COMEX)	289	-2,655036	-3,339026	0,171435	**4,50%**	0,682967	0,604540	0,804524
SV (Silver COMEX)	289	-3,029985	-3,807393	0,536055	8,65%	0,009832	0,898248	0,035412
GC (Gold COMEX)	289	-1,724372	-2,173198	0,105001	9,00%	0,005005	0,022996	0,001469
Foreign Exchange Rates	**AR(1)-HAR-RV**							
EUR/USD (EC)	289	-0,938610	-1,179771	**0,007220**	**5,19%**	0,891263	0,214108	0,457921
GBP/USD (BP)	289	-0,886966	-1,111539	**0,003910**	4,15%	0,488874	0,306938	0,467009
CAD/USD (CD)	289	-0,952570	-1,195246	**0,009237**	**5,19%**	0,892335	0,801553	0,960074

*The red color indicates rejection of the null hypothesis that the backtesting criterion is accurate; taking into consideration that red value is smaller than 5% significance level.

*The bold fond of MSE column denotes the lowest value for 5% significance level.

*The bold fond of observed exception rate column denotes the most suitable value around 5% significance level.

Table 7 shows the results for the 10-steps-ahead VaR forecasting of 95% significance level of GARCH and HAR-RV models, respectively. For this forecasting horizon, the Heterogeneous Autoregressive Realized Volatility specification (HAR-RV) does not appear to overperform the GARCH(1,1) specification. According to the Kupiec test, the GARCH and the HAR-RV model as well, produce an observed exception rate which is not statistically different to the anticipated failure rate of 5% for the majority of the nine asset markets. Especially, the corresponding figure for the GARCH specification is 8 out of 9 indices, apart from the Gold COMEX (0,018587). Moreover, the corresponding figure for the HAR-RV specification is 6 out of 9 indices, apart from the Kupiec values of Silver and Gold COMEX, which are rejected. The results of the Christoffersen test indicate that the VaR violations are all independently distributed under the AR(1)-GARCH-skT model. As far as the AR(1)-HAR-RV-skT model is concerned, the VaR violations are independently distributed for the 3 Stocks, the 3 FOREX and the one out of three COMEX; Silver and Gold reject the null hypothesis of Christoffersen test.

Although the forecasts are quite good both for GARCH and HAR-RV if we leave behind few exceptions, hence the modeling results are not robust enough across the different indices and markets tested. This is illustrated by the fact that MSE values are not small enough. To be more precise, GARCH specification produces a lower MSE for ES values only for Euro FOREX (EUR/USD) and for British Pound FOREX (GBP/USD), as well as HAR-RV produces the lowest MSE only for the whole three FOREX indices (Euro, GBP and CAD). At this point, the short memory GARCH specification is preferable since it is the more parsimonious model, if we take into consideration the column of Observed Exception Rate of Table 7, which indicates results almost around to 5% for GARCH, contrary to the HAR model that the percentage of violations is too high than that of 5% at the majority of the assets tested. By all accounts, the most suitable values around 5% are all the three Stock indices, Copper (HG) for COMEX category and CAD/USD for FOREX category, depending on the GARCH model, let alone for HAR-RV; Copper (HG) COMEX has a satisfactory range of forecast with 4.5% and from the FOREX category EUR/USD (5.19%) and CAD/USD (5.19%). As far as the Stock indices, if we take into consideration that the model runs only with a few observations (an average of 289 obs.) for 10-step-ahead forecasts, the results are pretty good for both models, and especially for GARCH due to the percentage of the observed exception rates, which are closely around 5%. For instance, if we compare the Figure 19 to Figure 20, we

Figure 19: Graph of daily log-XX_{50} Stock and 10-step-ahead $VaR^{95\%}$ of GARCH model.

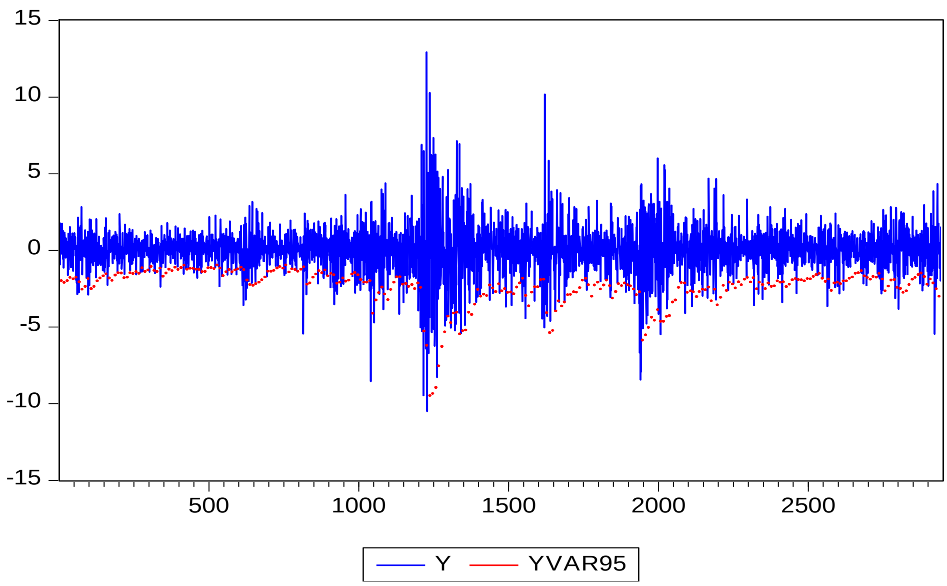
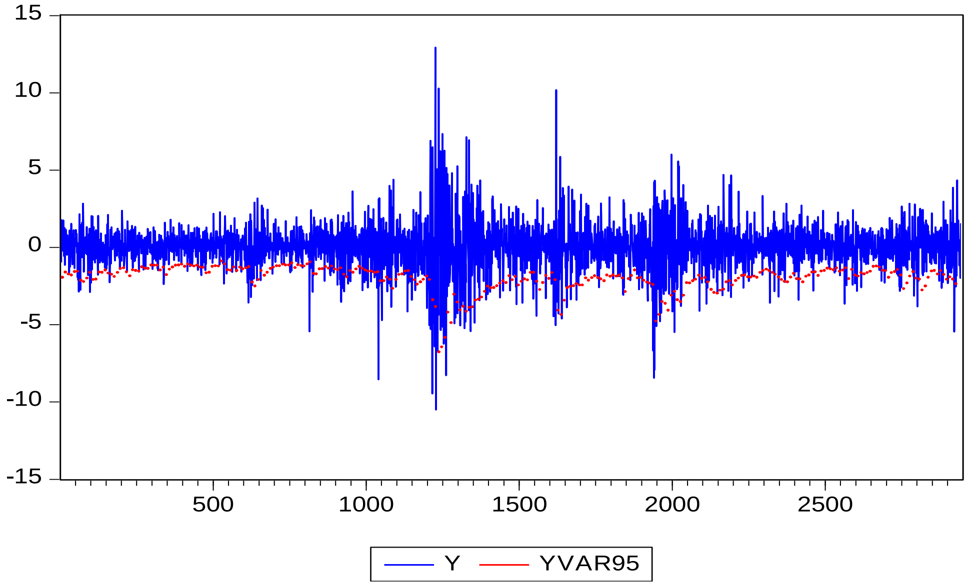


Figure 20: Graph of daily log-XX_{50} Stock and 10-step-ahead $VaR^{95\%}$ of HAR-RV model.



*The other graphs of daily log-returns are all available from the author on request.

can realize that the GARCH specification for the EurostoXX_{50} is preferable rather than HAR-RV, as the red dots of 10-steps-ahead forecasts of GARCH cover the losses when a violation occurs in a more accurate way. The previous claim is illustrated by the fact that the HAR specification does not cover all the negative peaks of the violated log returns, as we can observe in Figure 20.

Table 8: 20-step-ahead VaR$^{95\%}$ and ES modeling results.

Table 8: 20-Step-ahead VaR and ES Modeling Results (95%)								
Part A. GARCH-skT								
Index	**Number of 20-step-ahead VaR forecasts**	**Average VaR**	**Average ES**	**Average MSE**	**Observed Exception Rate**	**Kupiec p-value**	**Independence Christoffersen**	**Conditional Coverage Christoffersen**
Stock Indices	**GARCH-skT**							
S&P500	145	-1,829135	-2,478850	0,259767	5,52%	0,77922	0,439753	0,713368
EurostocXX50	147	-2,518926	-3,344469	0,367677	**5,44%**	0,81496	0,433228	0,715723
FTSE100	145	-1,899795	-2,582968	0,181323	6,90%	0,32645	0,155397	0,225217
Commodities	**GARCH-skT**							
HG (Copper COMEX)	144	-2,916934	-3,756540	0,294519	**4,17%**	0,625218	0,225305	0,425526
SV (Silver COMEX)	144	-3,626941	-4,649445	0,055227	8,33%	0,096048	0,137871	0,083262
GC (Gold COMEX)	144	-1,971987	-2,530160	0,198717	8,33%	0,096048	0,993925	0,993925
Foreign Exchange Rates	**GARCH-skT**							
EUR/USD (EC)	144	-1,024463	-1,299905	**0,002834**	6,94%	0,319503	0,219876	0,287056
GBP/USD (BP)	144	-0,941968	-1,196488	0,017590	4,17%	0,624566	0,468414	0,682070
CAD/USD (CD)	144	-0,980165	-1,256920	0,028583	**5,56%**	0,777733	0,330049	0,597984
Part B. AR(1)-HAR-RV-skT								
Index	**Number of 20-step-ahead VaR forecasts**	**Average VaR**	**Average ES**	**Average MSE**	**Observed Exception Rate**	**Kupiec p-value**	**Independence Christoffersen**	**Conditional Coverage Christoffersen**
Stock Indices	**AR(1)-HAR-RV-skT**							
S&P500	145	-1,504066	-1,895696	0,378001	6,21%	0,520418	0,569563	0,691951
EurostocXX50	147	-2,034986	-2,556575	0,516498	7,48%	0,003450	0,003450	0,006110
FTSE100	145	-1,629913	-2,046791	0,138419	8,97%	0,049006	0,437259	0,106532
Commodities	**AR(1)-HAR-RV**							
HG (Copper COMEX)	144	-2,632820	-3,308422	0,392020	6,25%	0,518486	0,573709	0,693010
SV (Silver COMEX)	144	-3,016907	-3,788454	0,183810	10,42%	0,009253	0,060497	0,005808
GC (Gold COMEX)	144	-1,712957	-2,159939	0,205898	10,42%	0,009253	0,589252	0,029235
Foreign Exchange Rates	**AR(1)-HAR-RV**							
EUR/USD (EC)	144	-0,943720	-1,186161	**0,002974**	6,25%	0,518486	0,271359	0,443355
GBP/USD (BP)	144	-0,884976	-1,110776	0,013882	**4,17%**	0,624566	0,468414	0,682080
CAD/USD (CD)	144	-0,949854	-1,193339	0,019583	6,25%	0,519129	0,271359	0,443639

*The red color indicates rejection of the null hypothesis that the backtesting criterion is accurate; taking into consideration that red value is smaller than 5% significance level.

*The bold fond of MSE column denotes the lowest value for 5% significance level.

*The bold fond of observed exception rate column denotes the most suitable value around 5% significance level.

Table 8 shows the results for the forecasting of 20-step-ahead VaR across the nine assets of Stocks, Commodities and Foreign Exchange Rate categories for both GARCH(1,1) and HAR-RV 95% significance level. For this longer time horizon the performance of the GARCH model slightly have improved comparing to the previous 10-step-ahead GARCH model. Furthermore, HAR seems to be less important at this time horizon, as there are many more rejections of the backtesting procedure comparing to the previous one. The Kupiec test results for the GARCH model suggest that the observed exception rate is not statistically different to the expected failure rate for all the 9 different assets. However, the Kupiec test results are little different for the HAR-RV model, as there are three rejections of the null hypothesis; particularly for EurostoXX$_{50}$, Silver (SV) and Gold (GC) commodities, in which the observed exception rates are statistically different to the expected one. As far as the 5th column of observed exception rate is concerned, it is an undeniable fact that the values indicate a widespread underestimation of the true VaR figure by both models, because almost all these values are much higher than the 5%. The most suitable values that do not underestimate the true VaR are HG (Copper COMEX) and GBP/USD (British Pound) with the percentage of 4.17% for the GARCH model, as well as only GBP/USD FOREX for the HAR-RV specification with 4.17% again.

Moreover, if we focus on the independence and conditional coverage of Christoffersen test, we will see that there isn't any rejection. The purpose of the test is to examine the null hypothesis that the VaR failures are independent and are spread over the whole estimation period, against the alternative hypothesis that the failures tend to be clustered. The main advantage of this test is that it can reject a model that generates too many or too few cluster exceptions, where in this case something like that does not happen. Finally, at Table 8 presented the MSE of ES, which measures the distance between actual returns and expected returns in the event of a VaR violation for 20-step-ahead period. As we can see, the lowest MSE values are EUR/USD (0.002834) for GARCH and EUR/USD (0.002974) for HAR, respectively.

To conclude, after checking the 1-step, 10-step and 20-step-ahead of 95% GARCH and HAR-RV models, we can infer that the results of VaR models are not much robust across different markets. As a consequence, the optimal model varies from one index to another. Hence, it is difficult to propose a clear-cut conclusion, concerning which one of the two models is the most accurate and reliable to forecast adequately the losses of a specific portfolio. More carefully, we observe that FOREX category acquire satisfactory forecasts with the method of Value-at-Risk both for GARCH and HAR as well, at all the time horizons; 1-step, 10-step and 20-step-ahead. This is undoubtedly a strong advantage; the fact that $VaR^{95\%}$ can forecast with high precision the losses of exchange rates assets for all the upcoming time horizons, let alone specifically for periods far ahead to the future, such as 10-days and 20-days-ahead.

Table 9: 1-step-ahead VaR$^{99\%}$ and ES modeling results.

Table 9: 1-Step-ahead VaR and ES Modeling Results (99%)							
Part A. GARCH-skT							
Index	**Number of 1-step-ahead VaR forecasts**	**Average VaR**	**Average ES**	**Average MSE**	**Observed Exception Rate**	**Kupiec p-value**	**Independence Christoffersen**
Stock Indices	GARCH-skT						
S&P500	2901	-3,397275	-4,338985	**0,003592**	0,48%	0,001845	0,712464
EurostocXX50	2949	-4,925017	-6,509659	**0,007183**	0,47%	0,001403	0,714722
FTSE100	2912	-3,303595	-4,102522	**0,003604**	**0,82%**	0,325364	0,193478
Commodities	GARCH-skT						
HG (Copper COMEX)	2897	-5,383969	-6,786839	0,025746	**0,83%**	0,337964	0,526584
SV (Silver COMEX)	2897	-8,150103	-11,04588	0,021580	0,52%	0,004049	0,001876
GC (Gold COMEX)	2897	-3,944656	-5,573181	0,002840	0,55%	0,008146	0,076609
Foreign Exchange Rates	GARCH-skT						
EUR/USD (EC)	2898	-1,645303	-1,970461	**0,000617**	0,72%	0,117176	0,579725
GBP/USD (BP)	2899	-1,513355	-1,802595	**0,002152**	**0,97%**	0,852560	0,274383
CAD/USD (CD)	2899	-1,510575	-1,784866	**0,000705**	1,07%	0,710622	0,342695
Part B. AR(1)-HAR-RV-skT							
Index	**Number of 1-step-ahead VaR forecasts**	**Average VaR**	**Average ES**	**Average MSE**	**Observed Exception Rate**	**Kupiec p-value**	**Independence Christoffersen**
Stock Indices	AR(1)-HAR-RV-skT						
S&P500	2901	-3,139733	-4,008740	**0,002972**	0,52%	0,003963	0,692885
EurostocXX50	2949	-4,449075	-5,871900	**0,004595**	0,37%	0,000087	0,774075
FTSE100	2912	-3,114106	-3,865971	**0,004902**	**0,82%**	0,325364	0,193478
Commodities	AR(1)-HAR-RV-skT						
HG (Copper COMEX)	2897	-5,041081	-6,358136	0,031726	**0,97%**	0,854016	0,459735
SV (Silver COMEX)	2897	-7,179038	-9,724841	0,045500	0,72%	0,117591	0,008409
GC (Gold COMEX)	2897	-3,567694	-4,626939	0,011326	0,59%	0,015434	0,088205
Foreign Exchange Rates	AR(1)-HAR-RV-skT						
EUR/USD (EC)	2898	-1,519651	-1,818650	**0,000835**	**0,97%**	0,854016	0,459735
GBP/USD (BP)	2899	-1,436972	-1,710775	**0,001820**	**1,17%**	0,362677	0,368925
CAD/USD (CD)	2899	-1,495670	-1,766665	**0,000386**	0,76%	0,172967	0,561809

*The red color indicates rejection of the null hypothesis that the backtesting criterion is accurate; taking into consideration that red value is smaller than 1% significance level.

*The bold fond of MSE column denotes the lowest value for 1% significance level.

*The bold fond of observed exception rate column denotes the most suitable value around 1% significance level.

Now at this point, the results for the one-day-ahead VaR forecasting of 99% across the nine assets; consisting of stock indices, commodities and exchange rates for both GARCH and HAR-RV specifications are shown in Table 9. At a first glance, the 99% VaR suggest better forecasting results for the two models and to begin with the AR(1)-GARCH(1,1)-skT, only two of the stocks and one out of three COMEX reject the null hypothesis of the Kupiec backtesting procedure. Additionally, the AR(1)-HAR-RV-skT model is clearly improved versus the HAR model of 95% VaR of the one-step-ahead, because the red values of 99% are much less than those of 95%. As far as the Christoffersen test for independence is concerned, the observed violation rate is statistically different only for Silver (SV) at both models.

Once again, the results of 99% VaR for one-step-ahead indicate that VaR models are not much robust across different markets. Especially, at this time horizon, both for GARCH and HAR models maybe there are some difficulties to forecast accurately VaR for the category of Stocks, as two out of three stock indices rejected, concerning the Kupiec test and for the category of Commodities, as well. (For example, look at Figure 21 and Figure 22 that shows a kind of overestimation of the true VaR). Exactly the same happens for HAR model. As a consequence, the optimal model varies from one index to another. Hence, the best category that again forecasts adequately VaR is FOREX, having a great percentage of violations near to the expected level of 1%.

Figure 21: Graph of daily log-S&P$_{500}$ Stock and 1-step-ahead VaR$^{99\%}$ of AR(1)-GARCH(1.1) model.

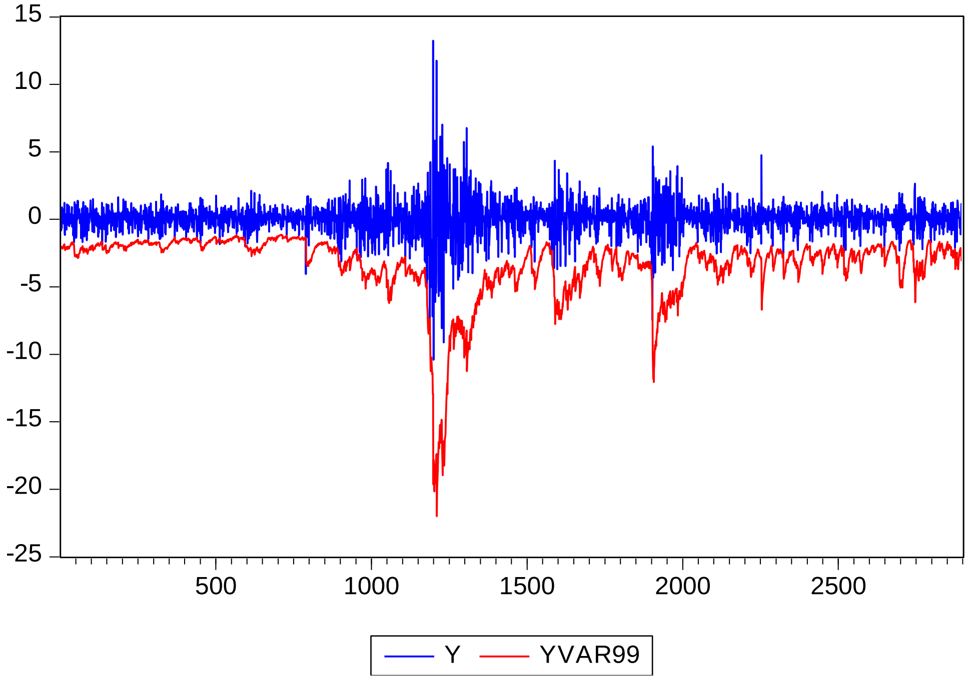
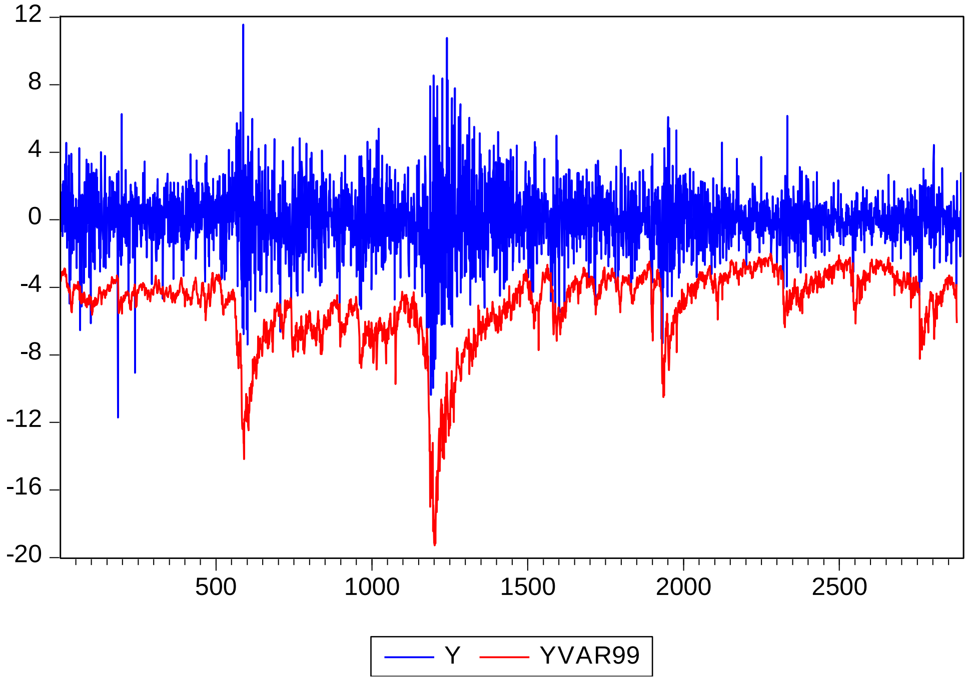


Figure 22: Graph of daily log-HG COMEX and 1-step-ahead VaR$^{99\%}$ of AR(1)-HAR-RV model.



*The other graphs of daily log-returns are all available from the author on request.

Table 10: 10-step-ahead VaR$^{99\%}$ and ES modeling results.

Table 10: 10-Step-ahead VaR and ES Modeling Results (99%)								
Part A. GARCH-skT								
Index	**Number of 10-step-ahead VaR forecasts**	**Average VaR**	**Average ES**	**Average MSE**	**Observed Exception Rate**	**Kupiec p-value**	**Independence Christoffersen**	**Conditional Coverage Christoffersen**
Stock Indices	**GARCH-skT**							
S&P500	290	-2,713784	-3,263684	0,015066	**1,03%**	0,953674	0,801911	0,967387
EurostocXX50	294	-3,609985	-4,295991	0,132889	**1,02%**	0,976256	0,803254	0,969012
FTSE100	291	-2,799076	-3,361837	0,107582	3,09%	0,004043	0,447644	0,012028
Commodities	**GARCH-skT**							
HG (Copper COMEX)	289	-4,239563	-4,914170	0,071091	1,73%	0,260592	0,674235	0,486194
SV (Silver COMEX)	289	-5,193212	-6,033968	0,212725	3,46%	0,001043	0,396282	0,003232
GC (Gold COMEX)	289	-2,825399	-3,266032	0,035635	3,08%	0,000255	0,349885	0,000804
Foreign Exchange Rates	**GARCH-skT**							
EUR/USD (EC)	289	-1,459795	-1,675273	**0,000814**	2,77%	0,013261	0,498933	0,037046
GBP/USD (BP)	289	-1,342207	-1,544417	**0,002941**	2,08%	0,109574	0,613341	0,244670
CAD/USD (CD)	289	-1,399029	-1,617769	**0,005508**	1,73%	0,260869	0,064120	0,095770
Part B. AR(1)-HAR-RV-skT								
Index	**Number of 10-step-ahead VaR forecasts**	**Average VaR**	**Average ES**	**Average MSE**	**Observed Exception Rate**	**Kupiec p-value**	**Independence Christoffersen**	**Conditional Coverage Christoffersen**
Stock Indices	**AR(1)-HAR-RV-skT**							
S&P500	290	-2,158911	-2,471835	0,089942	3,10%	0,003940	0,446834	0,011736
EurostocXX50	294	-2,908882	-3,316504	0,188173	3,06%	0,004407	0,450053	0,013047
FTSE100	291	-2,342712	-2,677232	0,127544	3,44%	0,001086	0,397975	0,003364
Commodities	**AR(1)-HAR-RV**							
HG (Copper COMEX)	289	-3,791532	-4,334332	0,107054	2,42%	0,040391	0,554799	0,102788
SV (Silver COMEX)	289	-4,311206	-4,928599	0,319464	4,84%	0,000002	0,701767	0,000013
GC (Gold COMEX)	289	-2,460248	-2,812279	0,045025	4,50%	0,000012	0,267495	0,000037
Foreign Exchange Rates	**AR(1)-HAR-RV**							
EUR/USD (EC)	289	-1,333491	-1,522888	**0,002357**	3,46%	0,001046	0,342256	0,002958
GBP/USD (BP)	289	-1,258021	-1,436047	**0,000618**	2,08%	0,109574	0,613341	0,244670
CAD/USD (CD)	289	-1,353754	-1,544206	**0,003655**	**1,38%**	0,538860	0,737114	0,782573

*The red color indicates rejection of the null hypothesis that the backtesting criterion is accurate; taking into consideration that red value is smaller than 1% significance level.

*The bold fond of MSE column denotes the lowest value for 1% significance level.

*The bold fond of observed exception rate column denotes the most suitable value around 1% significance level.

As the period of forecasting increases far ahead to the future, and particularly, in this occasion of the 99% of ten-days-ahead, we led to the inference that these two models; GARCH and HAR, do not forecast Value at Risk with the most appropriate way, combining inter-day data for the former model and for the latter one intra-day data (with realized volatility), let alone using the RV as an exogenous parameter for HAR specification. In more details, AR(1)-HAR-RV-skT model for 10-days-ahead does not appear to overperform the AR(1)-GARCH(1.1)-skT model for almost all the nine assets. As Table 10 presented, the category of Stocks and Commodities are getting worse by using the application of HAR-RV model in order to estimate the VaR measure. On the other hand, GARCH faced the rejection of Kupiec procedure only at $FTSE_{100}$ stock index and at two out of the three COMEX indices. As a result, GARCH is preferable model, comparing to HAR for the 10-steps-ahead VaR$^{99\%}$, and not to mention the fact that Stocks category plays a quite important role with GARCH forecasting rather than HAR. Almost for the same reasons, Christoffersen's values are better and of course, not rejected, at the GARCH specification. Moreover, the observed exception rates of GARCH and especially for the Stocks, is similar enough to the expected range of 1%.

Table 11: 20-step-ahead $VaR^{99\%}$ and ES modeling results.

Table 11: 20-Step-ahead VaR and ES Modeling Results (99%)								
Part A. GARCH-skT								
Index	**Number of 20-step-ahead VaR forecasts**	**Average VaR**	**Average ES**	**Average MSE**	**Observed Exception Rate**	**Kupiec p-value**	**Independence Christoffersen**	**Conditional Coverage Christoffersen**
Stock Indices	**GARCH-skT**							
S&P500	145	-2,884930	-3,542076	0,092655	2,07%	0,25827	0,036982	0,059928
EurostocXX50	147	-3,817402	-4,624643	0,211576	**1,36%**	0,67993	0,813661	0,893256
FTSE100	145	-2,993936	-3,685127	0,036307	2,76%	0,08114	0,632562	0,194896
Commodities	**GARCH-skT**							
HG (Copper COMEX)	144	-4,282625	-5,025230	0,149057	3,47%	0,020459	0,143264	0,023344
SV (Silver COMEX)	144	-5,265473	-6,177768	0,010146	3,47%	0,020430	0,547177	0,056779
GC (Gold COMEX)	144	-2,880967	-3,354845	0,097697	3,47%	0,020430	0,143264	0,023315
Foreign Exchange Rates	**GARCH-skT**							
EUR/USD (EC)	144	-1,481211	-1,708530	**0,000427**	2,78%	0,079897	0,631340	0,192314
GBP/USD (BP)	144	-1,363087	-1,575497	**0,005683**	2,08%	0,257866	0,719908	0,494416
CAD/USD (CD)	144	-1,429585	-1,666978	0,018167	**1,39%**	0,663893	0,811726	0,884455
Part B. AR(1)-HAR-RV								
Index	**Number of 1-step-ahead VaR forecasts**	**Average VaR**	**Average ES**	**MSE**	**Observed Exception Rate**	**Kupiec p-value**	**Independence Christoffersen**	**Conditional Coverage Christoffersen**
Stock Indices	**AR(1)-HAR-RV**							
S&P500	145	-2,147815	-2,460266	0,275627	2,76%	0,080162	0,080050	0,046759
EurostocXX50	147	-2,883045	-3,295188	0,398223	4,08%	0,004857	0,220042	0,008932
FTSE100	145	-2,313073	-2,640589	0,068693	4,14%	0,004551	0,470031	0,013763
Commodities	**AR(1)-HAR-RV**							
HG (Copper COMEX)	144	-3,750312	-4,279910	0,241818	3,47%	0,020459	0,143264	0,023344
SV (Silver COMEX)	144	-4,293261	-4,895013	0,061314	6,25%	0,000020	0,271359	0,000062
GC (Gold COMEX)	144	-2,445660	-2,797862	0,098816	4,17%	0,004432	0,225305	0,008362
Foreign Exchange Rates	**AR(1)-HAR-RV**							
EUR/USD (EC)	144	-1,342047	-1,532244	**0,000273**	3,47%	0,020459	0,547177	0,056850
GBP/USD (BP)	144	-1,255806	-1,429632	**0,003698**	**1,39%**	0,663893	0,811726	0,884455
CAD/USD (CD)	144	-1,351755	-1,544205	**0,010931**	2,08%	0,257866	0,719908	0,494416

*The red color indicates rejection of the null hypothesis that the backtesting criterion is accurate; taking into consideration that red value is smaller than 1% significance level.

*The bold fond of MSE column denotes the lowest value for 1% significance level.

*The bold fond of observed exception rate column denotes the most suitable value around 1% significance level.

Finally for the 20-steps-ahead of 99% VaR, there arose a clear-cut answer to the question of which of the two proposed models forecast losses of each portfolio more accurately. It is obvious, as we can see at Table 11 that the GARCH model is superior to the HAR specification, because the observed violation rates of all the nine assets from Stocks, Commodities and Exchange Rate classes are statistically different from the expected violation rate of 1%, according to the results of Kupiec test. Additionally, the results of Christoffersen test indicate that for GARCH model all the assets are independently distributed, by checking Christoffersen test with one and two degrees of freedom. Moreover, HAR-RV rejected the backtesting procedure null hypothesis both for Kupiec and Christoffersen values at $EurostoXX_{50}$, $FTSE_{100}$, Silver (SV) and Gold (GC). The HAR model for FOREX assets seems to be better and forecast more accurately the VaR and ES rather than the forecasts of Stocks and Commodities. As far as the column of the observed exception rate of Table 11 is concerned, we led to the inference that these percentages for both models suggest a widespread underestimation of the true Var figure, due to the fact that the true VaR presents high divergences from the observed one. Therefore, it is essential to take into consideration that at this procedure we forecast VaR for 20-days-ahead and as a consequence, the out-of-sample data have been reduced from 2900 to 144 observations, at the end. So, the high observed exception rates should not be considered as a serious drawback. The innovative part of this research is the superiority of the simple GARCH application rather than HAR for the 10-steps and 20-steps-ahead.

Chapter 4: Conclusion and Results

A widely common question with lots of interest is triggered through the financial literature and describes which model is the most appropriate to forecast the asset returns' volatility, particularly as the forecasting time horizon lengthens. It is well-known that investors are interested mainly in calculating Value-at-Risk (VaR) and forecasting volatility. Through this direction, the issue of choosing one superior model among all the potential models for all cases is complicated enough, because the results of many researches are confusing and conflicting. This happens as there is not a specific model that is deemed as adequate for all financial datasets, sample frequencies and applications, as well.

This study examines whether an intra-day or an inter-day model generate the most accurate forecasts for different datasets, among the 3 different asset categories; 3 stock indices ($S\&P_{500}$, $EurostoXX_{50}$, $FTSE_{100}$), 3 commodities (HG-Copper COMEX, SV-Silver COMEX, GC-Gold COMEX) and 3 Foreign Exchange Rates of Dollar (EUR/USD, GBP/USD, CAD/USD), under the framework of two financial model applications. I have been used the AR(1)-GARCH model followed by the skewed Student-t distribution and finally, the AR(1)-HAR-RV model followed again by the skewed Student-t distribution. As far as the methodology of each of the two models, there is a detailed description at the Subsections 3.2 and 3.3 of 3rd Chapter. The data used, capture a time horizon from 3rd of January, 2000 to 5th of August, 2015 and were conditioned to remove any non-trading days. By using different sample periods of out-of-sample observations, it will be more easily to investigate whether the risk management techniques are robust across various forecasting horizons. Furthermore, this procedure ensures that the observations of each sample would not repeat. As a result, the empirical analysis of this thesis presents forecasts at the 95% and 99% confidence level, for 1-day-ahead, 10-days-ahead and 20-days-ahead.

The modeling results suggest that the optimal model varies from one index to another and it depends on each forecasting horizon. To be more specific, as we can observe in Table 12 and taking into consideration all the above empirical results, there are different inferences for each period. It is clear that for one-step-ahead VaR, the combination of an autoregressive model for realized volatility HAR-RV model generates competitive VaR forecasts both for 95% and 99% confidence level, but just only for the one-day-ahead estimation. The superiority of the HAR-RV model for the one-step-ahead VaR forecasts, with a variety of different data frameworks (stocks, commodities and exchange rates), answer affirmatively to the question if the one-day-ahead volatility can be better estimated with a model using intra-day data rather than with a model using daily data.

Table 12: Pivot table of the final results.

	1-step-ahead	**10-step-ahead**	**20-step-ahead**
95%	AR(1)-HAR-RV-skT	AR(1)-GARCH-skT	AR(1)-GARCH-skT
99%	AR(1)-HAR-RV-skT	AR(1)-GARCH-skT	AR(1)-GARCH-skT

As far as the 10-steps and 20-steps-ahead forecasts are concerned, the results are definitely opposite from the one-step-ahead. In this occasion, the procedure to forecast daily volatility based on HAR-RV specification does not seem to overperform the VaR measure estimated by GARCH model both at 10-steps and 20-steps-ahead. In other words, GARCH model predicts more accurately and more effectively the losses of a portfolio when the time horizon of the estimation increases. This fact is in line with the literature, as a number of papers indicate that using intra-day data does not help when the criteria are based on daily frequency (Angelidis & Degiannakis, 2008). As a result, a Realized Volatility model, such as the AR(1)-HAR-RV, will not be able to transfer at risk managers all the appropriate information they need to calculate Value-at-Risk at high accuracy as the forecasting period lengthens.

Although HAR-RV specification incorporates three primary volatility components: the short-term with daily, the medium-term with weekly positions, and the long-term with a characteristic time of one or more months (Corsi F. , 2002), hence these models noticed not to forecast the long-term VaR adequately. Particularly, the empirical results of this dissertation about HAR-RV, found to suffer from excessive VaR violations, implying an underestimation of market risk for the most of the asset categories. Problematic enough were the Stock indices and the Commodities, in an attempt to use a Realized Volatility model in order to forecast the VaR measures. This is illustrated by the fact that expect for the one-day-ahead HAR forecasts that are quite satisfactory, at the other two time periods using HAR there were many rejections of the null hypothesis of Backtesting procedure by Kupiec and Christoffersen.

To summarize, the results indicate firstly that there is not a unique model for all cases that can be deemed an adequate one, and therefore investors should be extremely careful when they use one model in all cases. Secondly, from the empirical example at the previous chapter, it emerged a new innovative inference; the choice of AR(1)-GARCH(1,1), and in accordance to the literature, has been shown to produce reasonable one-day and multiple-days-ahead VaR forecasts under the skewed Student-t distribution, let alone and most importantly, across a variety of markets; stocks, COMEX and FOREX, respectively. Finally, as the literature indicates, many studies concluded that the use of a skewed instead of a symmetrical distribution for the standardized residuals produces superior VaR forecasts. As a consequence, the effects of the intra-day noise in the high frequency datasets are still an open area of study and require further investigation. Undoubtedly, from now on it is an undeniable fact that GARCH-skT specification is a safe model that predicts VaR adequately both for 95% and 99%, as the Basel Committee imposed to, not only at daily basis as we all know until this moment, but more importantly at 10-days and 20-days ahead.

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Appendix

A) Types of Distributions and the Density Function of them

In an attempt to estimate the vector of the unknown parameters, the density function is analyzed in the next paragraphs. The chosen density function, which was widely applied in finance, was $\{z_t\}_{t=0}^{T}$. However, Engle (1986) in his seminal paper; landmark for Risk Management, used the Standard Normal density function.

$$f(z_t) = \frac{1}{\sqrt{2\pi}} e^{-z_t^2/2} \qquad (104)$$

Bollerslev (1987) introduced the Student-t Distribution, due to the need of investigating fat-tailed financial assets. Its density function is given in the following equation.

$$f(z_t; \upsilon) = \frac{\Gamma((\upsilon+1)/2)}{\Gamma(\upsilon/2)\sqrt{\pi(\upsilon-2)}} \left(1 + \frac{z_t^2}{\upsilon-2}\right)^{-(\upsilon+1)/2} \qquad (105)$$

where $\Gamma(.)$ is the gamma function. As the υ tends to infinity, the Student-t tends to the Normal distribution.

However, the Student-t distribution is not the only fat-tailed distribution available in the literature. There is also the GED, Generalized Error Distribution, which was introduced in 1923 by Subbotin and finally, applied in the ARCH framework by Nelson (1991). Comparing to the Student-t distribution, GED is more flexible, as it could include both fat and thin-tailed distributions. The density function of GED is the following:

$$f(z_t; \upsilon, \lambda) = \frac{\upsilon exp\left(-0.5\left|{}^{z_t}/_{\lambda}\right|^{\upsilon}\right)}{\lambda 2^{(1+1/\upsilon)}\Gamma(\upsilon^{-1})} \qquad (106)$$

Where $\lambda \equiv \sqrt{2^{-2/\upsilon}\Gamma(\upsilon^{-1})\Gamma(3\upsilon^{-1})}$ and $\upsilon > 0$ are the tail-thickness parameters. For instance, for $\upsilon = 2$, z_t is standard Normally Distributed, let alone for $\upsilon < 2$, the distribution of z_t has thicker tails than the normal one.

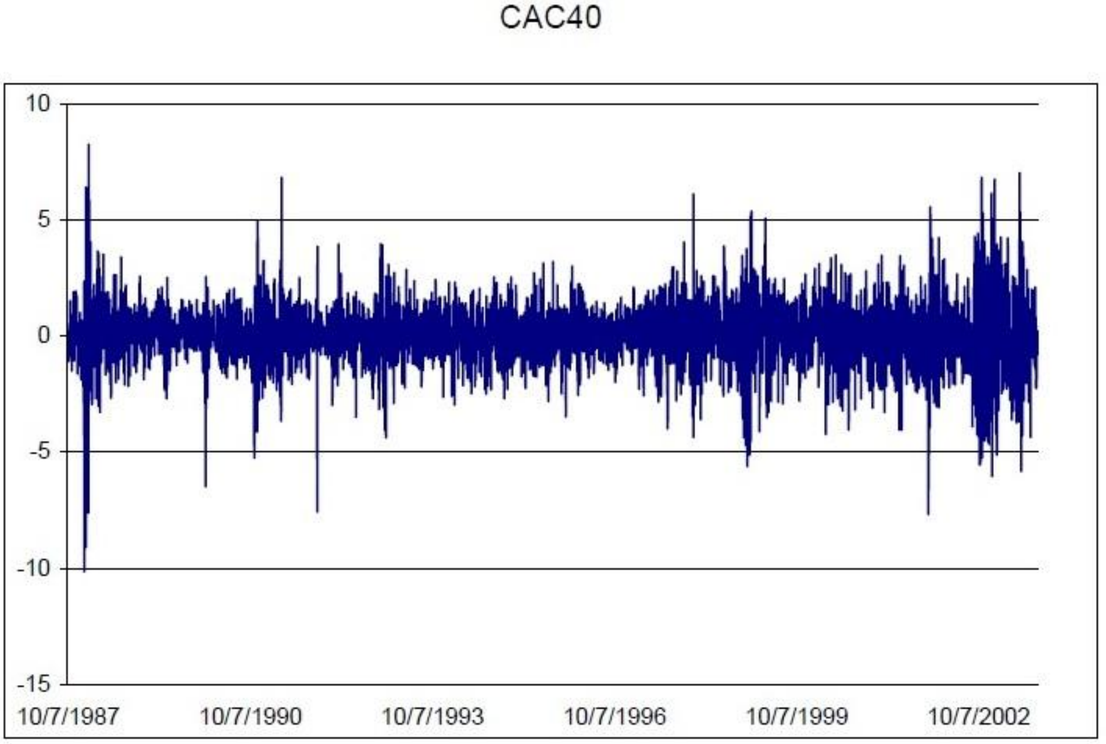
To conclude, there is another well-known distribution, that of skewed Student-t, which was introduced by Fernandez and Steel (1998) and was applied by Lambert and Laurent (2000) in the ARCH framework. Skewed Student-t distribution was significant, because it had to do with both long and short trading positions. The density function of skewed Student-t is:

$$f(z_t; \upsilon, g) = \frac{\Gamma((\upsilon+1)/2)}{\Gamma(\upsilon/2)\sqrt{\pi(\upsilon-2)}} \left(\frac{2s}{g+g^{-1}}\right) \left(1 + \frac{sz_t+m}{\upsilon-2} g^{-d^t}\right)^{-(\upsilon+1)/2} \qquad (107)$$

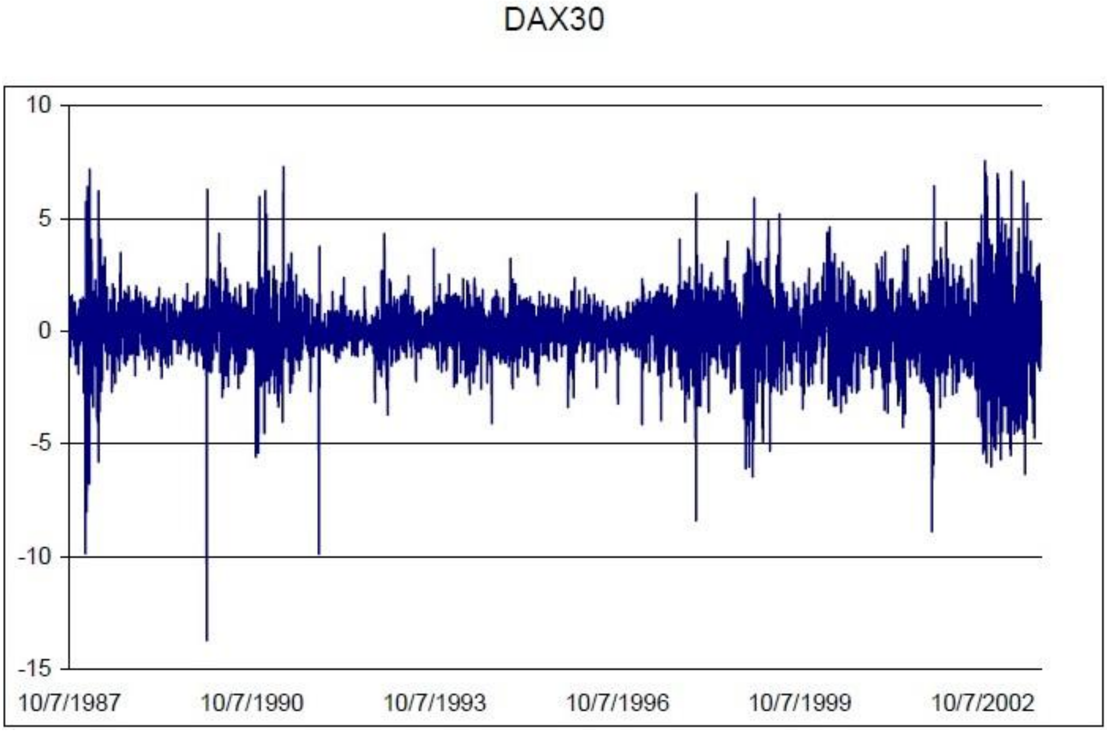
Where g is the asymmetry parameter, $\upsilon > 2$ denotes the degree of freedom of the distribution, $\Gamma(.)$ is the gamma function, $d_t = 1\ if\ z_t \geq -m/s\ \ and\ d_t = -1$ otherwise. Last but not least, Kuester (2006) noticed that there was substantial improvement in predicting VaR by using an asymmetric fat-tailed distribution than the normal one.

B) The CAC40, DAX30 and FTSE100 stock index daily returns in the period from July 10th, 1987 to June 30th, 2003.

Figure 23: CAC40, DAX30 and FTSE100 stock index daily returns in the period from July 10th, 1987 to June 30th, 2003.

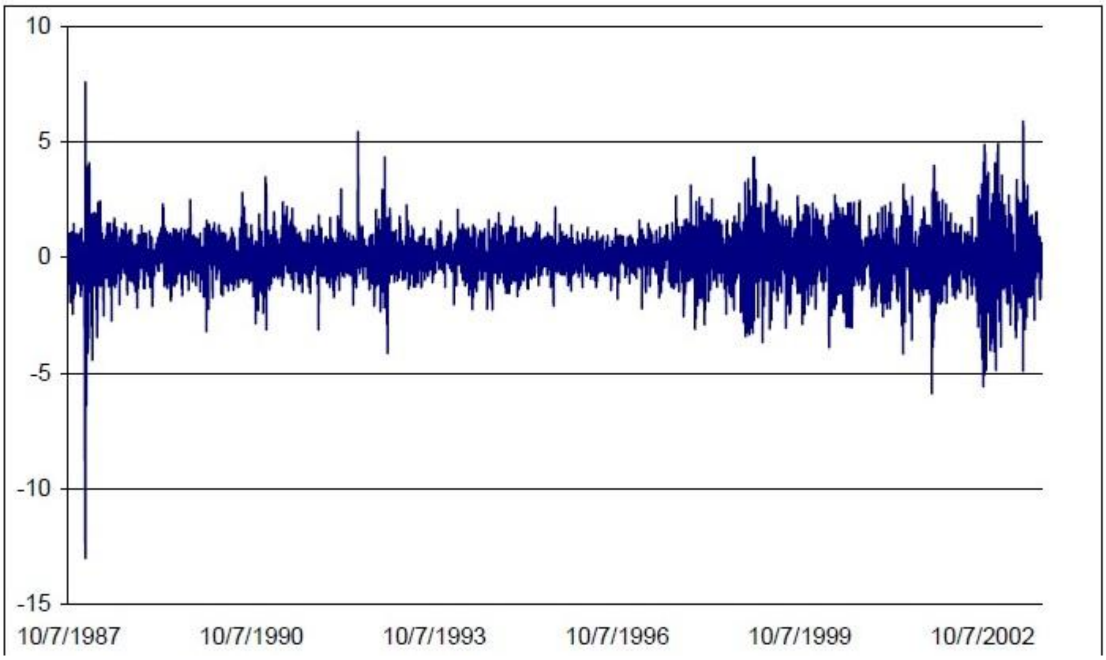


Source: Degiannakis S. (2004), Volatility Forecasting: Evidence from a FIAPARCH-skT Model, p.19/1-24.



Source: Degiannakis S. (2004), Volatility Forecasting: Evidence from a FIAPARCH-skT Model, p.19/1-24.

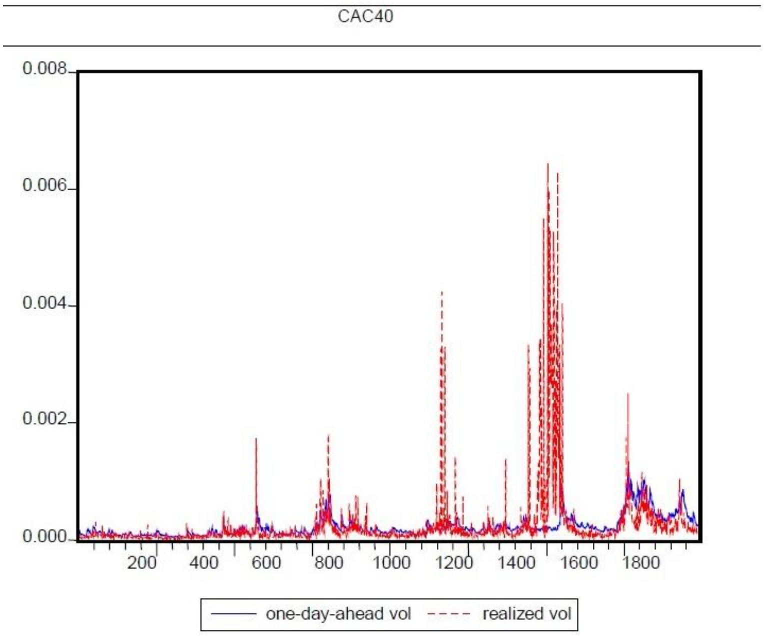
FTSE100



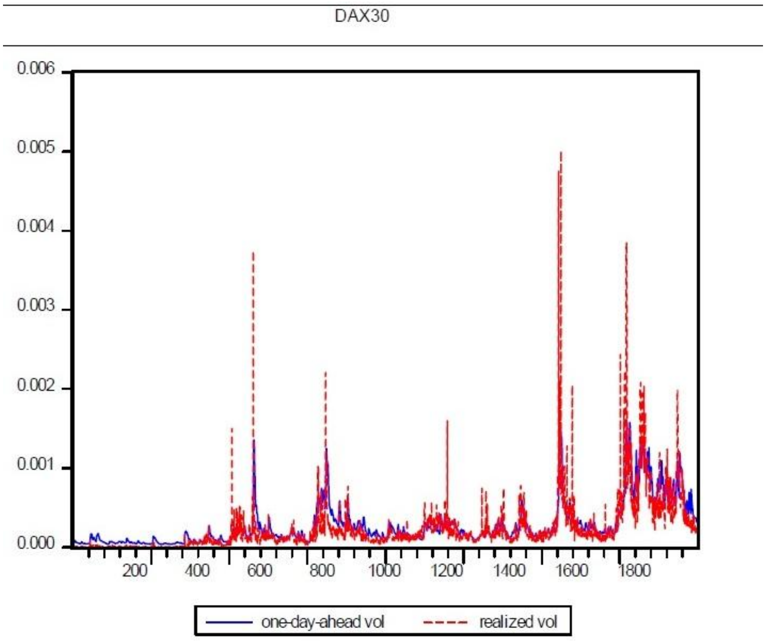
Source: Degiannakis S. (2004), Volatility Forecasting: Evidence from a FIAPARCH-skT Model, p.20/1-24.

C) The realized intra-day volatility and the relative one-day-ahead forecasts of the FIAPARCH(1,1)-skT model for the CAC40 (July 20th 1995 – June 30th 2003), DAX30 (July 11th 1995 – June 30th 2003) and FTSE100 indices (June 14th1995 – June 30th 2003).

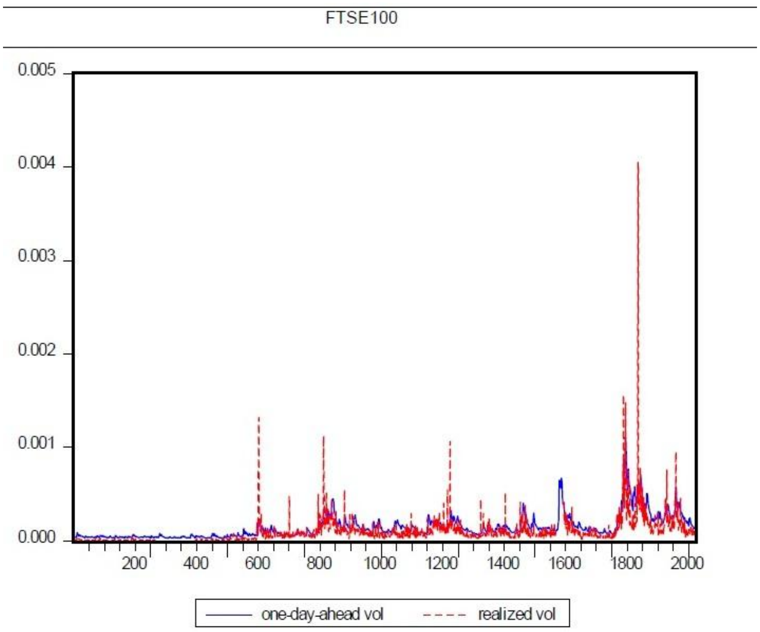
Figure 24: The realized intra-day volatility and the relative one-day-ahead forecasts of the FIAPARCH(1,1)-skT model for the CAC40 (July 20th 1995 – June 30th 2003), DAX30 (July 11th 1995 – June 30th 2003) and FTSE100 indices (June 14th 1995 – June 30th 2003).



Source: Degiannakis S. (2004), Volatility Forecasting: Evidence from a FIAPARCH-skT Model, p.21/1-24



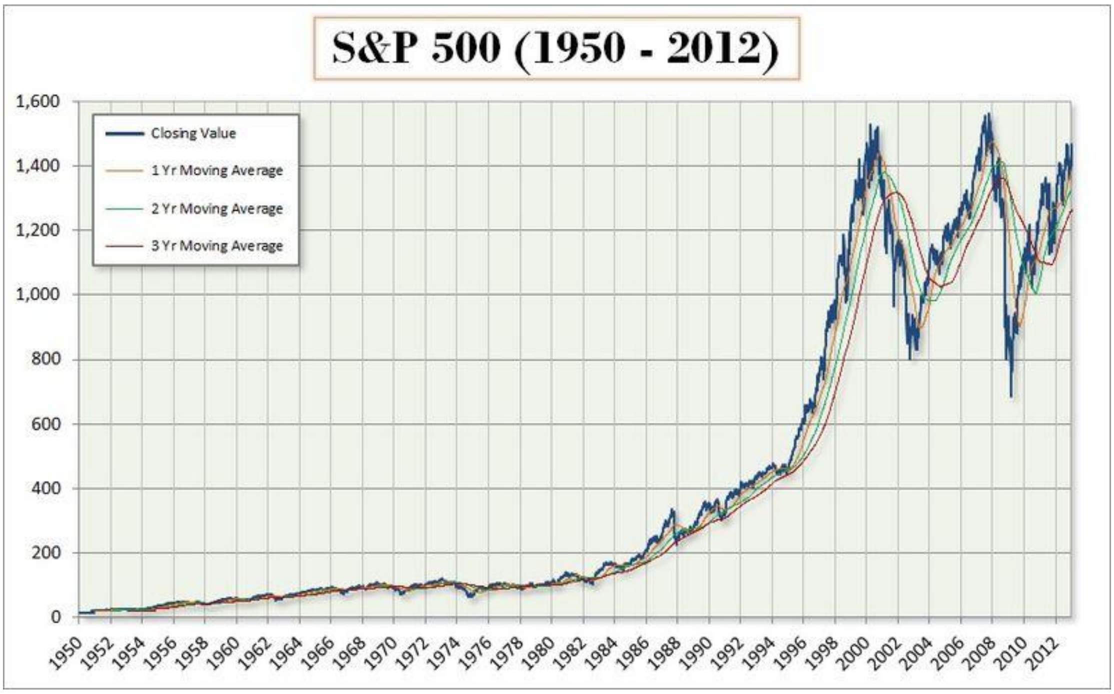
<u>Source</u>: Degiannakis S. (2004), Volatility Forecasting: Evidence from a FIAPARCH-skT Model, p.22/1-24.



<u>Source</u>: Degiannakis S. (2004), Volatility Forecasting: Evidence from a FIAPARCH-skT Model, p.23/1-24.

D) The closing values of indicators; S&P$_{500}$ and Gold Commodity through the years.

Figure 25: The closing values and moving averages of S&P$_{500}$ stock index from 1950 to 2012.



Source: From the link (http://www.fool.com/investing/general/2015/12/04/are-the-sp-500s-best-stocks-in-2015-still-worth-bu.aspx).

Figure 26: The closing values and moving averages of Gold commodity from 2009 to 2015.

Source: From the link of National Center for Scientific Research (http://www.cnbc.com/2015/12/03/unable-to-get-a-bid-gold-is-going-to-900-technician.html).