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Comments on the Paper by R.M. Goodwin *"A Growth Cycle"* [1]

by

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The purpose of this paper is, first, to explain and –to the extent that this is necessary for its comprehension– comment on Goodwin's paper *A Growth Cycle* and, second, to set forth and submit for the consideration of the reader certain estimations concerning its significance and value.

Together with Goodwin's paper, the reader should also read (at least) Chapter XXIII, Volume I of Marx's *Capital*. For Goodwin's paper is considered by many to be a mathematical formulation of the theory of economic fluctuations expounded by Marx in Chapter XXIII, Volume I of *Capital*.[2] Even Goodwin himself implies that his model expresses Marx's aforesaid theory in a mathematical, logically cohesive manner.

Because Goodwin himself, evidently for brevity's sake, omits the intermediate mathematical operations when setting out the results of his mathematical formulations, we consider that their presentation will facilitate a better understanding of the paper. To avoid overlapping, we shall not be presenting the entire model here, but rather only those points which require clarification and further analysis. Thus, in order to follow the explanations that follow, it will be necessary for the reader to refer to Goodwin's paper, which has been republished in this issue of Political Economy.

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1. R.M. Goodwin, "A Growth Cycle" in E.K. Hunt and Jesse G. Schwartz (eds.) *A Critique of Economic Theory*, Penguin, 1972, pp. 442-449.
2. See M. Desai, Growth Cycles and Inflation in a Model of the Class Struggle, *Journal of Economic Theory*, Vol. 6 (1973), pp. 527-545, E. Wolfstetter, Wert, Profitrate und Beschaftigung, Frankfurt/New York 1977, p. 142, J. Glombowski, Bemerkungen zur konjunkturellen Instabilitat, *Diskussionsbeitrage zur Politischen Okonomie*, Universitat Osnabruck, September 1978, pp. 29-62, Idem, Ein uberakkumulationstheoretisches Modell zyklischen Wachstums mit variabler Kapazitatsauslastung, *Argumment-Sonderband* Nr. 35, Berlin 1979, pp. 135-148.

We consider it expedient, first of all, to make the following clarifications regarding the symbols used in Goodwin's paper:

q real net product per production period. (Here, because q is a continuous function of time, the duration of a period is infinitesimal.),

k real constant capital, i.e. the used means of production,

w real wage per unit (= 'hour') of labour power, i.e. the real wage rate,

l employment (which is measured by the same measure used for measuring labour power, i.e. in 'hours'),

wl real wages in the period and

(q–wl) real profits in the period.

The magnitudes q, k, w, wl and (q–wl) are homogenous. This means that the economy produces only one good, using that very same good, and only that good, as a means of production.

In addition, the following symbols are used:

$$\sigma \frac{k}{q} = \text{constant}, \qquad (1)$$

the capital-output ratio, i.e. the capital coefficient, the inverse of l/σ,

$$\frac{1}{\sigma} = \frac{q}{k} = \text{constant}, \qquad (2)$$

the output-capital ratio, i.e. capital productivity, and

$$a = a_0 e^{\alpha t}\left(= \frac{q}{l}\right), \ \alpha = \text{positive constant}, \qquad (3)$$

the productivity of labour.

From (3) we get for the rate of increase of labour productivity $\hat{a}$:

$$\hat{a} = \frac{da}{dt}\frac{1}{a} = \alpha \qquad (4)$$

So, labour productivity increases at the constant rate α[3].

For the shares of wages and of profits in net product, the following holds, if we take into consideration (3), respectively:

3. Regarding the rules for calculating the rate of change of a variable as a function of the rates of change of variables, on which this variable depends, see the Appendix hereto.

$$\frac{wl}{q}=\frac{w}{a} \tag{5}$$

and

$$\frac{q-wl}{q}=1-\frac{w}{a} \tag{6}$$

If S symbolises the saving in the period, then because, according to the model, workers do not save at all, whereas capitalists save and invest all their profits, the following holds:

$$q-wl=\left(1-\frac{w}{a}\right)q=S \tag{7}$$

and

$$S=\frac{dk}{dt}=\dot{k} \tag{8}$$

where k is the absolute increase of capital per period, i.e. real –rather than planned– investment.

From (7) and (8) it follows that:

$$\left(1-\frac{w}{a}\right)q=\dot{k} \tag{9}$$

Because S is the *real* saving and $\dot{k}$ the *real* investment, the equality (8) does not mean that there is equilibrium in the market of commodities. Without ruling out the possibility of equilibrium in the market of commodities, it is also possible for supply to be less or greater than the demand for commodities. However, according to the model, capitalists do not consider possible deviations of supply from demand for commodities to be reasons for corresponding increases or reductions in capital accumulation and production.

But if this possibility of the model does indeed create problems of comprehension, then the reader may consider that $\dot{k}$ symbolises both the planned and the real investment, in which case (8) evidently means that in the model there is always equilibrium in the market of commodities.

Both of the above cases mean that the model does not deal with problems of profit realisation and, consequently, that the growth cycles which describes are not caused by such problems.

For the profit rate r, the following holds:

$$r=\frac{q-wl}{k} \tag{10}$$

and because of (2), (7), (8) and (9):

$$r=\frac{\dot{k}}{k}=\left(1-\frac{w}{a}\right)\frac{1}{\sigma} \tag{11}$$

According to (11), it holds that the profit rate is:

(a) equal to $\dot{k}/k(=\hat{k})$, that is, equal to the rate of increase of capital, which is a consequence of the premise that only capitalists save, and moreover all their profits, and

(b) equal to the product of the share of profits and 'capital productivity'.

Unlike the former, the latter equality holds generally, only here it is specialised in the sense that 'capital productivity' is considered invariable.

From (1) or (2) it follows that

$$\hat{\sigma}=\hat{k}-\hat{q}=0 \tag{12}$$

and from this

$$\hat{k}\left(\frac{\dot{k}}{k}\right)=\hat{q}\left(=\frac{\dot{q}}{q}\right), \tag{13}$$

i.e. that, because σ = constant, k and q increase at the same rate. From (11) and (13) we get:

$$r=\frac{\dot{k}}{k}=\frac{\dot{q}}{q}=\left(1-\frac{w}{a}\right)\frac{1}{\sigma} \tag{14}$$

i.e., that r is also equal to the rate of change of the net product. And this equality is clearly the consequence of the premises (a) that only capitalists save and moreover all their profits and (b) that the capital-output ratio, σ, is constant.

In Goodwin's paper, the following holds for the supply of labour n:

$$n=n_0e^{\beta t},\ \beta = \text{positive constant.} \tag{15}$$

From (15) it follows that:

$$\hat{n} = \beta$$

i.e., that the supply of labour increases at the constant rate β[4].

For q, the following evidently holds:

$$q = l a \tag{16}$$

and consequently for employment l:

$$l = \frac{q}{a} \tag{16α}$$

It is clear, that:

$$l \leq n$$

Because the evolution of labour productivity is given from (3), (16a) means that the evolution of employment l depends solely on the evolution of the net product q or, conversely, the evolution of q on the evolution of l.

In Goodwin's paper, $\frac{\dot{(q/l)}}{q/l}$, $\dot{q}/q$ and $\dot{l}/l$ are the rates of change of labour productivity (q/l), of the net product (q) and of employment (l) respectively. Thus the equality in Goodwin's paper derives directly from (3) and

$$\frac{\dot{(q/l)}}{q/l} = \frac{\dot{q}}{q} = \frac{\dot{l}}{l} = \alpha \tag{17}$$

Furthermore, the equality

$$\frac{\dot{l}}{l} = \frac{1 - w/a}{\sigma} - \alpha \tag{18}$$

in the same paper derives as follows: From (16a) we get:

$$\hat{l} = \hat{q} - \hat{a} = \hat{q} - \alpha \tag{19}$$

4. Goodwin's observation that "the labour force is continually growing both through natural increase and through men 'released' by technological progress" is not correct, because this released manpower is already included in the labour force, i.e. in the labour supply, as defined by Goodwin himself. The release of labour force as a result of technological progress may increase unemployment, but not the labour force.

and from this, taking into consideration (13), we get (18).

Lastly, the equality

$$\dot{v}/v = \frac{1-u}{\sigma} - (\alpha + \beta) \left[= r - (\alpha + \beta) \right], \tag{20}$$

where u is the share of wages, derives as follows: From

$$v = \frac{l}{n}, \tag{21}$$

where v is the grade of employment, we get, taking into consideration (16a)

$$v = \frac{q/a}{n},$$

From this we get

$$\hat{v} = \hat{q} - \hat{a} - \hat{n} = \hat{q} - \alpha - \beta$$

If in this latter equation we replace (14) and

$$u = w/a, \tag{22}$$

we get (20).

(20) shows that the rate of change of the grade of employment is equal to the profit rate less a constant (α + β). Therefore, the rate of change of the grade of employment increases (decreases) by the same amount by which the profit rate increases (decreases). As we shall show below, the constant (α + β) is the average value of the profit rate. Consequently, (20) shows that the rate of change of the grade of employment is equal to the difference between the profit rate and the average value of the profit rate and therefore that the grade of employment increases (decreases) by the amount that the profit rate is higher (lower) than its average arithmetical value.

From (20) we get, taking into consideration (13)

$$\dot{v}/v = \dot{k}/k - (\alpha + \beta). \tag{23}$$

As we shall show later, the average value of the rate of accumulation is also equal to (α + β). Consequently, (23) means that the grade of employment increases (decreases) by the amount that the rate of accumulation is higher (lower) than its average arithmetical value.

Goodwin expresses premise 7 mathematically as follows:

$$\hat{w} = -\gamma + \rho v \tag{24}$$

where

$$\gamma, \rho > 0$$

and

$$\gamma < \rho$$

From (24) we get

$$d\hat{w}/dv = \rho$$

Also from (24) we get, setting $v = 0$,

$$\hat{w}_0 = -\gamma$$

and setting $\hat{w} = 0$,

$$v_0 = \gamma / \rho$$

Thus, (24) gives a straight line with the slope ρ. This straight line intersects the $\hat{w}$ -axis at point $\hat{w}_0 = -\gamma$ and the v-axis at point $v_0 = \gamma / \rho$. Consequently, condition $\gamma < \rho$ means that this straight line intersects the v-axis at a point $v_0 < 1$. This point is located, according to Goodwin, near the point $v = 1$. Also according to Goodwin, γ and ρ are large. The fact that ρ is large means that the slope of (24) is large. However, because (24) intersects the v-axis at a point *before* the others and *near* the point $v = 1$, it is clear, given that ρ is large, that γ too is –and moreover in relation to ρ– large, though always smaller than ρ. Because if purely and simply (a) $v_0 = \gamma / \rho < 1$ and consequently $\gamma < \rho$ and (b) ρ is large and/or γ large but not large also in relation to ρ, then the slope of (24) is indeed large and (24) intersects the v-axis before the point $v = 1$, however it does not intersect it near the point $v = 1$ but clearly at a point *after* it and *near* the point $v = 0$.

(24) is depicted in Figure 1.

The fact that the slope of (24) is large means that when v, i.e. the grade of employment, increases (decreases) by a certain amount, then it is not necessarily the real wage rate that increases (decreases), but the –negative (when $v < \gamma / \rho$) or positive (when $v > \gamma / \rho$)– rate of change of the real wage rate and indeed by an amount greater than the amount by which the grade of employment increased (decreased). Specifically, in the case where $v < \gamma / \rho$ and consequently

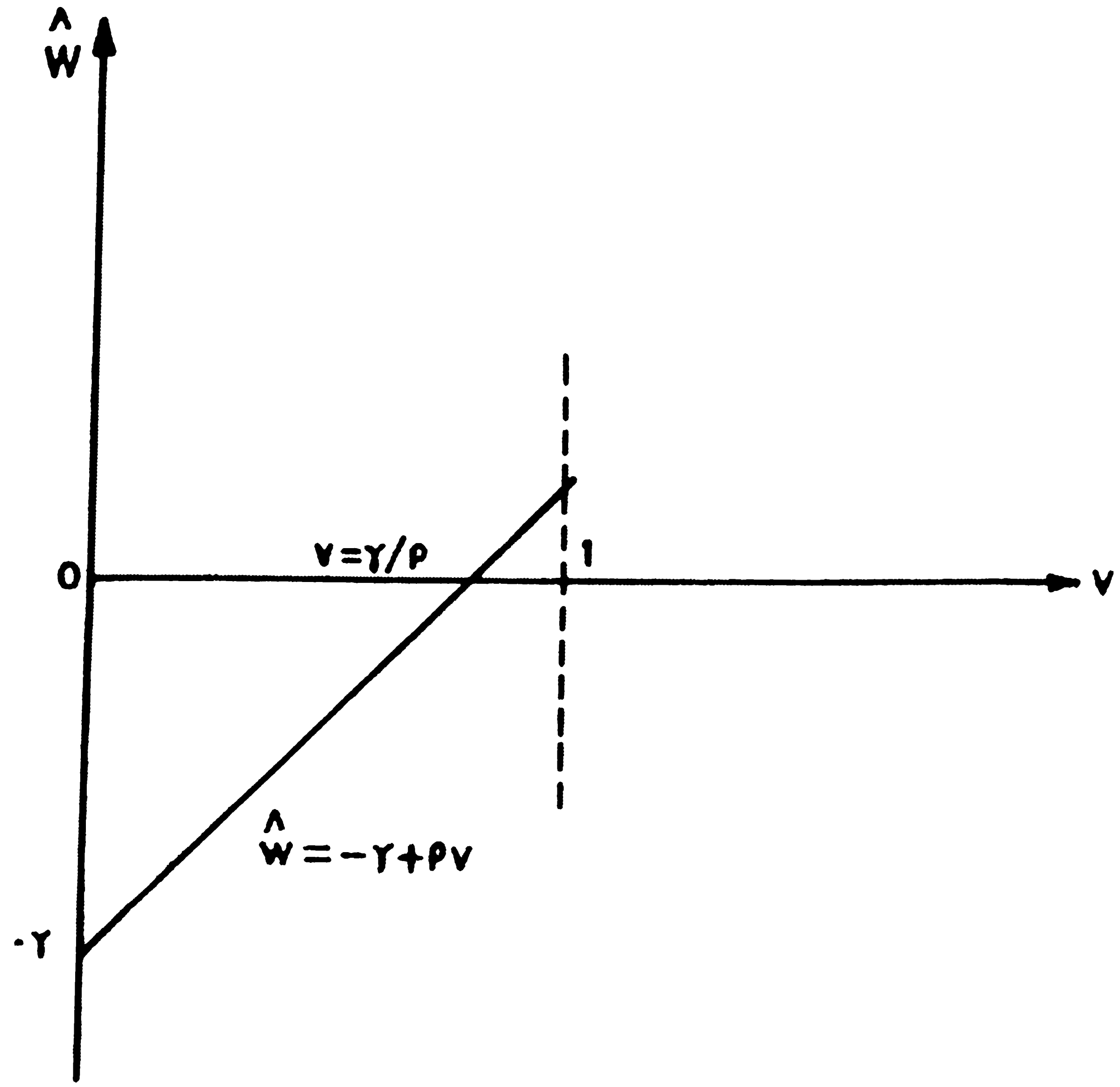

Figure 1

$\hat{w} < 0$, when v increases, then the –negative– $\hat{w}$ increases by a greater amount than v, however w *does not increase but decreases*, although, because the increasing $\hat{w}$ remains negative, it decreases at a *decreasing* rate. And, in the same case, when v decreases, then the –negative– $\hat{w}$ decreases by a greater amount than v, while w decreases too but at an *increasing* rate.

In the case where $v > \gamma / \rho$ and consequently $\hat{w} > 0$, when v increases, then the –positive– $\hat{w}$ increases by an amount greater than v and because $\hat{w}$

is positive and increases, w increases too at an increasing rate. And, in the same case, when v decreases, then the positive $\hat{w}$ indeed decreases by a greater amount than v, however w *does not decrease*, but, because the decreasing $\hat{w}$ remains positive, *increases at a decreasing* rate.

So the fluctuations of v do not necessarily cause fluctuations also of w, but of $\hat{w}$ and moreover by a greater amount than the fluctuations themselves. Thus, w does not increase (decrease) when v increases (decreases), but increases (decreases) only when $v > \gamma / \rho$ (when $v < \gamma / \rho$), irrespective of whether v increases or decreases. Hence, when $v > \gamma / \rho$, w increases not only with increasing but also with decreasing v, and, when $v > \gamma / \rho$, w decreases not only with decreasing but also with increasing v. The relations between $\hat{w}$ and v and between w and v in Goodwin's model are particularly important for comparing this model with the Marxian theory of economic crisis.

The

$$\dot{u}/u = \dot{w}/w - \alpha \tag{25}$$

in Goodwin's paper clearly follows from (22) and (4), and

$$\dot{u}/u = -(\alpha + \gamma) + \rho v \tag{26}$$

from (25) and (24).

From (21) we get

$$\dot{u} = \left[-(\alpha + \gamma) + \rho v\right] u, \tag{27}$$

i.e. Goodwin's equation (II).

From (20) it follows that

$$\dot{v} = \left[\frac{1}{\sigma} - (\alpha + \beta) - \frac{u}{\sigma}\right] v, \tag{28}$$

i.e. Goodwin's equation (I).

Eliminating the parameter t from (27) and (28), we get:

$$\frac{dv}{du} = \frac{\left[\frac{1}{\sigma} - (\alpha + \beta) - \frac{u}{\sigma}\right] v}{\left[-(\alpha + \gamma) + \rho v\right] u} \rightarrow$$

$$-(\alpha+\gamma)\frac{dv}{v}+\rho\, dv=\left[\frac{1}{\sigma}-(\alpha+\beta)\right]\frac{du}{u}-\frac{1}{\sigma}du$$

By integrating we get:

$$-(\alpha+\gamma)\log v+\rho v+c=\left[\frac{1}{\sigma}-(\alpha+\beta)\right]\log u-\frac{1}{\sigma}u \qquad (29)$$

(From (29) Goodwin's equation results:

$$\frac{1}{\sigma}u+\rho v-\left[\frac{1}{\sigma}-(\alpha+\beta)\right]\log u-(\gamma+\alpha)\log v=\text{constant})$$

From (29) we get

$$\varphi(u)=u^{\eta_1}e^{-\theta_1 u}=Hv^{-\eta_2}e^{\theta_2 v}=H\psi(v), \qquad (30)$$

i.e. Goodwin's equation (III), where

$$\theta_1=\frac{1}{\sigma},\quad \eta_1=\frac{1}{\sigma}-(\alpha+\beta),$$

$$\theta_2=\rho,\quad \eta_2=\gamma+\alpha$$

and

$H = e^c$ (c = the constant of equation (29)).

From (30) we get

$$\frac{d\varphi}{du}=-u^{\eta_1}\theta_1 e^{-\theta_1 u}+e^{-\theta_1 u}\eta_1 u^{\eta_1-1}=\left(-\theta_1+\frac{\eta_1}{u}\right)u^{\eta_1}e^{-\theta_1 u}$$
$$=\left(-\theta_1+\frac{\eta_1}{u}\right)\varphi$$

and

$$\frac{d\psi}{dv}=Hv^{\eta_2}\theta_2 e^{\theta_2 v}-He^{\theta_2 v}\eta_2 v^{\eta_2-1}=\left(\theta_2+\frac{\eta_2}{v}\right)Hv^{-\eta_2}e^{\theta_2 v}$$
$$=\left(\theta_2+\frac{\eta_2}{v}\right)\psi.$$

If we set

$$\frac{d\varphi}{du}=0 \text{ and } \frac{d\psi}{dv}=0,$$

we get (for $\varphi, \psi > 0$) respectively:

$$u=\bar{u}=\frac{\eta_1}{\theta_1} \text{ and } v=\bar{v}=\frac{\eta_2}{\theta_2}.$$

One can show (with the known method, namely by forming second differentials and investigating their signs), that at the point $d\phi / du = 0$, ϕ takes its maximum arithmetical value and at point $d\psi / dv = 0$, ψ takes its minimum arithmetical value.

The system has an unambiguous equilibrium solution, $(\bar{u},\bar{v})$, with

$$\bar{u}=\frac{\eta_1}{\theta_1}=1-\sigma(\alpha+\beta)$$

and

$$\bar{v}=\frac{\eta_2}{\theta_2}=\frac{\alpha+\gamma}{\rho}$$

Because by assumption $\alpha, \gamma, \rho > 0$ and consequently $\eta_2,\theta_2,\bar{v}>0$, it is clear that in order for this solution to be positive, i.e. in order also for $\bar{u}>0$, $\eta_1 > 0$ must hold or, which due to $\sigma > 0$ is the same thing, $\frac{1}{\sigma}>(\alpha+\beta)$. Goodwin presupposes not only that this solution is positive, but also that $\bar{u},\bar{v}<1$ holds. We also presuppose the same thing below.

If the initial arithmetical values of u and v, u_0 and v_0 are $u_0=\bar{u}$ and $v_0=\bar{v}$, then these remain invariable over time.

What happens though, when $u_0\neq\bar{u}$ and $v_0\neq\bar{v}$ holds for the initial arithmetical values of u and v? How then do u and v evolve over time? An initial reply to this question is provided by (27) and (28), if we write them as

$$\begin{aligned}\dot{u}&=\left[-(\alpha+\gamma)+\rho v\right]u\\ &=(-\eta_2+\theta_2 v)u\end{aligned}$$

$$= \left(-\frac{\eta_2}{\theta_2} + v \right) \theta_2 u$$
$$= (v - \bar{v}) \theta_2 u$$

and

$$\dot{v} = \left[\frac{1}{\sigma} - (\alpha + \beta) - \frac{u}{\sigma} \right] v$$
$$= (\eta_1 - u\theta_1) v$$
$$= \left(\frac{\eta_1}{\theta_1} - u \right) \theta_1 v = -(u - \bar{u}) \theta_1 v.$$

Because $\bar{v}, \bar{u}, \theta_2, \theta_1 > 0$, these equations give for v, u > 0, respectively

$$\operatorname{sign} \dot{u} = \operatorname{sign}(v - \bar{v}) \tag{27α}$$

and

$$\operatorname{sign} \dot{v} = -\operatorname{sign}(u - \bar{u}). \tag{28α}$$

(27a) and (28a) mean the following:
First of all, u increases (decreases) when v is greater (smaller) than $\bar{v}$, and v increases (decreases) when u is smaller (greater) than $\bar{u}$. This evidently means that when $v_0 \neq \bar{v}$ and $u_0 \neq \bar{u}$, the succession of points of the arithmetical values of v and of u trace in a v-u-graph –with vertical the v-axis and horizontal the u-axis– a clockwise motion. This motion may either lead asymptotically to the point of equilibrium $(\bar{u}, \bar{v})$ or move continuously away from that point, or produce a closed circle-like pattern around it. It can be shown that here, the latter occurs.[5]

So, ultimately the system has infinite solutions, each of which results for exogenously given initial values of v and u. The initial values of v and u are given for given H. Thus, at a given moment in time, i.e. for given t, the arithmetical values are given for all the other variables of the model. Because, given t, both n and a are determined. The exogenously given v and n determine l. Whilst l and the determined a determine q. The determined q and exogenously given u determine lw. The determined lw and l determine

5. See E. Wolfstetter, op. cit., p. 139c.

w. The given v determines $\hat{w}$. The given u and the known and invariable assumed σ determine r and consequently $\hat{k}$. The determined q and the known σ finally determine k. So, all the magnitudes are –for given initial values of v, u and t– unambiguously determined.

Their changes come about –when they come about, i.e. when the initial values of v and u are not equal to $\bar{v}$ and $\bar{u}$ respectively– as a consequence of changes in time and in a way that is determined, with respect to v and u, by (27) and (28).

Clearly, according to the mathematical formulation of the model, there are no causal relations between the variables. In the model, the changes in time appear in the end as a reason for the changes of all variables. However, because –as a consequence of changes in time– all the changes in the variables come about simultaneously, it cannot be said that the change in a certain variable constitutes the cause of a change in another variable, i.e. it is not possible to speak of causal relations between the variables.

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But before we describe the changes in the variables of the system over time, we consider it useful to comment on the long-term average arithmetical values of v and u, which are also the arithmetical values of those variables that define the equilibrium solution of the system, as well as on the long-term average values of $\bar{\hat{w}}, (1-\bar{u}), \bar{r}, \bar{\hat{q}}$ and $\bar{\hat{k}}$, the rate of change of the real wage rate, the share of profits, the profit rate, the rate of increase of the net product and the rate of increase of accumulation respectively.

In Goodwin's model:

$$\bar{v} = \frac{\eta_2}{\theta_2} = \frac{\alpha + \gamma}{\rho} \tag{31}$$

and

$$\bar{u} = \frac{\eta_1}{\theta_1} = 1 - (\alpha + \beta)\sigma \tag{32}$$

Therefore, because of (32),

$$(1-\bar{u}) = (\alpha + \beta)\sigma \qquad (33)$$

and, because of (14) and (32),

$$\bar{r} = \bar{\bar{q}} = \hat{\bar{k}} = (1-\bar{u})\frac{1}{\sigma} = (\alpha + \beta)\sigma\frac{1}{\sigma} = (\alpha + \beta) \qquad (34)$$

As is known, for the rate of change of the share of wages, the following holds

$$\hat{u} = \hat{w} - \alpha \qquad (25)$$

and, because of (24)

$$\hat{u} = -(\gamma + \alpha) + \rho v \qquad (26)$$

The average arithmetical value of the grade of employment, $\bar{v}$, is that arithmetical value of the grade of employment which would result if the share of wages, u, remained invariable, i.e. if the rate of change of the share of wages was equal to zero, and therefore the share of wages was equal to its average arithmetical value.

Consequently, if

$$u = \bar{u} \qquad (35)$$

and therefore

$$u = \hat{\bar{\bar{u}}} = 0 \qquad (36)$$

then

$$v = \bar{v} \qquad (37)$$

So, from (26), taking into consideration (36) and (37), we get:

$$\hat{u} = \hat{\bar{\bar{u}}} = -(\alpha + \gamma) + \rho\bar{v} = 0 \qquad (38)$$

and from this (31).

From (14), if we take into consideration (26) and (31), the following results:

$$\hat{r} = \hat{\hat{k}} = \hat{\hat{q}} = \widehat{(1-u)}$$

$$= -\frac{u}{1-u}\hat{u}$$

$$= \frac{u}{1-u}\left[(\alpha+\gamma)-\rho v\right]$$

$$= \frac{u}{1-u}\,\rho\,(\bar{v}-v) \qquad (39)$$

It is immediately apparent from (39) that $\bar{v}$ can be defined also as that grade of employment which would result if the rate of change of the profit rate, the rate of change of the share of profits (and consequently also the rate of change of the share of wages), the rate of change of the rate of increase of the net product and the rate of change of the rate of accumulation were always equal to zero and consequently the profit rate, the share of profits (and consequently also the share of wages), the rate of change of the net product and the rate of accumulation did not vary, which means: if all these magnitudes were always equal to their average arithmetical values. Because, for $\hat{r}=\hat{\hat{k}}=\hat{\hat{q}}=\widehat{(1-u)}=0$ (and consequently $\hat{u}=0$) and u, $\rho > 0$, (37) results from (39), i.e. $v=\bar{v}$.

Why is $\bar{v}$ equal to $(\alpha+\gamma)/\rho$? This question can be answered if we take into consideration the fact that $\bar{v}$ is that v, for which $u=\bar{u}=$ constant holds and consequently $\hat{u}=\hat{\bar{u}}=0$. But when does u not change? As emerges from (25), u does not change when the real wage rate increases always at the same rate at which labour productivity also increases. If the rate of increase α of labour productivity was equal to zero, then $\bar{v}$ would be, as emerges from (26) for $\hat{u}=0$ and $\alpha=0$, equal to γ/ρ, i.e. equal to that v which, as shown by (24) entails $\hat{w}=0$. When, however, the rate of increase α of labour productivity is positive, then, as emerges from (26) for $\hat{u}=0$ and $\alpha>0$, $\bar{v}$ is equal to $(\alpha+\gamma)/\rho$, i.e. greater by α/ρ than in the case where $\alpha=0$. So, α/ρ is the amount, by which, when labour productivity increases at rate α, v must increase beyond that value of v which entails $\hat{w}=0$ and consequently invariable w, in order for w to increase so much that u remains invariable, i.e. for w to increase by the same percentage as labour productivity. Indeed, if in (24) we set $v=(\alpha+\gamma)/\rho$ we get $\hat{w}=\alpha$.

The average value $\hat{w}$ of the rate of increase of the real wage rate is that which would result if always $u=\bar{u}$ and consequently $\hat{u}=\hat{\bar{u}}=0$. For the average value $\bar{\hat{w}}$ of $\hat{w}$, the following results from (25) for $\hat{w} = \hat{\bar{u}} = 0$:

$$\hat{u} = \hat{\bar{u}} = \bar{\hat{w}} - \alpha = 0 \rightarrow \bar{\hat{w}} = \alpha$$

We get the same result by replacing in (24) the average arithmetical value of v, i.e. (31). Hence from (24):

$$\bar{\hat{w}} = -\gamma + \rho\bar{v} = -\gamma + \rho\frac{\alpha + \gamma}{\rho} = \alpha$$

Therefore, the average value of the rate of increase of the real wage rate may be defined as that value which would result if the grade of employment was always equal to its average arithmetical value.

So, the average value of the increased rate of the real wage rate is, as we have seen, equal to the rate of increase of labour productivity and –because this is constant– constant.

As we saw above, when the real wage rate and labour productivity increase at the same rate, the share of wages remains invariable. Therefore, the average value of the share of wages may be defined as that value which would result if the real wage rate and labour productivity increased always at the same rate, or –which is the same thing– as that value which would result if the grade of employment was always equal to the average or –which is the same thing– if the rate of increase of the real wage rate was always equal to the average.

Lastly, the average values of the profit rate, of the share of profits, of the rate of increase of the net product and of the rate of accumulation may be defined as those values of the variables which would result if the rate of increase of the real wage rate was always equal to the average.

But why are the average values of the variables $u, (1-u), r, \hat{q}, \hat{k}$ what they are? Why, for example, is the average arithmetical value of r equal to $(\alpha + \beta)$? Because, as shown by (14), all these variables are interconnected, it is sufficient to reply to this question with respect to just one of the aforesaid values. The answer to the question "why is the average arithmetical value of a certain of these variables what it is?", clearly answers –in combination with (14)– also the question "why is the average value of each of the other variables what it is?".

We choose to answer the question "why is the average value $\bar{\hat{q}}$ of $\hat{q}$ equal to $(\alpha + \beta)$?" For the rate of increase $\hat{q}$ of q, it emerges from (16) that

$$\hat{q} = \hat{a} + \hat{l} = \alpha + \hat{l} \qquad (40)$$

The average arithmetical value $\bar{\hat{q}}$ of the rate of increase of q is:

$$\bar{\hat{q}} = \bar{\hat{a}} + \bar{\hat{l}} = \alpha + \bar{\hat{l}} \qquad (41)$$

As is known, $l \leq n$. However, in periods when the grade of employment decreases, in which case $\hat{v} = \hat{l} - \hat{n} < 0$ and consequently $\hat{l} < \hat{n}$, l increases by a lesser percentage than n and in periods when the grade of employment increases, in which case $\hat{v} = \hat{l} - \hat{n} > 0$ and consequently $\hat{l} > \hat{n}$, l increases by a greater percentage than n. Consequently, the average value $\bar{\hat{l}}$ of $\hat{l}$ may be equal to the constant rate of increase $\hat{n}(=\beta)$ of n.

And it is indeed equal to $\hat{n}(=\beta)$, because the average value $\bar{\hat{l}}$ of $\hat{l}$ is that which would result if v was always equal to its own constant average value $\bar{v}$. Because

$$v = \bar{v}\left(= \frac{\gamma + \alpha}{\rho}\right) = \text{constant}$$

means

$$\bar{\hat{v}} = \bar{\hat{l}} - \hat{n} = \bar{\hat{l}} - \beta = 0$$

and therefore

$$\bar{\hat{l}} = \hat{n}\ (=\beta)$$

where $\bar{\hat{l}}$ is the average value of $\hat{l}$. Because of (42), the following results from (41)

$$\bar{\hat{q}} = \alpha + n = \alpha + \beta$$

So why is the average value of $\hat{q}$ equal to $\alpha + \beta$? For obvious reasons q increases at the rate of $\alpha + \hat{l}$, i.e. at a rate equal to the aggregate of the rates of increase of labour productivity and of employment. Consequently, q increases in the long run at a rate equal to the aggregate of the rate at which productivity increases in the long run, and the rate at which employment increases in the long run. Productivity always increases, and consequently also in the long run, at rate α. Employment increases in the long run, because in the long run the grade of employment remains invariable, at the same rate as the labour force also increases, i.e. at rate β. This is why q in the long run increases at the rate of $(\alpha + \beta)$, that is, $\bar{\hat{q}} = (\alpha + \beta)$.

Because, according to (14) and for reasons we have already explained, in Goodwin's model $r = \hat{k} = \hat{q}$ and consequently $\bar{r} = \bar{\hat{k}} = \bar{\hat{q}}$, the aforementioned reasons why $\bar{\hat{q}} = (\alpha + \beta)$, are at the same time also the reasons why $\bar{r} = (\alpha + \beta)$ and $\bar{\hat{k}} = (\alpha + \beta)$.

Lastly, because –according to (14)– $(1-u) = \bar{\hat{q}}\sigma$ and consequently $(1-\bar{u}) = \hat{q}\sigma$ and $u = 1-\hat{q}\sigma$ and consequently $\bar{u} = 1-\bar{\hat{q}}\sigma$, the aforesaid reasons are also the reasons why $(1-\bar{u}) = (\alpha + \beta)\sigma$ and $\bar{u} = 1-(\alpha + \beta)\sigma$.

Because the average value of the share of wages and consequently also the average arithmetical value of the share of profits are constant, because, that is, in the long run the distribution of income remains invariable, real wages wl and real profits $(q - wl)$ in the long run increase at the same rate that q and k increase in the long run, i.e. at a rate equal to $\bar{r}$.

For the rate of change $(\widehat{\overline{wl}})$ of the average arithmetical value $\overline{wl}$ of wl we get:

$$\widehat{\overline{wl}} = \bar{\hat{w}} + \bar{\hat{l}} = (\alpha + \beta)(= \bar{r})$$

and for the rate of change $(\widehat{\bar{q}-\overline{wl}})$ of the average value $(\bar{q}-\overline{wl})$ of $(q - wl)$:

$$\widehat{(\bar{q}-\bar{w}\bar{l})} = \hat{\bar{q}}\frac{\bar{q}}{\bar{q}-\bar{w}\bar{l}} - (\hat{\bar{w}}+\hat{\bar{l}})\frac{\bar{w}\bar{l}}{\bar{q}-\bar{w}\bar{l}}$$
$$= (\alpha+\beta)(1-\bar{u}) - (\alpha+\beta)\bar{u}$$
$$= (\alpha+\beta)(=\bar{r}).$$

So, in the long run in Goodwin's model
(a) wages, profits, and because all the profits and only these are invested, investments and constant capital increase at the same rate that net product increases and
(b) the profit rate is equal to the rate of increase of capital.

The average value of the profit rate, which results in Goodwin's model, results on the condition that, as also in Goodwin all the profits and only the profits are invested, and in a model, which does not describe growth cycles but the Marxian falling tendency of the profit rate and in which, consequently, σ and, in the long run, (1 – u) do not remain, as in Goodwin, invariable, but increase, as the lowest limit of the continuously decreasing profit rate. In this model, σ increases continuously but without surpassing a limit equal to $1/(\alpha + \beta)$ and the profit rate continuously decreases without falling below a limit equal to $(\alpha + \beta)$.[6]

From what we have set out above regarding the average arithmetical values of the variables of Goodwin's model, it follows that the constancy of the average values of the variables $v, \hat{w}, u, (1-u), \hat{q}, \hat{wl}, (q-\hat{wl}), r$ and $\hat{k}$ is not due to actual economic causes, but to Goodwin's premises, according to which α, β, γ, ρ and σ are constants – premises which Goodwin states for the sake of convenience and not for actual economic reasons. Consequently, the constancy of the average arithmetical values of v, u, (1 – u) and r does not explain, as Goodwin himself maintains, the historic, i.e. the actual evolution of the magnitudes v, u, (1 – u) and r, but is purely and simply the logical consequence of the assumed –for the sake of convenience– constancy of the magnitudes α, β, γ, ρ and σ, constancy which is not a given in economic reality.

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6. See G. Stamatis, *Die "spezifisch kapitalistischen" Produktionsmethoden und der tendenzielle Fall der allgemeinen Profitrate bei Karl Marx*, Berlin 1977, pp. 279-281, and Γ. Σταμάτης, *Προβλήματα Μαρξιστικής Οικονομικής Θεωρίας*, Αθήνα 1986, σσ. 206-209.

Having concluded the above discussion of the average values of the variables of the model, let us now return to the question as to how these variables change over time.

As already noted above, in Goodwin's model all the variables are solely and exclusively functions of time and of the exogenously given initial values of v and u. The changes in the variables are solely and exclusively the results of time and changes in time. And because the changes (that come about as a consequence of the changes in time) in the variables all come about simultaneously, it is impossible in the framework of the model to speak about causal relations between the variables. Hence, it cannot be said that the changes in one or more variables constitute the cause for the changes in the rest. Thus, according to the model always, the economic interpretation of the cycle is impossible, since we could consider it equally correct that the changes in the real wage rate cause the changes in the rate of capital accumulation or, conversely, that the changes in the rate of capital accumulation are the cause for the changes in the real wage rate. The prime cause of the growth cycles is not determined by the model, but exogenously.

In order to be able to interpret the cycle from an economic viewpoint, we introduce time lags into the model. Because only thus can we describe the movement of Goodwin's cycle as the result of causal relations. As the prime cause we choose the profit rate and rate of capital accumulation which is determined therefrom, because it is the profit rate that decisively determines the economic activity of capitalists.

We begin from the point in the cycle, at which $u=\bar{u}$ and $v = v_{min}$. We assume, for the sake of convenience, that $v_{min} > \gamma / \rho$. At this point, $r=\bar{r}$ and $\hat{k}=\bar{\bar{k}}$. The magnitude r is increasing. In the next period, as a consequence of the increase of r and because all profits are invested, $\hat{k}$ increases, and because now $\hat{k}=\bar{\bar{k}}$, v also increases. As a consequence of the increase of v, and because, by assumption $v > \gamma / \rho$, w increases. However, because $v<\bar{v}$, w increases by a lesser percentage than labour productivity, with the consequence that u decreases, $(1-u)$ increases and therefore the profit rate also increases. This new increase of the profit rate causes, in the next period, anew the chain of reactions that we have just described. This is repeated from period to period until we reach a point very close to the point where

$v = \bar{v}, u = u_{max}$ and consequently $r = r_{max}$. Phase I of the cycle ends here.

Because $v > \gamma / \rho$, the increase of $\hat{k}$, which was caused by the last increase of r, increases v, which thus becomes greater than $\bar{v}$. This increase of v entails an increase of w. However, because now $v > \bar{v}$, w increases by a greater percentage than labour productivity with the consequence that u increases and therefore $(1 - u)$ and r decrease. The decrease of r entails in the next period an equal percentage reduction of $\hat{k}$. This reduction of $\hat{k}$ however does not entail a decrease, but an increase of v. The magnitude v increases with decreasing $\hat{k}$, because $\hat{k}$, though decreasing, remains higher than the average value of $\bar{\bar{k}}$. This movement continues up to a point near the point at which $v = v_{max}$ and $u = \bar{u}$ and consequently $r = \bar{r}$ and $\hat{k} = \bar{\bar{k}}$. Phase II ends at this point of the cycle.

The immediately ensuing decrease of $\hat{k}$ entails, because now $r < \bar{r}$ and $\hat{k} < \bar{\bar{k}}$, a decrease of v. The decrease of v entails a decrease of $\hat{w}$. But because $v > \gamma / \rho$ always holds, the decreasing $\hat{w}$ remains positive. Thus, w increases, but at a decreasing rate of increase. Because $v > \bar{v}$, w increases by a greater percentage than labour productivity. For this reason u increases and $(1 - u)$ and r decrease. The decrease of r entails in the next period an equal percentage reduction of $\hat{k}$, and thus the chain of reactions which we just described continues from period to period up to the point shortly before the point at which $v = \bar{v}$ and $u = u_{max}$ and consequently $r = r_{min}$ and $\hat{k} = \hat{k}_{min}$, which is where Phase III of the cycle ends.

The next decrease of $\hat{k}$ beyond this point entails the decrease of v to below $\bar{v}$. The magnitude $\hat{w}$ decreases, but w, because $v > \gamma / \rho$, increases. However, because now $v < \bar{v}$, w increases by a lesser percentage than labour productivity with the consequence that u decreases and $(1 - u)$ and r increase. The increase of r entails in the next period the increase of $\hat{k}$. The increase of $\hat{k}$ has the consequences which we have just described. This continues until just before the point at which $v = v_{min}$ and $u = \bar{u}$ and consequently $r = \bar{r}$ and

$\hat{k} = \bar{\bar{k}}$. Phase IV, the last of the cycle, ends here. With the next increase of $\hat{k}$ beyond $\bar{\bar{k}}$, we re-enter Phase I of the cycle, which we have already described.

In the above description of the cycle, we assumed for the sake of convenience that $v_{min} > \gamma / \rho$ and consequently that $v > \gamma / \rho$ always holds. This assumption consequently rules out those cases in which $v < \gamma / \rho$ and therefore $\hat{w}$ is negative so that with decreasing v not only $\hat{w}$ but also w decrease and with increasing v, although the –negative– $\hat{w}$ increases, w decreases. Evidently, these cases can only appear in Phases I and VI of the cycle, because only in these Phases is $v < \bar{v}\left[= (\gamma + \alpha)/\rho\right]$ and consequently is it possible for $v < \gamma / \rho$. In Phases II and III $v > \bar{v}\left[= (\gamma + \alpha)/\rho\right]$ and, for all the more reason, $v > \gamma / \rho$. As one may easily ascertain, nothing changes in the cycle if we accept that $v_{min} < \gamma / \rho$.

Of particular importance for understanding the growth cycles described by the model are the following:

First of all, v does not increase (decrease) always when $\hat{k}$ increases (decreases), but it increases (decreases) by as much as $\hat{k}$, irrespective of whether it increases or decreases, is higher (lower) than its average value. Thus, in Phases I and II, v increases, because $\hat{k}$ remains greater than $\bar{\bar{k}}$, even though in Phase I it increases and in Phase II it decreases. And in Phases III and IV it decreases, because $\hat{k}$, although it decreases in Phase III and increases in Phase IV, remains lower than $\bar{\bar{k}}$.

Also, u does not increase (decrease) always when v and consequently $\hat{w}$ increases (decreases), but increases (decreases) when v is, irrespective of whether it increases or decreases, is higher (lower) than the average value of $\bar{v}$, because only then is $\hat{w}$ higher (lower) than α. Thus, in Phases II and III, u increases because v, even though it increases in Phase II and decreases in Phase III, is in both Phases greater than $\bar{v}$. And u decreases in Phases IV and I because v, even though it decreases in Phase IV and increases in Phase I, is in both Phases smaller than $\bar{v}$.

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Following the above analysis of the trade cycle described by Goodwin's model, we may now venture certain estimations regarding its importance and its relation to the Marxian theory of overaccumulation crisis.

In Goodwin's model only one commodity is produced, which is used as a means of consumption, but also as a means of production. According to Goodwin, all the economic magnitudes of his model are real, not nominal, magnitudes. However, in his model, $\dot{k}$ symbolises not only the surplus product, i.e. a real economic magnitude, but also profit, i.e. the nominal economic magnitude corresponding to the surplus product. As a consequence of this, it is clear that the other real economic magnitudes are at the same time also nominal magnitudes – which means that the price of the produced commodity is always equal to unity.

Likewise, w symbolises not only the real but also the nominal wage rate. This is equivalent to a linking of wages with the price index, such that in the case where the price of the commodity changes over time, the nominal wage rate increases by the percentage given by (24) plus the rate of change of the price of the commodity, and thus the real wage rate increases by the percentage given by (24).

So, relation (24) is a type of Phillips' curve[7] in the case where workers have no money illusions, regarding the consequances of price increases in the real wage rate. This is immediately apparent, if (24) is written as

$$\hat{w} = (\rho - \gamma) - \rho(1 - v) \qquad (24\alpha)$$

where $(1 - v)$ is the degree of unemployment and $\hat{w}$ the rate of change of the real and nominal wage rate.[8]

Relation (24a) is clearly of neoclassical origin.

7. See A.W. Phillips, The Relation between Unemployment and the Rate of Change of Money Wage Rates in the United Kingdom, 1861-1957, Economica, Vol. 25 (1958), pp. 283-299.

8. Here, (24) is a straight and not, like the Phillips' curve, a curve that bends towards the beginning of the axes, because Goodwin, as he himself says, depicts the relation between v and not, as he would like, with the right branch of a parabola, but for the sake of convenience, by way of approximation with (24), i.e. with a straight line.

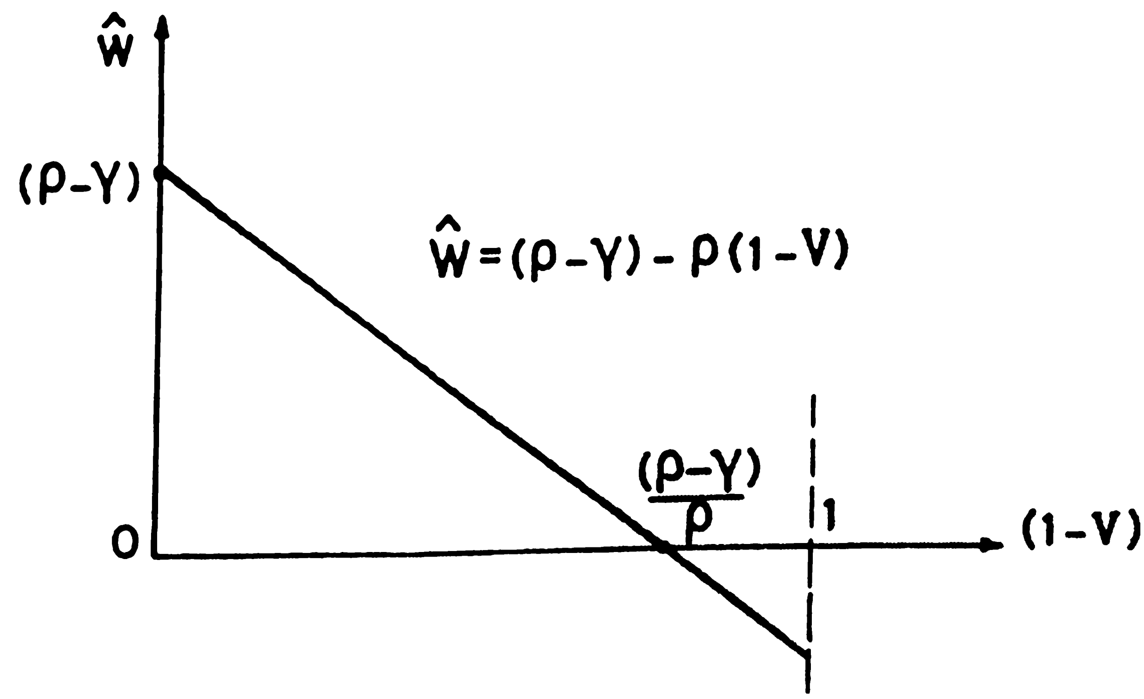

Figure 2

As we have shown, Goodwin's system has one equilibrium solution with $u = \bar{u}$ and $v = \bar{v}$. In general, however, the solution of the system consists of infinite 'circles' around the point of the equilibrium solution, each of which results for different exogenously given initial values of u and v. The circle which results for certain exogenously given initial values of u and v represents, for these exogenously determined initial values of u and v, the solution of the system. A corollary of the fact that the solution depends on the initial values is that the model does not allow anything to be said about the intensity of the fluctuations. Because the intensity of the fluctuations depends on the magnitute of the cycle, which depends on the assumed initial values of v and u.

With regard to the correctness of Goodwin's conviction that his model constitutes a mathematical formulation of Marx's views concerning growth cycles, we shall confine ourselves to an examination of whether and to what extent the explicit assumptions of Goodwin's model coincide with corresponding assumptions of the Marxian theory.

According to Goodwin's assumption no. 1, technical progress is "disembodied". This means that the percentage increase of labour productivity is independent of the percentage increase of constant capital (and constant).

According to Marx, in contrast, labour productivity depends on the quantity of means of production per worker, i.e. on the technical composition of capital: Labour productivity increases, because the technical composition of capital increases, and moreover by a lesser percentage than the latter. Thus, according to Marx, each percentage increase of labour productivity presupposes a greater percentage increase of the technical composition of capital. This clearly implies that in Marx, the capital-output ratio calculated in value terms increases with increasing labour productivity.[9] In Goodwin, in contrast, this ratio remains invariable.

In Goodwin's model, the following technical composition of capital T holds:

$$T = k/l = a(k/q) = a\sigma, \text{ where } \sigma = \text{constant}$$

and for its percentage change:

$$\hat{T} = \hat{a}(=\alpha)$$

So, in Goodwin's model, the technical composition of capital changes by the same percentage by which labour productivity also changes. The consequence of this is that in this model the capital-labour ratio calculated in value terms, i.e. the ratio of objectified labour in the means of production to direct (living) labour, remains invariable. The objectified labour in means of production k, the labour value, that is, of means of production k, is equal to k (1/a), where 1/a symbolises the value of one unit of means of production,[10] and direct labour is equal to l. Consequently, their ratio (Q) is:

$$Q = \frac{k}{al} = \frac{1}{a}T$$

For the percentage change of this ratio we get

$$\hat{Q} = \hat{T} - \hat{a}$$

and – because of $\hat{T} = \hat{a}$

$$\hat{Q} = \hat{a} - \hat{a} = 0$$

9. See G. Stamatis, *Die "spezifisch kapitalistischen" Produktionsmethoden...*, op cit., pp. 30-40 and 47-54, and Γ. Σταμάτης, *Προβλήματα Μαρξιστικής Οικονομικής Θεωρίας*, Αθήνα 1986, σσ. 116-129.

10. See G. Stamatis, *Die "spezifisch kapitalistischen" Produktionsmethoden...*, op cit., p. 51c.

So, in Goodwin's model Q remains invariable. (This also results directly from the invariability of σ and from the fact that Q = σ. Because

$$Q = \frac{k}{al} = \frac{1}{q} = \sigma)$$

In Goodwin's model, u and consequently (1 – u) remain constant in the long run – which means that the real wage rate increases in the long run at the same rate that labour productivity also increases. The fact that in Goodwin 1 – u remains in the long run constant, in combination with the fact that σ too is constant, evidently entails that r too remains constant in the long run.

In Marx, in contrast, in the long run u increases and 1 – u decreases, because in the long run the rate of exploitation $m'(= 1 - u)/u$ increases. This is a consequence of the fact that according to Marx, although the real wage rate increases in the long run, it does so at a lower rate than the rate at which labour productivity increases.[11]

However, in Marx the profit rate does not increase in the long run, nor does it remain invariable but decreases – despite the increasing rate of exploitation. If we disregard variable capital, then according to Marx the profit rate is:

$$r = \frac{1-u}{u} : E = \frac{m'}{E},$$

where E is the value composition of capital, and its percentage change is:

$$\hat{r} = \hat{m}' - \hat{E} < 0 \rightarrow \hat{m}' < \hat{E}$$

So, the profit rate decreases in Marx, because with increasing labour productivity although m' $(\hat{m}' > 0)$ increases, at the same time $E(\hat{E} > 0)$ also increases and moreover at a faster rate than $m'(\hat{E} > \hat{m}')$.[12] According to

11. See G. Stamatis, *Die "spezifisch kapitalistischen" Produktionsmethoden...*, op cit., pp. 62-97, and Γ. Σταμάτης, *Προβλήματα Μαρξιστικής Οικονομικής Θεωρίας*, Αθήνα 1986, σσ. 133-138.

12. See G. Stamatis, *Die "spezifisch kapitalistischen" Produktionsmethoden...*, op cit., pp. 143-159, 221-236.

Marx, E increases because Q and m′ increase. And indeed, because the following holds for the relation between E and Q

$$E = Q(1 + m')$$

and consequently for the relation between $\hat{E}$ and $\hat{Q}$

$$\hat{E} = \hat{Q} + \hat{m}' \frac{m'}{1 + m'},$$

when $\hat{Q}, m' > 0$, then $\hat{E} > 0$, i.e. when Q and m′ increase, E also increases, and moreover –because, as results from the latter equation, $\hat{E} > \hat{Q}$ – faster than Q.

With regard to Goodwin's assumptions 2 to 5, we can accept that they are not incompatible with the Marxian theory.

The same appears to be true also for assumption 7, according to which "the real wage rate rises in the neighbourhood of full employment"). But this is not the case. Because although this holds in Marx, it does not hold in this mild form in Goodwin's model. As we have already seen, in Goodwin's model the increase by a certain amount of the grade of employment beyond the point $v = \gamma / \rho$ leads to an increase not only of the real wage rate but also of the rate of increase of the real wage rate and moreover by an amount that is greater than the amount of increase of the grade of employment. Also, in Goodwin's model, for v, $0 < v < \gamma / \rho$, when v increases, the real wage rate *decreases*.

This is connected, as we have shown, with neoclassical views regarding the relation between wage rate and employment that is expressed by Phillips' curve, but in no way with the corresponding views of Marx.

A further difference between Marx and Goodwin is that, while in Goodwin the degree of employment of capital is equal to unity (= full employment of capital, irrespective of which phase the cycle is in), in Marx the degree of employment of capital varies during the economic cycle.[13]

Furthermore, in Marx one does not encounter Goodwin's premise, according to which all profits are invested. On the contrary, it appears that

13. The changes in the degree of employment of capital in the different phases of the cycle are taken into consideration by J. Glombowski in his two papers cited above.

according to Marx capitalists in the long run – in order to achieve a certain and constant increase in labour productivity, invest an increasing percentage of their profits.[14]

We believe that in general, Goodwin's knowledge of the Marxian theory is sorely lacking. This is quite apparent from the fact that his references to Marx himself are vague or even incorrect. Towards the end of his paper, after analysing the growth cycles described by his model, Goodwin writes:

"This is, I believe, essentially what Marx meant by the contradiction of capitalism and its transitory resolution in booms and slumps. It is, however, un-Marxian in asserting that profitability is restored not (necessarily) by a fall in real wages but rather by their failing to rise (at the same rate - Tr. N.) with productivity".

Clearly, Goodwin believes that according to Marx, in the recovery phase, the profit rate increases not because the real wage rate increases at a slower rate than labour productivity, but because it decreases. It is not necessary to stress just how mistaken this view is. Suffice it for us to note that according to Marx, the real wage rate, in the medium and long run, never decreases, but increases –always however at a slower rate than labour productivity.[15] (The result of this in Marx is that the average arithmetical value of the share of wages decreases and does not, as in Goodwin, remain invariable).

Another crucially important difference between Goodwin and Marx is the following: As already pointed out above, Goodwin's model does not describe causal relations between the economic magnitudes. Thus, this model does not address what for Marx was the extremely important question of whether the wage rate depends on the rate of accumulation, or, conversely, the rate of accumulation depends on the wage rate. As is known, Marx underlines that the rate of accumulation does not depend on the wage rate but, on the contrary, the wage rate on the rate of accumulation.[16]

If in Goodwin's model we introduce value increases and assume that for each given v the rate of increase of the nominal wage rate is equal to that defined by (24) plus a percentage less than that of the price increase, then

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14. See G. Stamatis, *Die "spezifisch kapitalistischen" Produktionsmethoden...*, op cit., pp. 232-236, and Γ. Σταμάτης, *Προβλήματα Μαρξιστικής Οικονομικής Θεωρίας*. Αθήνα 1986, σσ. 148-205.
15. See K. Marx, *Das Kapital*, Bd. III, MEW Bd. 25, p. 631.
16. See K. Marx, *Das Kapital*, Bd. I, op cit., p. 648.

the model has a global stable equilibrium solution: the growth cycles decrease continuously over time and tend to disappear.[17]

If, lastly, in Goodwin's original model one takes into consideration the fact that labour productivity does not always increase at a constant rate, but in boom phases at a rate higher than in slump phases, then the intensity of the growth cycles decreases in relation to the original model: v and $\hat{w}$ vary as a consequence of the changes of $\hat{k}$ less than in the original model.

We noted above that the constancy of the average values of the variables in Goodwin's model is not due to economic reasons, but is the logical consequence of the assumed constancy of α, β, γ, ρ and σ. But the magnitudes of the average values of certain variables such as v and u also depend on the size of these parameters. The size of $\bar{v}$ and $\bar{u}$ cannot therefore be interpreted by the model.

For the same reasons, the steady state equilibrium of the system cannot be interpreted as its economic equilibrium, i.e. it is not possible for the average values of the variables of the system which define its steady state equilibrium to be interpreted as those values of the variables in the case of economic equilibrium. Because, if $\bar{v}$ for example is equal to 0.5 –and the model presents no reason whatsoever why $\bar{v}$ should necessarily be approximately equal to unity, for example equal to 0.98–, then it is clearly somewhat difficult for us to consider this 0.5 as that grade of employment, which means that there is equilibrium in the labour market. Unless of course "equilibrium in the labour market" does not mean "very high grade of employment" but simply "constant grade of employment".

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Goodwin's model and its analysis which we have attempted here provide a basis for an estimation of the scientific worth of the usual mathematical formulations of economic theory. It may emerge from such an estimation that numerous mathematical formulations of economic theory, on the pretext of precise formulation that is logically cohesive and free of contradictions but also, very often, on the pretext of 'sleek' formulation, brush aside with very little regard the issue of the completeness of the

17. See E. Wolfstetter, op cit., p. 147 et seq.

theory, thereby expelling from economic science those questions – and there are many –which the aforesaid formulations themselves are unable to treat. Just like some individual– not particularly skilful but with a very well developed sense of neatness – who, after packing his suitcase, within the limits of his ability, takes a pair of scissors and diligently cuts away all those parts of its content which are hanging out.

Appendix

Rules for calculating the rate of change of a variable

The rate of change, i.e. the percentage change $\hat{a}$ of a variable (a) in time (t) is equal to the ratio of the absolute change of the variable (da/dt) to the arithmetical value (a) of this variable at the point in time of the change:

$$\hat{a} = \frac{da}{dt}\frac{1}{a}$$

When a variable is a function of other variables, which in turn are functions of time, then the rate of change is a function of these variables and of their rates of change.

Below we set out those rules which enable us to calculate the rate of change of such a variable as a function of the variables, on which it itself depends, as well as the rates of change of these variables.

Rule 1:

Let

$$a = bc$$

where b and c are functions of time. Then, for the rate of change of a we get:

$$\hat{a} = \hat{b} + \hat{c}$$

Rule 2:

Let

$$a = \frac{b}{c}$$

where b and c are functions of time. Then, for the rate of change of a we get:

$$\hat{a} = \hat{b} - \hat{c}$$

Rule 3:

Let

$$a = b^{c}$$

where c = constant and b is a function of time. Then, for the rate of change of a we get:

$$\hat{a} = c\hat{b}$$

Rule 4:

Let

$$a = b + c$$

where b and c are functions of time. Then, for the rate of change of a we get:

$$\hat{a} = \frac{b}{b+c}\hat{b} + \frac{c}{b+c}\hat{c} = \frac{b}{a}\hat{b} + \frac{c}{a}\hat{c}$$

Rule 5:

Let

$$a = \bar{a}$$

where $\bar{a}$ = constant. Then, for the rate of change of a it is self-evident that

$$\hat{a} = 0$$

Rule 6:

Let

$$a = a_0 e^{\alpha t}$$

where a_0 and α = constants. Then, for the rate of change of a the following holds

$$\hat{a} = \frac{da}{dt}\frac{1}{a} = \alpha a_0 e^{\alpha t}\frac{1}{a} = \alpha a \frac{1}{a} = \alpha$$

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