# The Role of 'Circulation' in the Reproduction of the Economic System and in the Production of Surplus Value and Profit 

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We shall show here why in capitalist commodity production it is not possible for the circulation sector to get surplus value or, correspondingly, profit, which was produced by other sectors, and we shall explain why each sector gets only the surplus value or, correspondingly, the profit, produced by that same sector or, conversely, itself produces the surplus value or, correspondingly, the profit it gets. We shall also show that, just as labour, which is expended in 'material' production is, depending on whether it produces reproductive or non-reproductive use values, reproductive or non-reproductive labour, so too the labour which is expended in 'circulation' is, depending on whether it produces reproductive or non-reproductive use values, reproductive or non-reproductive labour.

A use value is reproductive (non-reproductive) when it enters (does not enter) directly or indirectly into the production of all the use values produced. So, a reproductive use value enters directly or indirectly into the production of all the -reproductive and non-reproductive- use values, while a non-reproductive use value, on the contrary, either does not enter into the production of any use value or enters into the production of some or all of the nonreproductive use values. A reproductive use value is, therefore, precisely because it enters into the production of all the use values produced, absolutely necessary for the reproduction of the economic system as a whole. If it is not produced, then no use value whatsoever can be produced and consequently the economic system cannot be reproduced. Thus, each reproductive use value is absolutely necessary for the reproduction of the economic system as a whole. A non-reproductive use value, on the contrary, because it does not enter into the production of all the commodities, is not necessary for the reproduction of the economic system as a whole. Because, if it is not produced, there can be no production of only those non-reproductive use values, into the production of
which it itself enters directly or indirectly (if it enters into the production of certain non-reproductive use values).

A simple way to ascertain whether a use value is reproductive or nonreproductive is to investigate whether that use value enters or does not enter directly or indirectly into the production of use values consumed by workers. If it enters, it is reproductive; if it does not enter, it is a non-reproductive use value. The reason for this is that wage commodities -because they enter directly into the reproduction of labour power, which enters directly into the production of all commodities, and consequently they too enter at least indirectly into the production of all commodities- are reproductive commodities, so that, when a use value enters directly or indirectly into their production, it too -like them- enters into the production of all the use values produced and is consequently reproductive. If, on the contrary, a use value does not enter directly or indirectly into the production of wage commodities, it does not enter into the reproduction of labour power, therefore it does not enter into the production of all commodities and consequently it is non-reproductive.

So, if certain use values of those produced in the 'circulation' sector enter into the production of all commodities and consequently are reproductive use values, then the overall system cannot be reproduced if these use values are not produced, i.e. certain parts of the circulation sector are absolutely necessary for the reproduction of the economic system.

We shall deal with the above issues with the help of a model of capitalist production using three (though in reality, as we shall see later on, six) commodities, commodities 1,2 and 3 , of which commodity 1 is a means of production, commodity 2 a wage commodity and commodity 3 a commodity consumed only by capitalists.

The production technique of these three commodities is described by the matrix A of technical coefficients,

$$
\mathrm{A}=\left|\begin{array}{ccc}
\mathrm{a}_{11} & \mathrm{a}_{12} & a_{13} \\
0 & 0 & 0 \\
0 & 0 & 0
\end{array}\right|,
$$

and the vector $l$, $l=\left[l_{1}, 1_{2}, 1_{3}\right]$, of labour coefficients.

Thus, the production of one unit of commodity 1 requires $a_{11}$ units of commodity 1 itself (where of course $\mathrm{a}_{11}<1$ ) and $\mathrm{l}_{1}$ units of labour power, the
production of one unit of commodity 2 requires $\mathrm{a}_{12}$ units of commodity 1 and $l_{2}$ units of labour power and the production of one unit of commodity 3 requires $a_{13}$ units of commodity 1 and $l_{3}$ units of labour power.

We also assume that the real wage rate, i.e. the quantity of wage commodity 2 purchased by workers with the nominal wage which they receive for one unit (=hour) of labour power, i.e. with the labour value, when the commodities are exchanged at their labour values, or with the price, when they are exchanged at prices which differ from labour values, of one unit of labour power, i.e. with the nominal wage rate expressed in labour values or, correspondingly, in prices, is $\mathrm{a}_{20}$ units of commodity 2 . Consequently, if we symbolise the vector of prices with p , the labour value of one unit of labour power with $\gamma$ and the price of one unit of labour power with $\delta$, the following holds

$$
\gamma=\omega_{2} \mathrm{a}_{20}
$$

and

$$
\delta=\mathrm{p}_{2} \mathrm{a}_{20} .
$$

The fact that the real wage rate is given allows us to replace -in the description of the production technique- the vector of labour coefficients with the vector of real wage coefficients. That is, instead of saying that the production of one unit of each of the three commodities requires $l_{1}, l_{2}$ and $l_{3}$ units of labour power respectively, we can say that we need $l_{1} \alpha_{20}, l_{2} \alpha_{20}$ and $l_{3} \alpha_{20}$ units of the wage commodity, i.e. of commodity 2 , respectively.
If we then set

$$
\begin{aligned}
& 1_{1} \alpha_{20}=\alpha_{21} \\
& 1_{2} \alpha_{20}=\alpha_{22}
\end{aligned}
$$

and

$$
1_{3} \alpha_{20}=\alpha_{23},
$$

then $\alpha_{21}, \alpha_{22}$ and $\alpha_{23}$ symbolise respectively the inputs in commodity 2 that are required for the production of one unit of commodity 1 , of commodity 2 and of commodity 3 . Of course $\mathrm{a}_{22}<1$ holds, i.e. to produce one unit of commodity 2 we need to pay as real wages less than one unit of commodity 2 , because otherwise the production of commodity 2 would be pointless for the capitalists of sector 2 which produce it. ${ }^{1}$

[^0]According to the above, matrix $\overline{\mathrm{A}}$,

$$
\bar{A}=\left[\begin{array}{ccc}
a_{11} & a_{12} & a_{13} \\
a_{21} & a_{22} & a_{23} \\
0 & 0 & 0
\end{array}\right],
$$

now alone describes the given production technique. Because vector 1 of labour coefficients has now been incorporated in it in the form of the vector [ $\alpha_{21}, \alpha_{22}$, $\alpha_{23}$ ] of wage commodity coefficients. Each column of matrix shows the inputs in means of production and wage commodities that are necessary for the production of one unit of a certain commodity: the first column the inputs in means of production and wage commodities that are necessary for the production of commodity 1 , the second the inputs in means of production and wage commodities that are necessary for the production of one unit of commodity 2 and the third the inputs in means of production and wage commodities that are necessary for the production of one unit of commodity 3 .

The production technique that is defined by matrix $\overline{\mathrm{A}}$ is surplus productive, i.e. it is capable of producing any exogenously given strictly positive or non-negative surplus product, producing in the first of the two cases an also strictly positive and in the second a strictly positive or non-negative gross product, only when the maximal eigenvalue of $\overline{\mathrm{A}}$ is positive and smaller than unit.

If we symbolise the exogenously given surplus product with vector Y and the respective gross product with vector X , then the following evidently holds

$$
\mathrm{X}-\overline{\mathrm{A}} \mathrm{X}=\mathrm{Y} \Rightarrow(\mathrm{I}-\overline{\mathrm{A}}) \mathrm{X}=\mathrm{Y}
$$

In the case that Y is strictly positive $(\mathrm{Y}>0), \mathrm{X}$ is strictly positive $(\mathrm{X}>0)$, only when

$$
(\mathrm{I}-\overline{\mathrm{A}})^{-1} \geq 0 .
$$

In the case that Y is non-negative $(\mathrm{Y} \geq 0), \mathrm{X}$ is strictly positive $(\mathrm{X}>0)$ or non-negative ( $\mathrm{X} \geq 0$ ) also only when

$$
(\mathrm{I}-\overline{\mathrm{A}})^{-1} \geq 0 .
$$

The condition

$$
(\mathrm{I}-\overline{\mathrm{A}})^{-1} \geq 0
$$

is evidently satisfied, when the maximal eigenvalue $\lambda_{m}$ of $\bar{A}$ is positive and less than unit. $\lambda_{\mathrm{m}}$ is positive because $\overline{\mathrm{A}}$ is non-negative $(\overline{\mathrm{A}} \geq 0)$. Let us see when $\lambda_{\mathrm{m}}$ is less than unit.

The eigenvalues $\lambda$ and $\overline{\mathrm{A}}$ are given by the characteristic equation of matrix $\overline{\mathrm{A}}$, that is by

$$
\begin{aligned}
& -\overline{\mathrm{A}}-\lambda \mathrm{I}=0 \Rightarrow \\
& \left|\begin{array}{ccc}
\left(\mathrm{a}_{11}-\lambda\right) & \mathrm{a}_{12} & a_{13} \\
a_{21} & \left(\mathrm{a}_{22}-\lambda\right) & a_{23} \\
0 & 0 & -\lambda
\end{array}\right|=0 \Rightarrow \\
& -\left(\mathrm{a}_{11}-\lambda\right)\left(\mathrm{a}_{22}-\lambda\right) \lambda+\mathrm{a}_{12} a_{21} \lambda=0 \Rightarrow \\
& \left(\mathrm{a}_{11}-\lambda\right)\left(\mathrm{a}_{22}-\lambda\right) \lambda-\mathrm{a}_{12} a_{21} \lambda=0 \Rightarrow \\
& \left(\lambda-\mathrm{a}_{11}\right)\left(\lambda-\mathrm{a}_{22}\right) \lambda-\mathrm{a}_{12} a_{21} \lambda=0 .
\end{aligned}
$$

One value of $\lambda$ is clearly the $\lambda_{1}, \lambda_{1}=0$. The other two are given by the equation

$$
\left(1-a_{11}\right)\left(1-a_{22}\right)-a_{12} a_{21}=0
$$

In order for the maximum of these two values of $\lambda$ and consequently the maximal eigenvalue $\lambda_{\mathrm{m}}$ of $\overline{\mathrm{A}}$ to be smaller than unit, the following must obviously hold

$$
\left(1-a_{11}\right)\left(1-a_{22}\right)-a_{12} a_{21}>0 .
$$

We postulate that this inequality is satisfied and consequently that the production system can produce each strictly positive or non-negative surplus product.

As is directly apparent from matrix $\overline{\mathrm{A}}$, commodities 1 and 2 are reproductive commodities, while commodity 3 is a non-reproductive commodity.

Commodity 1 is purchased by sectors 2 and 3 (although sector 1 uses it, it does not purchase it, because it itself produces it), commodity 2 is purchased by workers in all three sectors and commodity 3 is purchased by the capitalists of all three sectors.

We now postulate that users or consumers, apart from the case of own consumption, do not purchase any of the 3 commodities directly from producers, but from commerce, to which the three producers, i.e. sectors 1, 2 and 3 , sell their commodities (apart from the quantity which they themselves use as means of production). We also postulate that there are three sectors of commerce, sector 4 which markets commodity 1 , sector 5 which markets commodity 2 and sector 6 which markets commodity 3 .

Each of the sectors 4,5 and 6 uses commodity 1 as a means of production and also uses labour power, that is, after the conversion of labour power into a wage commodity which we performed above, commodity 2 as a wage commodity, in order to transport, store and distribute commodities 1,2 and 3 respectively to buyers. Consequently, sector 4 uses inputs consisting of commodities 1 and 2 , where its inputs to commodity 1 per unit of commodity that it sells consist of a quantity of commodity 1 that it uses as a means of production and of a second quantity of commodity 1 which it must have, in order to be able to market one unit of commodity 1 . This latter quantity is equal to at least one unit of commodity 1. It is greater than one unit of commodity 1 when sector 4 has wastage in commodity 1 that it markets, so that in order to sell one unit it needs more than one unit of that commodity. Sector 4 produces a product 4 which, although it is the same as commodity 1 from a physical viewpoint, from an economic viewpoint it differs from the latter: Commodity 4 is not commodity 1 at the door of the plant which produced it, but is commodity 1 at the store of the merchant of sector 4 . Sector 4 produces, that is, a new commodity, commodity 4.

The same holds correspondingly for sectors 5 and 6 . Sector 5 (sector 6 ) uses in order to sell one unit of commodity 2 (of commodity 3 ), that is, to produce one unit of commodity 5 (of commodity 6 ), a certain quantity of commodity 1 as a means of production, a certain quantity of commodity 2 as a wage commodity and at least one unit of commodity 2 (of commodity 3 ), which it markets.

The production technique is now described by the matrix $\overline{\mathrm{B}}$,

$$
\overline{\mathrm{B}}=\left[\begin{array}{cccccc}
\mathrm{b}_{11} & 0 & 0 & b_{14} & 0 & 0 \\
0 & 0 & 0 & 0 & b_{25} & 0 \\
0 & 0 & 0 & 0 & 0 & b_{36} \\
0 & b_{42} & b_{43} & b_{44} & b_{45} & b_{46} \\
b_{51} & b_{52} & b_{53} & b_{54} & b_{55} & b_{56} \\
0 & 0 & 0 & 0 & 0 & 0
\end{array}\right] .
$$

Each column of $\bar{B}$ symbolises the inputs of the respective sector in quantities of commodities of that sector and of the other sectors, which are necessary for the production of one unit of the commodity that it produces.

Evidently

$$
\begin{aligned}
& \mathrm{b}_{11}=\mathrm{a}_{11}(<1) \\
& \mathrm{b}_{51}=\mathrm{a}_{21} \\
& \mathrm{~b}_{42}=\mathrm{a}_{12} \\
& \mathrm{~b}_{52}=\mathrm{a}_{22}(<1)
\end{aligned}
$$

and

$$
\mathrm{b}_{43}=\mathrm{a}_{13} .
$$

While for obvious reasons

$$
\mathrm{b}_{44}<1
$$

and

$$
\mathrm{b}_{55}<1
$$

If $1_{4}, 1_{5}$ and $1_{6}$ symbolise respectively the direct labour that is necessary for the production of one unit of commodity 4,5 and 6 , then clearly

$$
\begin{aligned}
& b_{54}=l_{4} b_{50}=l_{4} a_{20}, \\
& b_{55}=l_{5} b_{50}=l_{5} a_{20}
\end{aligned}
$$

and

$$
b_{56}=l_{6} b_{50}=l_{6} a_{20} .
$$

The technique, which is described by $\overline{\mathrm{B}}$, and the technique described by $\overline{\mathrm{A}}$, consequently have the following common elements;
(a) The production processes of commodities 1, 2 and 3 are exactly the same and
(b) The real wage rate is in both techniques the same, apart from the fact that in the technique described by $\overline{\mathrm{A}}$ it consists of a certain quantity, the quantity $\alpha_{20}$, of commodity 2 , while in the technique described by $\overline{\mathrm{B}}$ it consists of an equal quantity of $b_{50}$ of commodity 5 . This means that workers now purchase the wage commodity not as commodity 2 from sector 2 , but as commodity 5 from sector 5 .

In addition, the two techniques differ with respect to the following;
(a) Because, when the real wage rate is the 'same' in both, the wage comcommodity corresponding to the technique described by $\bar{B}$ is more expensive than the wage commodity corresponding to the technique described by $\overline{\mathrm{A}}$ (commodity 5 is evidently more expensive than commodity 2 , if we assume that the latter is in both techniques equally expensive), the nominal wage rate calculated either in values or prices is in the technique described by $\overline{\mathrm{B}}$ higher than that in the technique described by $\overline{\mathrm{A}}$, and
(b) The technique described by $\bar{B}$ contains three production processes, those of sectors 4,5 and 6 , which are not contained in the technique described by $\overline{\mathrm{A}}$.

We postulate that the technique described by $\overline{\mathrm{B}}$ is surplus productive, i.e. it can produce each strictly positive or non-negative surplus product Y , and consequently that the maximal eigenvalue of $\bar{B}$ is less than unit.

Assuming that the surplus product of the production system is $\mathrm{Y}, \mathrm{Y}>0$, then for the gross product X the following holds

$$
\begin{aligned}
& \mathrm{X}-\overline{\mathrm{B}} \mathrm{X}=\mathrm{Y} \Rightarrow \\
& (\mathrm{I}-\overline{\mathrm{B}}) \mathrm{X}=\mathrm{Y}
\end{aligned}
$$

and, because, firstly, $\mathrm{Y}>0$ and, secondly, the maximal eigenvalue of $\overline{\mathrm{B}}$ is ex hypothesis less than unit and consequently $(\mathrm{I}-\overline{\mathrm{B}})^{-1} \geq 0$,

$$
\mathrm{X}=(\mathrm{I}-\overline{\mathrm{B}})^{-1} \mathrm{Y}>0 .
$$

Each component of vector Y does not depict the surplus product of a sector, but rather the quantity of one of the commodities, which comprise the surplus product of the overall system: that quantity, by which this commodity is
represented in the surplus product of the overall economy. But what is the surplus product of each sector? What is the surplus product of sector 1 for example? The surplus product of the sector is clearly equal to the difference between the outputs and the total inputs of that sector, i.e. equal to

$$
\left[\begin{array}{c}
\mathrm{X}_{1} \\
0 \\
0 \\
0 \\
0 \\
0
\end{array}\right]-\left[\begin{array}{c}
\mathrm{X}_{1} \mathrm{~b}_{11} \\
0 \\
0 \\
0 \\
\mathrm{X}_{1} \mathrm{~b}_{51} \\
0
\end{array}\right]=\left[\begin{array}{c}
\mathrm{X}_{1}\left(1-\mathrm{b}_{11}\right) \\
0 \\
0 \\
0 \\
-\mathrm{X}_{1} \mathrm{~b}_{51} \\
0
\end{array}\right] .
$$

What surplus product did this sector produce? A surplus product that consists of $X_{1}\left(1-b_{11}\right)$ units of commodity 1 and of $-X_{1} b_{51}$ units of commodity 5.

The surplus product of sector 1 consequently contains positive and negative quantities of commodities. (The quantity $\mathrm{X}_{1}\left(1-\mathrm{b}_{11}\right)$ is positive, because $\mathrm{b}_{11}=\mathrm{a}_{11}$ and $a_{11}<1$ ). And what is the surplus product of sector 2 ? It is equal to $X_{2}$ units of commodity 4 and $-\mathrm{X}_{2} \mathrm{~b}_{42}$ units of commodity 5 . And the surplus product of this sector too contains positive and negative quantities of commodities. This latter holds also for the surplus product of each of the other sectors: the surplus product of these sectors too consists of positive and negative quantities of commodities.

Thus, the surplus product of sector 3 is equal to
$\mathrm{X}_{3}$ units of commodity 3 and
$-\mathrm{X}_{3} \mathrm{~b}_{43}$ units of commodity 4 and
$-\mathrm{X}_{3} \mathrm{~b}_{53}$ units of commodity 5 .
The surplus product of sector 4 is equal to
$\mathrm{X}_{4}\left(1-\mathrm{b}_{44}\right)$ units of commodity 4 and
$-\mathrm{X}_{4} \mathrm{~b}_{14}$ units of commodity 1 and
$-\mathrm{X}_{4} \mathrm{~b}_{54}$ units of commodity 5 .
The surplus product of sector 5 is equal to
$\mathrm{X}_{5}\left(1-\mathrm{b}_{25}\right)$ units of commodity 5 and
$-\mathrm{X}_{5} \mathrm{~b}_{25}$ units of commodity 2 and
$-\mathrm{X}_{5} \mathrm{~b}_{45}$ units of commodity 4 .

And the surplus product of sector 6 is equal to
$\mathrm{X}_{6}$ units of commodity 6 and
$-\mathrm{X}_{6} \mathrm{~b}_{34}$ units of commodity 3 and
$-\mathrm{X}_{6} \mathrm{~b}_{46}$ units of commodity 4 and
$-\mathrm{X}_{6} \mathrm{~b}_{56}$ units of commodity 5.
Can we compare the surplus products of the different sectors with each other? Apparently not. But the most important thing is the following: Precisely because the surplus product of each sector contains also negative quantities of commodities, it is not possible to speak in terms of which sector produced what part of the surplus product of the overall economy. This, i.e. the fact that the surplus product of each sector contains also negative quantities of commodities, as well as the aforementioned consequence of this, is the result of the social division of labour. Social division of labour means, first of all, that each producer (here: each sector) does not himself produce what he needs for his production, i.e. the inputs of his production process, but takes them from other producers (here: sectors), which is why these inputs appear as negative quantities of commodities in his net outputs, i.e. in the net product or, correspondingly, in the surplus product which he produces. ${ }^{2}$ So, as a consequence of the social division of labour it is not possible to say which producer or which sector produced what part of the total net product or of the total surplus product of the economy. So, what is happening here is the same that happens on account of intraplant division of labour. Because also in the production process of a certain producer (=plant) the workers produce under the conditions of intraplant division of labour, it is not possible to say -even if one could say (which, as we saw, one cannot) what is the net product or the surplus product of the plant, i.e. of all the workers as a whole- which part of that net product or surplus product was produced by a certain worker.

The only thing that can be said is that the total net product or surplus product of the economy was produced by the aggregate of its producers and consequently by the aggregate of its workers (and this again, only when this economy is, as in the case here, 'closed', i.e. it does not have exchange relations with other economies).

[^1]But how can one ascertain what a certain sector produced or what it took from the total surplus product of the economy?

In capitalism, the social division of labour is mediated by commodity production. The products of labour become commodities. Each producer takes from the others what he needs for his production, giving something equivalent in return. This also applies to aggregates of producers of the same commodity, i.e. to production sectors. The (by means of money) exchange of the products of labour converts the different (as use values) commodities into things that are homogeneous and commensurate. Thus, the marketing of products of labour, i.e. commodity exchange, allows the ascertainment of the part of the total surplus product produced by a sector as an ascertainment of the part of the total surplus value (if commodities are exchanged at their labour values) or of part of the total profit (if commodities are exchanged at prices which differ from their labour values) produced by that sector.

Let us assume that the commodities are exchanged at their labour values. Then, in the place of each quantity of commodities, its labour value equivalent enters, i.e. the labour value of those commodities, and consequently in the place of the surplus products of the various sectors, the surplus values of those sectors. For this to be possible, it is of course necessary to know the labour value of one unit of each commodity. The vector of labour values can be calculated on the basis of the given technique. It is

$$
\omega=\ell^{(\mathrm{B})}(\mathrm{I}-\mathrm{B})^{-1},
$$

where

$$
\mathrm{B}=\left[\begin{array}{cccccc}
\mathrm{b}_{11} & 0 & 0 & \mathrm{~b}_{14} & 0 & 0 \\
0 & 0 & 0 & 0 & \mathrm{~b}_{25} & 0 \\
0 & 0 & 0 & 0 & 0 & b_{36} \\
0 & \mathrm{~b}_{42} & \mathrm{~b}_{43} & \mathrm{~b}_{44} & \mathrm{~b}_{45} & \mathrm{~b}_{46} \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{array}\right]
$$

and

$$
\ell^{(B)}\left(\ell_{1}, \ell_{2}, \ell_{3}, \ell_{4}, \ell_{5}, \ell_{6}\right) .
$$

B results from $\bar{B}$, if we set to zero all the inputs in wage commodities, i.e. if we set

$$
\mathrm{b}_{51}=\mathrm{b}_{52}=\mathrm{b}_{53}=\mathrm{b}_{54}=\mathrm{b}_{55}=\mathrm{b}_{56}=0 .
$$

Because

$$
(\mathrm{I}-\mathrm{B})^{-1} \geq 0,{ }^{3}
$$

and

$$
\ell^{(B)}>0,4
$$

then also

$$
\omega>0
$$

that is, the labour values of all the commodities are positive.
Then the surplus value produced by sector 1 is evidently equal to

$$
\omega\left[\begin{array}{c}
X_{1}\left(1-b_{11}\right) \\
0 \\
0 \\
0 \\
-X_{1} b_{51} \\
0
\end{array}\right]=\omega_{1} X_{1}\left(1-b_{11}\right)-\omega_{5} X_{1} b_{51}
$$

And the surplus value that was produced by sector 2 , sector 3 , sector 4 , sector 5 and sector 6 is correspondingly equal to

$$
\begin{aligned}
& \omega_{2} X_{2}-\omega_{4} X_{2} b_{42}-\omega_{5} X_{2} b_{52}, \\
& \omega_{3} X_{3}-\omega_{4} X_{3} b_{43}-\omega_{5} X_{3} b_{53}, \\
& \omega_{4} X_{4}\left(1-b_{44}\right)-\omega_{1} X_{4} b_{14}-\omega_{5} X_{5} b_{54}, \\
& \omega_{5} X_{5}\left(1-b_{55}\right)-\omega_{2} X_{5} b_{25}-\omega_{4} X_{5} b_{45}
\end{aligned}
$$

and

$$
\omega_{6} X_{6}-\omega_{3} X_{6} b_{36}-\omega_{4} X_{6} b_{46}-\omega_{5} X_{6} b_{56} .
$$

3. The relationship $(I-B)^{-1} \geq 0$ holds, because $(I-\bar{B})^{-1} \geq 0$ and $B \leq \bar{B}$ hold.
4. This means that no production process is fully automated.

What surplus values were taken by the various sectors? Precisely those which they produced. The fact that each sector takes precisely this surplus value, which it produces, is because the surplus value, which it produces, is calculated on the basis of commodity exchange, i.e. calculated as surplus value, which it produces. And indeed, the surplus value produced by a sector is calculated, as we saw, as the difference between the earnings reckoned in labour value terms and the expenses reckoned in labour value terms, i.e. indeed as the surplus value which it takes. In order for it to be possible for a sector to take more or less surplus value than it produces, the exchange of certain commodities must not have been an equivalent exchange, i.e. there must be two labour values for each of these commodities: a set of labour values, according to which it is calculated how much surplus value was produced by the sector, and a second -different- set of labour values, according to which it is calculated how much surplus value was taken by the sector. But this is impossible, because, as we saw, due to the social division of labour and its mediation by commodity exchange, the quantity of surplus value that was produced by a sector is calculated as the difference between the earnings reckoned in terms of labour values and the expenses also reckoned in labour terms of values, i.e. in terms of labour values, on the basis of which the surplus value -which it takes- is calculated. Consequently, the surplus value, which a sector produces, and the surplus value, which that sector takes, are calculated at the same labour values, at those labour values which indeed apply during exchange. So, the labour values, on the basis of which the surplus value that a sector produces is calculated and the labour values, on the basis of which the surplus value which that sector takes is calculated, are the same. Which is why the surplus value produced by a sector and the surplus value taken by that sector is the same.

It now remains to be shown that the surplus value produced and taken by a sector is positive. The proof is as follows: As is known, the value of the net product, i.e. the aggregate of the value of labour power and of surplus value, of a sector is equal to the living labour used by that sector. For, if $x_{i}$ and $y_{i}$ is the vector of the gross product and of the net product of sector $\mathrm{i}, \mathrm{i}=1,2,3,4,5,6$, then from

$$
\omega=\ell^{(B)}(\mathrm{I}-\mathrm{B})^{-1}
$$

we get

$$
\omega \mathrm{y}_{\mathrm{i}}=\ell^{(\mathrm{B})}(\mathrm{I}-\mathrm{B})^{-1} \mathrm{y}_{\mathrm{i}}
$$

From this equation we get, because of $x_{i}=(I-B)^{-1} y_{i}$ :

$$
\omega \mathrm{y}_{\mathrm{i}}=\ell^{(\mathrm{B})} \mathrm{x}_{\mathrm{i}}
$$

where $\omega y_{i}$ is the labour value of the net product $y_{i}$ and $\ell^{(B)} \mathrm{x}_{\mathrm{i}}$ is the direct labour that was expended for the production of the respective gross product $\mathrm{x}_{\mathrm{i}}$. The surplus value results, if from the labour value $\ell^{(B)} \mathrm{x}_{\mathrm{i}}$ of the net product $\mathrm{y}_{\mathrm{i}}$ we deduct the labour value of the quantity $\ell^{(\mathrm{B})} \mathrm{X}_{\mathrm{i}}$ of labour power that was expended for the production of the respective gross product $\mathrm{x}_{\mathrm{i}}$. The labour value of one unit of labour power is equal to $\omega_{5} \mathrm{~b}_{50}$. Consequently, the labour value of $\ell^{(\mathrm{B})} \mathbf{x}_{\mathrm{i}}$ units of labour power is equal to $\boldsymbol{\ell}^{(\mathrm{B})} \mathbf{x}_{\mathrm{i}} \omega_{5} \mathrm{~b}_{50}$. Thus, the surplus value of sector $i$ is equal to

$$
\ell^{(\mathrm{B})} \mathrm{x}_{\mathrm{i}}-\ell^{(\mathrm{B})} \mathrm{x}_{\mathrm{i}} \omega_{5} \mathrm{~b}_{50} .
$$

This surplus value is clearly positive, when

$$
1-\omega_{5} b_{50}>0
$$

or

$$
\omega_{5} b_{50}>0,
$$

i.e. when the labour value $\omega_{5} b_{50}$ of one unit of labour power is less than unit. It can be shown that in systems, such as the given system, which are able to produce a strictly positive (or semi-positive) surplus product, the labour value of one unit of labour power is less than unit. Consequently, the surplus values of all sectors are positive.

Nothing changes in the above, if we assume that commodities are not exchanged at their labour values, but at prices which differ from the latter. But in this case, it is not possible to speak about the surplus value that a sector produces and takes, rather we must speak in terms of the profit that a sector produces and takes. The profit of a sector results if we multiply the vector of the surplus product of that sector not by the vector of labour values $\omega$, but by the vector of production prices $p$. The vector of prices can, in the given case where the real wage rate is given, be fully determined, if we normalise it by setting the price of a commodity or of a bundle of commodities equal to a positive constant. With the vector of prices, the general rate of profit is also uniquely determined. One can show that the vector of prices is strictly positive
(and in certain cases, where the real wage rate is equal to zero, non-negative) and the rate of profit is always positive. Because the general rate of profit is, due to the fact that the technique is surplus productive, positive, the profit of each sector is also positive. And this is because the rate of profit is equal to the ratio of profit to the price-calculated nominal value of means of production. However, because this nominal value is -due to the positiveness of pricespositive and because the rate of profit is positive, profit is also positive.

And again nothing changes in the above, if the commodities are not exchanged at their labour values or at their production prices, but are exchanged (as happens in reality) at market prices, which differ from both labour values and production prices.

It emerges from our elaboration that each sector takes precisely the surplus value or, correspondingly, the profit, which it itself produces. Consequently, the surplus value or, correspondingly, the profit taken by each of the sectors of circulation (here: of commerce) is produced by the sector itself. And because these sectors produce surplus value or, correspondingly, profit, they are productive sectors, and the labour, which they employ and which produces this surplus value or, correspondingly, this profit, is productive labour.

Furthermore, certain sectors of circulation produce reproductive commodities. In our example, all the sectors except sectors 3 and 6 produce, as one may easily ascertain, reproductive commodities. Only sectors 3 and 6 produce non-reproductive commodities. It follows from this that the sectors of commerce which trade in reproductive commodities (here: sectors 4 and 5 which trade in reproductive commodities 1 and 2) produce reproductive commodities, while the sectors of commerce which market non-reproductive commodities (here: sector 6 which markets the non-reproductive commodity 3 ) produce non-reproductive commodities. Thus, not only are all the sectors of circulation, because and to the extent they use labour power that is not their own, productive sectors, but some of them are also reproductive, i.e. absolutely necessary for the reproduction of the overall economic system.

Our elaboration above constitutes a good basis for evaluating the soundness of certain theories such as the theory of unequal exchange, the theory of surplus transfer from one sector of the economy (the agricultural sector) to another sector of the economy (the industrial sector) as well as, lastly, the theory according to which the sector of 'circulation' does not
produce surplus value or, correspondingly, profit, and consequently is a nonproductive sector, but takes the surplus value which it appropriates from the sector of 'production', which alone produces surplus value or, correspondingly, profit and consequently is the only productive sector of the economy. According to our elaboration above, it is not possible for unequal exchange to exist and also, it is not possible -through exchange- for there to exist surplus transfer from one sector to another sector, either from the agricultural sector to the industrial sector or from the sector of 'production' to the sector of 'circulation'. Lastly, according to the above, in a capitalised economy, the sector of 'circulation' is as a whole not only productive but certain segments of it, those which trade reproductive commodities, are reproductive.

Concluding, we should like to avert a possible misconception. The ascertainment that a sector takes exactly what it produces, applies to all capitalist sectors in their interrelations, but not to the so-called 'factors of production', i.e. to labour and capital in their interrelation. It does not follow from the above that both 'capital' and 'labour' produce what they take, i.e. the former, surplus value or, correspondingly, profit and the latter, the labour value or, correspondingly, the price of total labour power. Because both, not only the surplus value or, correspondingly, the profit but also the labour value or, correspondingly, the price of total labour power, and consequently also their aggregate, the net product as a whole calculated at labour values or, correspondingly, at prices, are solely and exclusively products of labour. So why then is it not possible to say that capitalists take part of the labour value or, correspondingly, of the price of the net product that was produced not by themselves but by workers? Because in capitalist reality, workers do not produce a certain net product for their own account, part of which is then taken by capitalists without having produced it, but rather capitalists produce for their account the total net product, part of which they then pay to workers in return for the labour power which they purchased from them and used for production. Not only this part, but also the remaining part, the surplus value or, correspondingly, the profit which is left for them, was produced by themselves for their own account. The question is not whether they produced it or not, but how they produced it and consequently how they took it without working, since each part of the net product is the product of labour and only labour. The answer to this question is that they produced it, because in its production they used inter alia (means of production, etc.) also something, the
labour value or, correspondingly, the price of which is less than the labour value or, correspondingly, the price of the net product which this produces, because, that is, in its production they use labour power that is not their own. A producer, on the other hand, who produces without using labour power that is not his own, indeed produces, just as the capitalist produces and the paid worker does not produce, for his own account. But also he does not produce, as the capitalist produces, surplus value or, correspondingly, profit. The capitalist does not differ from this producer, who does not use labour power that is not his own, in that the capitalist also takes something which he himself did not produce for his own account, while the producer takes only what he produced for his own account. For neither of them take anything which they did not produce for their own account. But rather they differ in that, because the producer does not use labour power that is not his own, no part of his product takes the form of surplus value or, correspondingly, profit, while part of the product of the capitalist, precisely because he uses labour power that is not his own, takes the form of surplus value or, correspondingly, of profit. In a word: the part of the product reaped by the capitalist is not produced by the worker and then taken by the capitalist, but is produced by the capitalist for his own account, which is precisely why he himself takes it, except that, just as this part results because the capitalist uses labour power that is not his own, the labour value or, correspondingly, the price of which is smaller than the labour value or, correspondingly, the price of the product which he creates, so too this same part is created not by his own, but by the labour power which he uses and is not his own. The fact that what the capitalist takes and what the worker takes are both products of the worker's labour is not contradicted by the fact that both these parts of the net product are produced by the capitalist. The apparent contradiction vanishes if one adds: using however labour power that is not his own (for which he pays less than the labour value or, correspondingly, the price of the net product which this produced).

So, the fact it cannot be said that capitalists take part of the product that was produced by workers, is due to the fact that workers produce absolutely nothing for their own account, part of which could then be taken by the capitalists. The entire net product is produced by capitalists for their own account using labour power that is not their own. They pay a part of this as remuneration to workers, whose labour power they have used, while the rest remains with them. They do not take it from anyone, since they themselves
produced it, just like the first part, for their account, using of course labour power that is not their own, which means that these two parts are at the same time products of the expending of precisely this labour power.


[^0]:    1. If sector 2 used commodity 2 also as a means of production, then naturally the quantity of commodity 2 it used as a means of production and as a wage commodity for the production of one unit of commodity 2 must have been less than one unit of commodity 2 .
[^1]:    2. Net outputs are equal to the net product, when wage commodities are not contained in intermediate inputs, and are equal to the surplus product, when they are contained, as here, in intermediate inputs.
