

Subsystems of Production

or

Feld'man's vertically integrated sectors

by
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In this paper we wish to show that Feld'man's article "*On the Theory of Growth Rates of National Income*" (Feld'man, 1928, 1964, 1969) published in 1928, introduces in the framework of a model of economic growth for the first time in economic science the notion of 'subsystem of production' or, as such a subsystem has come to be known, 'vertically integrated sector of production'.

It may be useful, before proceeding to a presentation of the notion of the 'vertically integrated sector of production' introduced by Feld'man, to refer briefly to the life and work of the man. The following biographical and work particulars are cited from Ottomar Kratsch (Kratsch 1969).

Feld'man was born in Rostov in 1884. He studied electrical engineering among other places in Germany also. From 1912, he worked as an electrical engineer in St. Petersburg and Moscow. After the October Revolution, he collaborated with the Supreme Council for National Economy of the USSR, the central planning body for the Soviet economy. He worked under G.M. Krshishanowski along with 200 other scientists in drawing up the GOELRO plan in the 'Electricity' department. In February 1923, Feld'man was invited by G.M. Krshishanowski to work on the State Planning Commission, which was set up in 1921, where he was occupied in the 'Konjunktur and World Economy' department. There, he was mainly involved with the comparative analysis of the long run structure and dynamic of the economies of the USA and the USSR. In one of his relevant papers ("Reflections on the Structure and Dynamic of the USA economy in the Years from 1850 to 1925 and of the USSR economy in the Period from 1926/27 to 1940/41", *Planowoje chosjaistwo*, Vol. 1927, No. 7) he formulates and sets out for the first time thoughts about economic growth.

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The following year, Feld'man published his main work, his paper "On the Theory of Growth rates of National Income" (Feld'man 1928, 1964, 1969). In Part I "Feld'man's study [...] sets out his own growth model and the core equations relating its variables. The author shows how, in order to achieve given changes in the growth rate of output, 'accumulation' (investment) should be allocated either to increase efficiency in use of existing capital or to change the structure of the economy, i.e., the ratios of his sectors E (producers' goods) and V (consumers' goods)." (Spulber 1964, p. 4). In Part II "Feld'man stressed [...] the interdependence between the rates of growth of the capital stock and their utilization in his sector U (consumers' goods) and E (producers' goods). After examining theoretically the impacts of variations in the pattern of allocation of investments as between his two sectors, and after indicating the variant which would result in the quickest increase in the rate of growth of V, Feld'man drew the attention of the Soviet policy makers to the immediate importance of increasing the *effectiveness of capital utilization* until the ratio of the capital stocks of the two sectors (K_E/K_U) could be raised, i.e. 'until a much higher degree of industrialization has been attained'." (Spulber 1964, p. 281).

Also published in the same journal, "Planowoje chosjaistwo", in issues 2 and 12 of 1929, were the articles "On the limits of the Industrialization" and "The Analytical Method of the Perspective Planning".

In the second article, Feld'man deals with some of the issues which he had set out in his paper of 1928 (a translation of certain excerpts from this paper by Feld'man is contained in Feld'man 1969, pp. 111-122). This was the last work published by Feld'man on growth theory. In 1931 he left the State Planning Commission and worked as an electrical engineer. A number of relevant patents are registered in his name. He died in Moscow in 1958.

Feld'man's works in the field of the theory of economic growth are probably the first in the field of the *modern* theory of economic growth. Although they were not to remain obscure, they became known only when the modern theory of economic growth had already been developed and was following its own path. Thus, as far as we know, Feld'man did not influence it in any way.

As has already been pointed out, in this paper we shall not be dealing with Feld'man's theory of economic growth, but rather with the notion of the 'vertically integrated sector of production', which he introduced within the framework of his theory of economic growth.

Feld'man's economic growth model is a two sector model. The economy that he describes consists of two sectors. Feld'man begins with the two production sectors of Marx, the production sector of means of production and the production sector of means of consumption. He ascertains however, quite correctly, that this marxian division of the economy into the aforesaid two sectors is not a suitable one for the mathematical treatment of problems of economic growth which he himself wishes to raise and solve. The division of the economy that is suitable for his purposes, which Feld'man introduces by starting with the corresponding marxian division, is identical to introducing the notion of the sraffian 'subsystem' or, as such a subsystem has come to be called, the 'vertically integrated production sector'.¹

The criterion of the usual distinction between different sectors of production, i.e. the criterion of the usual definition of a sector of production, is the gross product which that sector produces. This same criterion was used by Marx to define the two sectors of production into which he divides the economy, namely the production sector of means of production and the production sector of means of consumption, and to distinguish one from the other. The first produces –as its gross product– all the means of production which are contained in the gross product of the economy and only those means, while the second produces –as its gross product– all the means of consumption which are contained in the gross product of the economy and only those means. These two sectors, which like Feld'man we shall call sector A and sector B respectively, constitute all the sectors of the economy. Consequently, the aggregate of their gross products X_A and X_B , $X_A \geq 0$ and $X_B \geq 0$, form the gross product X of the economy. We therefore have

$$X_A + X_B = X,$$

where X_A , X_B and X are $n \times 1$ vectors.

Assuming then that the economy produces n commodities, of which the commodities 1 to κ are means of production and the commodities $\kappa+1$ to n are means of consumption. Then the $n \times n$ matrix of technical coefficients A , $A \geq 0$, of the economy has the form

1. See P. Sraffa, *Production of Commodities by Means of Commodities*, Cambridge 1960, Appendix A.

$$A = \begin{bmatrix} A_{11} & A_{12} \\ 0 & 0 \end{bmatrix}$$

where A_{11} , $A_{11} \geq 0$, the $\kappa \times \kappa$ irreducible non singular matrix of inputs of used up means of production per unit of produced means of production and A_{12} , $A_{12} \geq 0$, the $\kappa \times (n-\kappa)$ matrix of inputs of used up means of production per unit of produced means of consumption.² Because the technique used by the economy is productive, the following holds

$$(0 <) \lambda_A = \lambda_{A_{11}} < 1,$$

where λ_A the maximum eigenvalue of A and $\lambda_{A_{11}}$ the maximum eigenvalue of A_{11} . Consequently

$$(I - A)^{-1} \geq 0 \quad (1)$$

and

$$(I - A_{11})^{-1} > 0. \quad (2)$$

Assuming that the economy produces the net product Y , $Y > 0$. For Y the following holds by definition

$$Y = X - AX, \quad (3)$$

where X the gross product of the economy.

From (3) we get for X , taking into consideration (1):

$$X = (I - A)^{-1}Y. \quad (4)$$

X is the aggregate of the gross products of all sectors of production of the economy, i.e. the aggregate of the gross product X_A of sector A and of the gross product X_B of sector B . X_A by definition contains only means of production, i.e. only the first κ commodities, and X_B by definition contains only means of consumption, i.e. only the last $n-\kappa$ commodities. Consequently:

$$X_A = \begin{pmatrix} \bar{X}_A \\ 0 \end{pmatrix},$$

where \bar{X}_A , $\bar{X}_A > 0$, a $\kappa \times 1$ vector,

2. From this point onwards, for brevity's sake, we shall omit the term 'used up'.

$$X_B = \begin{pmatrix} 0 \\ \bar{X}_B \end{pmatrix},$$

where $\bar{X}_B, \bar{X}_B > 0$, a $(n-\kappa) \times 1$ vector, and

$$X = X_A + X_B = \begin{pmatrix} \bar{X}_A \\ \bar{X}_B \end{pmatrix}.$$

For the net product Y_A of the production sector of means of production A the following by definition holds

$$\begin{aligned} Y_A &= X_A - AX_A \Rightarrow \\ X_A &= (I - A)^{-1} Y_A \Rightarrow \\ \bar{X}_A &= (I - A_{11})^{-1} \bar{Y}_A, \end{aligned} \tag{5}$$

with

$$Y_A = \begin{pmatrix} \bar{Y}_A \\ 0 \end{pmatrix},$$

where \bar{Y}_A is a $\kappa \times 1$ vector.

Because $\bar{X}_A > 0$ and $(I - A_{11})^{-1} > 0$, as emerges from (6), $\bar{Y}_A \geq 0$. However, because by assumption $Y > 0$, eventually $\bar{Y}_A > 0$, and consequently $Y_A \geq 0$. So the production sector of the means of production A produces a semi-positive net product Y_A , which contains all the commodities 1 to κ , i.e. all the means of production, in positive quantities. Consequently, sector A is viable, just as the economy as a whole is viable. For, sector A, in producing the semi-positive gross product X_A , produces a corresponding semi-positive net product Y_A . The fact that sector A, by producing the semi-positive gross product X_A , produces a corresponding semi-positive net product Y_A , means that the sector itself produces the means of production AX_A which are directly necessary for the production of the gross product of X_A or, which is the same, the means of production AX_A which are directly and indirectly necessary for the production of the net product of Y_A .

For AX_A we get, taking into consideration (5):

$$AX_A = A(I - A)^{-1}Y_A.$$

The expression $A(I - A)^{-1}Y_A$ represents the means of production directly and indirectly necessary for the production of the net product Y_A , i.e. the means of production which are directly and indirectly necessary for the production of Y_A . Since for $A(I - A)^{-1}Y_A$ we get:

$$\begin{aligned} A(I - A)^{-1}Y_A &= A(I + A + A^2 + A^3 + \dots)Y_A = \\ &= AY_A + A(AY_A) + A(A^2Y_A) + A(A^3Y_A) + \dots, \end{aligned}$$

where AY_A is the direct means of production of the net product Y_A , $A(AY_A)$ the direct means of production of the direct means of production AY_A of the net product Y_A , $A(A^2Y_A)$ the direct means of production of these latter means of production A^2Y_A , and so on, and consequently $AY_A + A(AY_A) + A(A^2Y_A) + A(A^3Y_A) + \dots$ the direct and indirect means of production of the net product Y_A of sector A. Sector A is therefore a sraffian 'subsystem' or, as such a subsystem is usually called, a vertically integrated production sector, i.e. a sector of production which produces a semi-positive (or positive) net product and therefore it itself produces the means of production which are directly and indirectly necessary for the production of its net product.

Let us now take a look at what happens with sector B. For the net product Y_B of sector B the following holds by definition

$$\begin{aligned} Y_B &= X_B - AX_B \Rightarrow \\ Y_B &= \begin{pmatrix} 0 \\ \bar{X}_B \end{pmatrix} - A \begin{pmatrix} 0 \\ \bar{X}_B \end{pmatrix} \Rightarrow \\ Y_B &= \begin{pmatrix} 0 \\ \bar{X}_B \end{pmatrix} - \begin{pmatrix} A_{12}\bar{X}_B \\ 0 \end{pmatrix} \Rightarrow \\ Y_B &= \begin{pmatrix} -A_{12}\bar{X}_B \\ \bar{X}_B \end{pmatrix}. \end{aligned}$$

Consequently, the net product Y_B of sector B contains also negative

quantities of commodities, the quantities $-A_{12}X_B$. The quantities of commodities $A_{12}X_B$ are merely the means of production AX_B , which are directly necessary for the production of the gross product X_B of sector B or –which is the same– the means of production AX_B , which are directly and indirectly necessary for the production of the net product Y_B of sector B. For

$$X_B = (I - A)^{-1}Y_B$$

and consequently

$$\begin{aligned} AX_B &= A(I - A)^{-1}Y_B = \\ &= A(I - A + A^2 + A^3 + \dots)Y_B = \\ &= AY_B + A(AY_B) + A(A^2Y_B) + A(A^3Y_B) + \dots \end{aligned}$$

These means of production however are not produced by sector B itself, but by sector A, which supplies them to sector B. It is for this reason that their quantities appear with a minus sign in the net product Y_B of sector B. Sector B therefore is not viable. It cannot exist and reproduce alone, i.e. without sector A. In contrast, as we saw, sector A can exist and reproduce alone, without sector B, because it is not supplied with anything from sector B.³

For the net product Y of the economy as a whole we get

$$\begin{aligned} Y &= Y_A + Y_B = \begin{pmatrix} \bar{Y}_A \\ 0 \end{pmatrix} + \begin{pmatrix} -A_{12}\bar{X}_B \\ \bar{X}_B \end{pmatrix} = \\ &= \begin{pmatrix} \bar{Y}_A - A_{12}\bar{X}_B \\ \bar{X}_B \end{pmatrix}. \end{aligned}$$

When

$$\bar{Y}_A - A_{12}\bar{X}_B = 0,$$

3. Of course this happens only because real wages are here not included in the inputs of each sector. If however matrix A did not represent only the inputs of used up means of production but also inputs of real wages per unit of produced commodity, then because real wages consist of means of consumption, sector A would not be viable either and would not be able to exist and reproduce without sector B, which would be supplying it with its real wages. In this case, the surplus product of sector A would also contain quantities of commodities with a minus sign and specifically, the quantities of wage commodities not produced by sector A, but supplied from sector B.

then

$$Y = \begin{pmatrix} 0 \\ \bar{X}_B \end{pmatrix},$$

i.e. the net product of the economy Y consists only of means of consumption and the economy reproduces simply.

And when

$$\bar{Y}_A - A_{12} \bar{X}_B \geq 0,$$

then sector A produces as its net product in addition to the quantities $A_{12} \bar{X}_B$ of means of production, which sector B needs to replace its used up means of production, also the means of production quantities $\bar{Y}_A - A_{12} \bar{X}_B$ which are used to increase the economy's stocks of means of production and consequently for the expanded reproduction of the economy. In this case

$$Y = \begin{pmatrix} \bar{Y}_A - A_{12} \bar{X}_B \\ \bar{X}_B \end{pmatrix} \geq 0$$

and consequently Y contains not only means of consumption but also means of production in positive quantities.

The conclusion which emerges from the above is that, if the sectors are defined using the criterion of their gross product, then they are not all viable. In the case in question this is quite apparent, for matrix A is reducible and consequently sector A constitutes the basic subsystem and sector B constitutes the non-basic subsystem. But this is also the case when matrix A is irreducible. In fact, when matrix A is irreducible, as a rule no sector is viable. A necessary but not also sufficient condition for a sector to be viable in this case, is that it produces all the commodities produced by the economy as a whole. Because however this condition is necessary but not also sufficient, there are sectors which produce all the commodities produced by the economy as a whole but despite this are not viable, i.e. they do not constitute vertically integrated sectors of production.

Feld'man assumes a «closed» viable national economy, i.e. a viable (and therefore productive) production system, for which $X > 0$ and $Y \geq 0$ holds. As we have already mentioned, he initially starts out with the marxian division of

the system of production into two sectors: sector A which produces as its gross product the means of production and sector B which produces as its gross product the means of consumption. He shows however in an exceptionally simple way that this division is not suitable for dealing with issues of economic growth with which he himself intends to deal. Feld'man writes:

«The idea suggested itself of following Marx's example by introducing data on capital invested in the production of consumers' goods and of producers' goods as basic indicators of the magnitude and structure of the economy. However, more detailed analysis indicates that this «principle of division» is inadequate to accomplish *by mathematical methods* the particular concrete objective stated above.

To the extent that the rate of growth of production depends on the rate of growth of the equipment of the labour force, and productive equipment is made in sector A (producers' goods sector), it may be stated outright that the increase of the rate of growth of production depends on the increase of the capital of sector A as compared with the increase of the capital of sector B (consumers' goods sector).

With expanding reproduction, sector A must supply sector B not only with producers' goods required to continue production at the current level of output, but also with *additional* fixed and circulating capital necessary for expansion of reproduction, given constant efficiency of capital utilization.⁴

This gives rise to the idea of dividing the capital of sector A into two sections, of which one (A_2) supplies sector B with the means of production required to sustain output at a given level, and the other (A_1) supplies all industries in both sectors with *additional* capital to enable reproduction to expand. Given constant efficiency of capital utilization, A_2 must be proportional to B, while the *magnitude* of A_1 is determined entirely by the rate of growth of production as a whole, and of its separate parts.

Since capital consists of constant and variable parts, consistent application of the foregoing principle of classification requires the transfer to sector A of that portion (B_1) of sector B which provides the increments of

4. By efficiency of capital utilization we mean the ratio of the value of net output per unit of time to the value of the fixed and circulating capital in a given enterprise or sector. Both value of net output and capital must be expressed in terms of the cost of reproduction as of the same time.

variable capital, leaving in sector B only the portion (B_2) required to maintain consumption at any given level. Hence the specific expression on which the rate of growth of total consumption depends will be the ratio

$$\frac{A_1 \text{ and } B_1}{A_2 \text{ and } B_2}.$$

The numerator includes everything that provides the basis of expanded reproduction, while the denominator includes everything that serves current direct consumption.⁵

From the viewpoint of the proposed division, there is no basis for including the capital invested in a weaving factory in sector B, and capital invested in cotton plantations which produce cotton for the manufacture of yarn, or capital invested in spinning factories, in sector A, since in either case both spinning and weaving will take place in the same factory, and weaving divides itself into a number of consecutive productive processes which yield semifinished products that are themselves means of production for subsequent stages of production.

A specific structural division with quantitatively interrelated components must therefore be formulated before the dependence of the rate of growth of national income on these structural relationships can be determined. The basic step is a precise economic division of production, in accordance with the principal objective of this work. An *absolute and precise criterion* is necessary in order to determine the exact extent of the capital required to produce consumer goods at a level sufficient to satisfy current consumer needs.

From the viewpoint of the capacity of the productive apparatus to expand reproduction, there is, therefore, no reason to separate from sector B any portion of production concerned in one way or another with producing final products, and particularly consumer goods, up to the level sufficient to satisfy current needs. It must, therefore, be concluded that in the formulation of the problem it is appropriate to place in sector B all the industries

5. This division is realizable only by the accounting method and does not correspond to the actual breakdown of production according to enterprises. Analytical evidence of the practicability of the proposed division is not introduced in this article, but in another one specifically devoted to this subject. [Feld'man, «Analiticheskii metod postroeniia perspektivnykh planov», *Planovoe khoziaistvo*, 1929, No. 12.

concerned in any way with creating the values of consumer goods up to the level sufficient to satisfy current needs.

In this sector [B], in which Marx's schema places the entire value of consumers' goods, must be included also all of the capital used in producing the consumers' goods. It is understood that this includes neither the *increase* of fixed and circulating capitals in sector B nor their replacement when they become technically obsolete.

This capital can be obtained only from sector A. The value of the output of sector B can include only the value of raw materials and that portion of the equipment and producers' goods actually used up in the production of consumers' goods. In sector B must not be included the value of producers' and consumers' goods accumulated for expanded production, which will only later, as they wear out and are used up, enter into the value of consumers' goods produced with the expanded capital. Thus the wear and tear of productive equipment in sector B must, by definition, be made good within that sector.

Thus defined, sector B possesses the remarkable property of being capable of existence without sector A, but only for purposes of simple reproduction. Thus, starting from an analysis of what is required for a more precise division of output -from the viewpoint of determining the value of consumer goods required to satisfy the existing level of needs- we have arrived at a confirmation of the above idea: that production must be divided into *sector B*, capable of maintaining consumption at a given level even with a cessation of the inflow of producers' and consumers' goods from sector A to be added to the capital of sector B, and *sector A*, which provides both sector B and itself with all the capital required for expansion of reproduction.

Thus, starting from Marx's division, we have arrived at a new division which corresponds, however, to another division, the Marxian simple and expanded reproduction, the «production of income» and the «production of capital». To avoid confusion, the letter E will henceforth be used for what has been developed from Marx's sector B, and the letter U for the remaining part of production, developed from Marx's sector A.» (Feld'man 1964, pp. 174-178).

Thus, Feld'man divides the economy into two sectors, sector E and sector U, in which sector E produces as its net product the means for increasing the stocks of the economy, i.e. the means for expanding production, in other words for economic growth, while sector U produces as

its net product the means of consumption. The criterion for distinguishing sectors E and U is therefore their net product.

Because sectors E and U constitute the economy as a whole, the aggregate of their net products Y_E and Y_U respectively is equal to the net product of the economy Y:

$$Y_E + Y_U = Y,$$

where Y_E , Y_U and Y are $n \times 1$ vectors.

According to Feld'man, Y_E consists of:

- a) the means of production for increasing the already existing stocks of means of production of both sectors E and U of the economy,
- b) the (as real wages advanced) means of consumption for increasing the respective stocks of both sectors⁶ and
- c) the means of production and the wage commodities (chiefly the former) for replacing the 'morally depreciated' parts of the stocks of means of production and of wage commodities of both sectors.⁷

Thus, if F represents the stocks of the economy, then

$$Y_E = \Delta F,$$

where ΔF and consequently F consist, according to Feld'man, of precisely the same three commodities that Y_E consists. In reality, however, Y_E , and consequently also ΔF and F, consist only of the first two commodities.⁸

6. Parts a) and b) constitute what we now call net investment.

7. Here, Feld'man explicitly considers the quantities of commodities, which are used to replace the 'morally depreciated' parts of the economy's stocks of means of production and wage commodities, to be part of the net product of sector E and part of the net product of the economy. This is obviously incorrect and is the reason for certain paradoxical results obtained by Feld'man particularly in the 2nd part of his article. The correct view is that the above quantities of commodities constitute in their entirety not part of the net product of the economy, but part of its gross product and specifically of its depreciation. The reader may, if he/she so wishes, eliminate this error by setting 'moral depreciation' equal to zero. In addition, if the reader has difficulties understanding that the remaining parts a) and b) constitute what we now call net investment (because today we often mistakenly consider net investment to consist only of means of production), he/she may also set part b) equal to zero.

8. Compare footnote 7.

For the gross product X_E of sector E the following holds

$$\begin{aligned} X_E &= Y_E + AX_E \Rightarrow \\ X_E &= (I - A)^{-1}Y_E. \end{aligned}$$

But because $Y_E \geq 0$ and $(I - A)^{-1} \geq 0$, the following holds:

- a) if $Y_E = 0$,⁹ then $X_E = 0$ and
- b) if $Y_E \geq 0$, then $X_E \geq 0$.

In the second case, which is the only one to which reference may be made concerning sector E, sector E produces not only a semi-positive net product Y_E , it also produces the used means of production $AX_E [= A(I - A)^{-1}Y_E]$ which are directly and indirectly necessary for the production of this net product Y_E , i.e. the means of production AY_E and $A(AY_E)$ and $A(A^2Y_E)$ and so on.

Consequently it is a viable sector, i.e. a vertically integrated sector, which can exist and reproduce alone, independent of sector U.¹⁰

The net product Y_U of sector U consists of the means of consumption which are contained in Y and which are consumed in the current period. Consequently, it contains also the wage commodities which wage labourers receive and consume in the current period.¹¹ The other uses of Y_U are set out in detail by Feld'man.

For the gross product X_U of sector U the following holds

$$\begin{aligned} X_U &= Y_U + AX_U \Rightarrow \\ X_U &= (I - A)^{-1}Y_U. \end{aligned}$$

Because however $Y_U \geq 0$ and $(I - A)^{-1} \geq 0$, then $X_U \geq 0$. Thus sector U produces a semi-positive net product and consequently also the used means of production $AX_U [= A(I - A)^{-1}Y_U]$ which are directly and indirectly necessary for the production of this net product, i.e. the means of production AY_U and $A(AY_U)$ and $A(A^2Y_U)$ and so on. Consequently it is a viable sector,

- 9. When $Y_E = 0$ and consequently $X_E = 0$, then we have simple reproduction, i.e. zero growth of the economy.
- 10. Compare however footnote 3. Apparently, only for the reasons to which we refer in footnote 3, Feld'man avoids making the observation that sector E can exist and reproduce independent of sector U.
- 11. It is assumed that wage labourers consume their entire wage.

i.e. a vertically integrated sector, which can exist and reproduce independent of sector E – and of course independent of the fact that here matrix A represents only inputs of used up means of production and not inputs of means of production and real wages per unit of commodity produced. Regarding this characteristic of sector U, Feld'man writes:

«All parts of sector U consist merely of the stages of a single productive process [i.e. production process -G.S.] resulting in the consumers' goods required to satisfy the existing level of [consumers' – G.S.] needs» (Feld'man 1964, p. 178).

According to Feld'man, sectors E and U produce roughly the same commodities, and consequently X_E and X_U contain roughly the same commodities. This is absolutely correct.

If both new means of production and new means of consumption are not produced, if, that is, sector E alone does not produce new types of means of production in addition to those which it uses and uses up and consequently sector U produces in the current period, as well as new types of wage commodities in addition to those produced by sector U in the current period, then sector U also, like sector E, uses all the n production processes and produces all the n commodities. Consequently, not only $X_E > 0$, but also $X_U > 0$.

If however *sector E* alone produces new means of production and new consumer commodities in addition to those produced by sector U in the current period,¹² then $X_E > 0$ and $X_U \geq 0$ and X_U contains also zero components, which relate to these new commodities, and certain (non-zero) columns and the corresponding (zero) rows of A relate to these new commodities produced alone by sector E and not sector U.¹³

The defined –using the criterion of net product– sectors E and U of Feld'man are sraffian 'subsystems' or, which is the same, vertically integrated sectors of production. Thus, the first economist to introduce the notion of the vertically integrated sector of production in economic theory was Feld'man.

The distinction between sectors A and B made by Marx is an economic

12. Which, of course, will be used in the following period. Thus, sector E is apparently also the sector of technological innovations.

13. Apparently these commodities, because they are not used in this period but rather in subsequent periods, are non-basic and consequently the rows of A, which relate to them, are zero.

distinction, which aims –without entirely succeeding– at responding to the needs of analysing the reproduction process. The distinction between sectors E and U made by Feld'man, who starts out with Marx's above distinction, is an economic distinction, which aims at responding –and it responds to a much greater degree than the marxian distinction, although not entirely– to the needs of analysing the reproduction process, i.e. the process of economic growth. For on the one hand, one of the two vertically integrated sectors of Feld'man, namely sector E, produces as its net product only the means for the expanded reproduction of the system, but on the other, it is vertically integrated only on the condition that matrix A does not represent also the inputs of wage commodities per unit of produced commodity.¹⁴ Consequently, the distinction between the two production sectors E and U of Feld'man would fully respond to the needs of analysing the process of economic growth only if it was made under the condition that matrix A represents also the inputs of wage commodities per unit of produced commodity.

Despite this however, Feld'man's distinction allows theoretical issues of economic growth to be dealt with, even in its given form, without having reference to the gross products of the various sectors and the gross product of the economy overall, but with reference only to the corresponding net products.¹⁵

The following emerges from this advantage of Feld'man's distinction: If one calculates real magnitudes instead of prices in terms of marxian labour values, then, because these material magnitudes are net products of sectors or of the economy, one does not need to know the labour values of the single commodities in order to calculate the labour value of the net product of a

14. Because although sector E produces the wage commodities for increasing the stocks of the two sectors E and U in wage commodities, it does not produce the wage commodities received by the workers which it employs. These latter commodities are produced as part of its gross product by sector U, which supplies them to sector E.

15. One would obtain the same result however in the case where there was no variable capital, i.e. in the case where wages are not advanced in part or in whole, if the economy was divided into three sectors, one of which produces as its net product the new means of production, that is, net investment, the second produces as its net product the means of consumption of capitalists and the third produces wage commodities as its net product. See such a model in G. Stamatis, *Die "spezifisch kapitalistischen" Produktionsmethoden und der tendenzielle Fall der allgemeinen Profitrate bei Karl Marx*, Berlin 1977.

sector or of the economy as a whole, because this latter is always equal to the living labour used by this sector or the economy as a whole.¹⁶

Feld'man himself makes use of this possibility.

The dividing of the economy as a whole into two vertically integrated sectors, which Feld'man introduced, has not been used, as far as we know, in modern theory of economic growth. However, the notion of the vertically integrated production sector, which he first introduced to economic theory, is extremely important for the theory of linear production systems.¹⁷ Thus, for example, the normalization subsystem is a vertically integrated production sector of Feld'man.¹⁸

Despite Feld'man's view to the contrary, the dividing of the economy into two vertically integrated sectors E and U is probably unsuitable for economic planning and for empirical research in general. Feld'man is of the view that, for the purposes of economic planning, this distinction must be given greater depth, i.e. that the same criterion should be applied to further divide sectors E and U, and the gathering of statistical material adapted to this distinction.

16. *Proof*: Taking $[A, \ell_E]$ as the technique of sector E, where ℓ_E is the vector of living labour per unit of produced commodity of sector E. For the vector of values ω , $\omega = \ell_E(I - A)^{-1}$ holds. For the value of the net product Y_E of sector E, $\omega Y_E = \ell_E(I - A)^{-1} Y_E = \ell_E X_E$ holds, where X_E is the gross product of sector E and consequently $\ell_E X_E$ the living labour of sector E. The above holds also for the case of footnote 15.

17. See L. Pasinetti, "The Notion of Vertical Integration in Economic Analysis", *Metroeconomica*, vol. XXV (1973), pp. 1-29, "Sraffa's Circular Process and the Concept of Vertical Integration", *Political Economy, Studies in the Surplus Approach*, vol. 2 (1981), pp. 3-6, "Growing subsystems, vertically hyper-integrated sectors and the labour theory of value", *Cambridge Journal of Economics*, vol. 12 (1988), pp. 125-134, and "Vertical integration and capital theory: a comment", *Journal of Postkeynesian Economics*, vol. 13 (1990), pp. 65-70.

18. See G. Stamatias, *Sraffa und sein Verhältnis zu Ricardo und Marx*, Göttingen 1983 and, also by Stamatias, *Das Normwaresubsystem und die w-r-Relation*, Kritiki Publications, Athens 1988, and G. Stamatias, «On the Position and the Slope of the w-r-Curve», *Political Economy*, Issue 2, Spring 1998.

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ABSTRACT

This paper presents in a mathematical way the notion of ‘subsystems of production’ which G.A. Feld’man first introduced in 1928 starting from marxian schemes of reproduction. Sraffa made the notion of ‘subsystem of production’ known some 30 years later. This notion was further developed by Pasinetti and became widely known as the ‘vertically integrated sector of production’. Nowadays, it is commonplace in economic theory, particularly neoricardian.

KEYWORDS: Feld’man, Sraffa, Pasinetti, linear production theory, marxian economic theory.