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On V.K. Dmitriev's Contribution to the so-called "Transformation Problem" and to the Profit Theory

by
*Theodore Mariolis**

I. Introduction

As is well known (see Howard and King (1987), Stamatis (1989)), Wolfang Muehlpfordt (1893), (1895), was the first economist that *correctly formulated* the *so-called* problem of transforming values into production prices. However, Vladimir Korpovich Dmitriev, was the first economist that *correctly solved* the said problem, in his pioneering essay on the Ricardian theory of value ([1898] 1974).

It should, of course, be pointed out that in fact Dmitriev does not deal with the quantitave relation between labour values and production prices, but with the ricardian problem of calculating the uniform profit rate and production prices, for a given linear technique of *single production* and for a given real wage rate. Specifically, Dmitriev proves the following propositions:

P_1. The calculation of the quantities of labour "embodied" in the different commodities (or labour values) is absolutely possible, "...without any digressions into the prehistoric times of the first inception of technical capital" (Dmitriev (1974), p. 44).

P_2. Given the technical conditions of production, commodity prices can be reduced to what, after Sraffa (1960), Ch. VI, we call "dated quantities of labour". Consequently, the relative prices constitute explicit functions of the profit rate (Dmitriev (1974), p. 49).

P_3. Prices are never proportional (or equal) to labour values. Exceptions: the case in which the profit rate is zero and the case in which the "organic composition of capital" is equal in all sectors (Dmitriev (1974), pp. 53-56, 69-73).

* Panteion University, Department of Public Administration, Athens, Greece.

P_4. Irrespective of the *numéraire* chosen, the nominal wage rate constitutes a strictly decreasing function of the profit rate (Dmitriev (1974), p. 57).

P_5. Since the price of the bundle of commodities going to the workers as real wage can be reduced to "dated quantities of labour", it follows that the level of the real wage rate constitutes a strictly decreasing function of the profit rate. The shape of this function is determined (see Dmitriev (1973), pp. 60-61 and 73) *exclusively* by the technical conditions of production in the wage goods industries and in those industries that directly or indirectly provide them with means of production (namely, by the production conditions of the *reproductive* commodities[1]).

P_6. *Starting from the data of Ricardo's approach* (: technical conditions of production, composition and level of the real wage rate – for a thorough exposition, see Kurz and Salvadori (1995), Ch. 1, (1999)), the profit rate and the relative commodity prices can be determined "simultaneously". Consequently, the ricardian system is complete and the well-known Walras's objection ("one equation cannot be used to determine two unknowns", see Dmitriev (1974), pp. 51-52) is untenable (Dmitriev (1974), p. 59).

P_7. The profit rate is positive when, and only when, the value of the real wage rate is less than unit (: *particular profitability condition*). However, "...it is theoretically possible to imagine a case in which all products are produced exclusively by the work of machines, so that no unit of *living labour* (whether human or of any other kind) participates in production, and nevertheless an industrial profit may occur in this case under certain conditions; this is a profit which will not differ essentially in any way from the profit obtained by present-day capitalists using hired workers in production[2]" (Dmitriev (1974), p. 63). Thus, generally, the profit on capital is positive when, and only when, the production of one unit of that commodity, to which the production cost of all the other commodities is

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1. When the composition of the real wage rate is exogenously given in a production system (and hence real wages are part of the produced inputs), we call *reproductive* (*non-reproductive*) those commodities which enter (do not enter) directly or indirectly the production of all commodities. Thus, a *basic* (*non-basic*) commodity is (is not) always reproductive (non-reproductive). Regarding the concepts of *basic* and *non-basic* commodities, see Sraffa (1960), §6-8 and the concepts of *reproductive* and *non-reproductive* commodities, see Marx (1969), pp. 71-73 and Steedman and Metcalfe (1973).
2. See also Denis (1968), pp. 261-263, Nuti (1974), pp. 18-19, Bidard (1991), p. 65.

reduced, requires a smaller *total* (direct and indirect) quantity of that same commodity (: *general profitability condition* –see Dmitriev (1974), pp. 62-66).

Dmitriev proves these propositions within the framework of an "Austrian" model of production[3] (i.e. the series of dated labour inputs is finite and hence the maximum profit rate tends to infinity[4]). In the following, starting from a linear and *indecomposable* technique of single production, we shall gradually develop the model that Dmitriev uses in order to prove the propositions $\mathbf{P}_1$-$\mathbf{P}_7$ (*Part II*). Furthermore, we shall present Dmitriev's contribution to the so-called "transformation problem" and to the profit theory (*Part III*). At the end, we shall determine the *inner limits* of its approach (*Part IV*).

II. Dmitriev's model and solution

We assume a linear technique of single production $[A, \ell]$. The matrix A, $A \equiv [\alpha_{ij}] \geq 0$, symbolises the square nxn matrix of technical coefficients[5], the element α_{ij} of which represents the quantity of commodity i required to produce one unit of commodity j (as gross product), with i, j = 1, 2..., n, while the

3. We note that $\mathbf{P}_1$ is proved within the framework of a technique that produces *basic* and *non-basic* commodities. As is known, Georg Charasoff (1910), p. 147, develops an "algorithm for the calculation of the labour values, because, it seems, he overlooks the fact that Marx's determination of labour values is equivalent to the formulation of a system of n independent consistent equations with n unknowns, which unambiguously determines the labour values of the n commodities" (Stamatis (1988), p. 43 – for Charasoff's contribution, see also Kurz and Salvadori (1995), pp. 387-390). Nevertheless, Dmitriev does not investigate the issue of the positiveness of the labour values. It could, *perhaps*, be said that Dmitriev did not use in the second part of his essay a model with *basic* and *non-basic* commodities, because he was not in the position to solve the said issue or/and he had understood that when there are *non-basic* (or *non-reproductive*) commodities, which enter directly or indirectly its own production, then the production prices are not always strictly positive (take into account some similarity between Dmitriev (1974), pp. 67-69, and the non-positive solutions that appear in the *Part IV* of the present paper).
4. Regarding Dmitriev's "Austrian" model, see Nuti (1974), pp. 12-18, Samuelson (1975), Kurz and Salvadori (1995), pp. 86-87, 176-177, and the *Part II* (*Case 3*) of the present paper. About the "Austrian" models in general, see Burmeister (1974) and Bidard (1991), Ch. 4, 12, 14, 15. In particular, for a reappraisal of the "Austrian" capital theory through the work of P. Sraffa, see Hageman and Kurz (1976), Howard (1980), and Ravix (1990).
5. If all elements of a matrix (or vector) A are greater than those of B, we write A>B, if they are greater or equal, we write $A \geqq B$; we write A≥B, if $A \geqq B$, and A≠B.

vector ℓ, $\ell \equiv [\ell_j] > 0$, symbolises the 1xn vector of inputs of direct homogeneous labour, the component ℓ_j of which represents the quantity of labour required to produce one unit of commodity j (as gross product).

If we introduce the usual assumptions, as well as the assumption that wages are paid at the beginning of the production period, the 1xn vector of relative production prices is determined by the following equations:

$$p = (pA + w\ell)(1 + r), \quad w \equiv p(cd) \tag{1}$$

equivalently (and if $\lambda_A \neq 1$)

$$p = pHr + w\omega(1 + r), H \equiv A[I - A]^{-1}, \omega \equiv \ell[I - A]^{-1} \tag{2}$$

equivalently

$$p = pB(1 + r) \tag{3}$$

where (take into account the Perron-Frobenius Theorems for semipositive matrices) p is the 1xn vector of relative production prices, r(w) the *by assumption* uniform profit rate (nominal wage rate), d the (semi-) positive nx1 vector of the by assumption exogenously given composition of the real wage rate, of which the level is symbolised by the real number c (≥ 0), H ($\geq 0 \Leftrightarrow \lambda_A < 1$) the vertically integrated technical coefficients matrix (Pasinetti (1973), pp. 2-9), I the nxn identity matrix, ω ($> 0 \Leftrightarrow \lambda_A < 1$) the vector of the quantities of labour "embodied" in the different commodities (or labour values), $B \equiv A + (cd\ell) \equiv [b_{ij}]$ the augmented matrix of inputs and λ_A (>0) the Perron-Frobenius eigenvalue of A.

If we use X to symbolise the nx1 vector of the levels of operation of the processes (X>0), Y(>0) the nx1 vector of the net product of the system [A, ℓ, (IX)], U, S the surplus product and the surplus value, respectively, of the system [B, (IX)] and $L \equiv \ell X$, then by definition it holds:

$$Y \equiv [I - A]X \tag{4}$$

$$U \equiv [I - B]X \equiv Y - (cdL) \tag{5}$$

$$S \equiv \omega U \equiv [1 - (\omega cd)]L \tag{5a}$$

We distinguish three cases:

Case 1: *The technique is indecomposable and viable*

The fact that a single production technique is indecomposable means that all the commodities produced are *basic* and that it is viable means that $\lambda_A < 1$. From (1) and (3), (5a) and the Perron-Frobenius Theorems, it follows that:

$$\{p > 0, c \geq 0 \text{ and } r \geq 0\} \Leftrightarrow r \in [0, R] \tag{6}$$

$$\{p > 0 \text{ and } 0 \leq c \leq (1/\omega d)\} \Leftrightarrow r \in [0, R] \tag{6a}$$

$$r = 0 \Leftrightarrow \lambda = 1 \Leftrightarrow \omega cd = 1 \tag{6b}$$

$$r \in (0, R) \Leftrightarrow \lambda_A < \lambda < 1 \Leftrightarrow \omega cd < 1 \tag{6c}$$

$$r = R \Leftrightarrow \lambda_A = \lambda \Leftrightarrow c = 0 \tag{6d}$$

$$S \gtreqless 0 \Leftrightarrow (\omega cd) \lesseqgtr 1 \Leftrightarrow r \gtreqless 0 \Leftrightarrow \lambda \lesseqgtr 1 \tag{6e}$$

$$c = 1/[\ell(1+r)C(r)d], \ \forall c \in (0, 1/\omega d] \tag{7}$$

$$p = (pcd)(1+r)\sum_{t=0}^{\infty} \ell[A(1+r)]^t, \ \forall r \in [0, R) \tag{8}$$

where $R \equiv (1 - \lambda_A)/\lambda_A$ is the maximum (economically significant) profit rate, $r = (1-\lambda)/\lambda$, λ the Perron-Frobenius eigenvalue of B, p the left eigenvector of B associated with its eigenvalue λ and $C(r) \equiv [I - A(1+r)]^{-1}$.

Case 2: *The technique is decomposable à la Sraffa and viable*

For the sake of simplicity, let us assume that the following hold:

$$B = \begin{bmatrix} B_{11} & B_{12} \\ 0 & B_{22} \end{bmatrix} = \begin{bmatrix} A_{11} + (cd_I\ell_I) & A_{12} + (cd_I\ell_{II}) \\ 0 & A_{22} \end{bmatrix} \tag{D_S}$$

where: a) $[A_{11}, \ell_I]$, $[A_{12}, A_{22}, \ell_{II}]$ is the technique of the s $(1 \leq s < n)$ basic, and the n-s non-basic, processes respectively, b) the matrix A_{22} is indecomposable, c) $d = [d_I, 0]^T$, d) $\lambda = \max\{\lambda_I, \lambda_{II}\}$ *and* $\lambda < 1$ (λ_I, λ_{II} the Perron-Frobenius eigenvalue of B_{11} and of $B_{22} = A_{22}$ respectively). Consequently, the surplus value of the system is positive:

$$S \equiv [1 - (\omega_I cd_I)]L, \ \omega_I \equiv \ell_I[I_1 - A_{11}]^{-1} \tag{9}$$

$$S \gtreqless 0 \Leftrightarrow (\omega_I cd_I) \lesseqgtr 1 \Leftrightarrow \lambda \lesseqgtr 1 \tag{10}$$

where ω_I is the 1xs vector of the labour values of the basic commodities and I_1 the sxs identity matrix.

Finally, the systems of determining the prices of the s basic (p_I) and the n-s non-basic (p_{II}) commodities are as follows:

$$p_I = p_I B_{11}(1+r) \tag{11}$$

$$p_{II} = (p_I B_{12} + p_{II} B_{22})(1+r) \tag{11a}$$

Case 3: The technique is decomposable à la Dmitriev

Let us assume that there is no commodity that enters (directly or indirectly) its own production. This implies that there are no basic commodities *and* that at least one commodity is not a means of production. Consequently, matrix A can take the form:

$$A = [\alpha_{ij}] \geq 0 \text{ and } \alpha_{ij} = 0, \forall i \geq j \qquad (D_D)$$

Namely, matrix A is *strictly upper triangular* (all its eigenvalues are equal to zero) and the following hold:

$$\exists \mu \leq n : A^{\mu} = 0 \qquad (12)$$

$$\forall v \in \mathbf{R} : [I - Av]^{-1} = \sum_{t=0}^{\mu-1} (Av)^t \equiv C_D(v) \qquad (12a)$$

From (1), (12a), and if we introduce a normalization equation (of prices) of the form:

$$pu = e \qquad (13)$$

where u is a (semi-) positive nx1 vector and e a positive constant, it follows that:

$$p = (pcd)(1+r) \sum_{t=0}^{\mu-1} \ell[A(1+r)]^t \equiv (pcd)(1+r)\ell C_D(r) \qquad (14)$$

$$c = 1/[\ell(1+r)C_D(r)d] \qquad (15)$$

$$w = e/[(1+r)\ell C_D(r)u], \ \dot{w} < 0, \ r \to \infty, \text{ when } w \to 0 \qquad (16)$$

$$(\omega cd) \lesseqgtr 1 \Leftrightarrow r \gtreqless 0 \qquad (17)$$

where $\dot{w}$ is the derivative for the function w(r).

If, taken further, and in agreement with Dmitriev, we assume that f (to the n-th) "higher order" means of production, g "lower order" means of production and h means of consumption are produced, then matrix A and vectors d, ℓ have the form:

$$A = \begin{bmatrix} O_{ff} & A_{fg} & O_{fh} \\ O_{gf} & A_{gg} & A_{gh} \\ O_{hf} & O_{hg} & O_{hh} \end{bmatrix} \qquad (D_D^*)$$

$$\ell = [\ell_f \ , \ \ell_g \ , \ \ell_h]$$

$$d = [O_f \ , O_g \ , \ d_h]^T, \ d_h > (\geq) 0$$

where A_{gg} is a strictly upper triangular matrix, all of whose elements above the principal diagonal are positive, A_{fg} is a positive matrix and A_{gh} is a (semi-) positive matrix. If A_{gh} is positive, then all the "lower order" means of production are *reproductive* commodities. Thus, in Dmitriev's model, *matrix B* is reduced to a decomposable matrix of the form (D_S) (see *Case 2*), the only difference being that the corresponding A_{22} is zero (if A_{gh} is positive) or strictly upper triangular (if A_{gh} is semipositive). Obviously, given the abovementioned form of A, the following hold (take into account (12a)):

$$\omega = \{[\ell_f A_{fg} + \ell_g][I_g + A_{gg} + A_{gg}^2 + \ldots + A_{gg}^{g-1}]A_{gh}\} + \ell_h \tag{18}$$

$$p_f = w\ell_f(1+r) \tag{19}$$

$$p_g = [w\ell_f A_{fg}(1+r)^2 + w\ell_g(1+r)][I_g - A_{gg}(1+r)]^{-1} \tag{20}$$

$$p_h = \{[w\ell_f A_{fg}(1+r)^3 + w\ell_g(1+r)^2][I_g - A_{gg}(1+r)]^{-1} A_{gh}\} + w\ell_h(1+r) \tag{21}$$

where (19) corresponds to f "point input – point output" processes, (20) to g "flow input – point output" processes of g+1 production periods and (21) to h "flow input – point output" processes of g+2 production periods.

The definition equation of the values in (2) (and the equation (18)) constitutes the proof of the proposition $\mathbf{P}_1$. $\mathbf{P}_2$ and $\mathbf{P}_3$ are proved by means of (14). Equation (16) constitutes the proof of $\mathbf{P}_4$. $\mathbf{P}_5$ is proved by means of (15) (or (21), with $w \equiv p_h(cd_h)$ and $d_h \geq 0$). With the real wage rate given from outside the system, the profit rate and the relative prices are *unambiguously determined* by (15) and (14) respectively (: $\mathbf{P}_6$). Finally, condition (17) constitutes the *particular profitability condition* ($\mathbf{P}_7$) and the quantity ωcd expresses (*in accordance with Dmitriev's general profitability condition*) the *total* quantity of the labour-power commodity required to produce one unit of the labour-power commodity (see also the *Part III* of the present paper).

It is obvious that the *logic* of Dmitriev's solution is completely applied to the case in which A is indecomposable (see mainly (6e), (7), (8)). As we shall see (*Part IV*), however, the same does not hold in the case in which A is decomposable à la Sraffa. In the following (*Part III*), we shall prove that also an indecomposable technique can be analysed on the basis of Dmitriev's profit theory.

III. Generalization

We assume a linear and indecomposable technique of single production. Let[6] m_{ij} be the total (direct and indirect) quantity of commodity i required to produce one unit of commodity j (as gross product), on the basis of matrix[7] B. The quantities m_{ij} are determined by the system:

$$M \equiv \tilde{M}B \tag{22}$$

equivalently

$$[m_{k1}, m_{k2}, ..., m_{kn}][I-B] \equiv (1-m_{kk})b_k \Leftrightarrow$$

$$[m_{k1}, m_{k2}, ..., m_{kn}][I-B_k] \equiv b_k, \ k = 1, 2, ..., n \tag{22a}$$

where $M \equiv [m_{ij}]$, $\tilde{M}$ the nxn matrix, which derives from the nxn matrix M, when we replace all the elements of its principal diagonal with unit, b_k the k-th row of B and B_k the nxn matrix, which derives from B, when we replace all the elements of its k-th row with zero. Consequently:

a) If $\sigma_{ii} \neq 0$, it follows that:

$$1 - m_{ii} \equiv (\det[I-B]) / \sigma_{ii} \tag{23}$$

$$m_{ij} \equiv \sigma_{ji} / \sigma_{ii}, \ i \neq j \tag{23a}$$

where $\det[I-B]$ is the determinant of $[I-B]$ and σ_{ii}, σ_{ji}, the cofactors of the element ii and ji (respectively) of the matrix $[I-B]$.

b) If $\lambda \neq 1$, it follows that:

$$[m_{ij} / (1-m_{ii})] \equiv h^*_{ij} \tag{23b}$$

where $H^* \equiv [h^*_{ij}] \equiv B[I-B]^{-1}$ is the vertically integrated augmented matrix of inputs.

c) If we use q to symbolise the right eigenvector of B associated with λ, then from (22) we obtain:

$$[M-(\lambda \tilde{M})]\, q \equiv 0 \tag{24}$$

6. Note that the following analysis can analogously be applied to matrix A in order to examine the existence of a positive maximum profit rate.

7. If, for example, n = 2 (and $b_{ii} < 1$), then:

$$m_{ij} \equiv b_{ij} + m_{ij} b_{jj} \ \Rightarrow \ m_{ij} \equiv b_{ij}(1 + b_{jj} + b^2_{jj} + ...)$$

and

$$m_{ii} \equiv b_{ii} + m_{ij} b_{ji} \ \Rightarrow \ m_{ii} \equiv b_{ii} + [(b_{ij} b_{ji})/(1-b_{jj})]$$

The application of the Perron-Frobenius Theorems to the equations (22a) and (24) leads to the following results:

$$\lambda \leq 1 \Leftrightarrow \{m_{ij} > 0,\ m_{ii} \leq \lambda,\ \forall i,j\} \tag{25}$$

$$\exists k: \{m_{kj} > 0,\ \forall j \textit{ and } m_{kk} \gtreqless 1\} \Rightarrow \lambda \gtreqless 1 \tag{26}$$

$$\exists k: \{m_{kj} > 0,\ \forall j \neq k \textit{ and } \lambda > 1\} \Rightarrow m_{kk} > \lambda \tag{27}$$

$$\exists k: m_{kk} = 1 \Rightarrow \{m_{ii} = \lambda = 1,\ \forall i\} \Rightarrow M[I-B] = 0$$

$$\Rightarrow m_{ij} = p_j^* / p_i^*,\ \forall i,j \tag{28}$$

where p_j^*, p_i^* the components j and i (respectively) of the left eigenvector p^* of B associated with[8] λ.

Lastly, and if we suppose that $\lambda \leq 1$ holds, from systems (3) and (22a) it follows that the "production cost" of *each* commodity can be reduced to the "production cost" of the k-th commodity, as follows (compare with Dmitriev (1974), pp. 58-63):

8. Note that the system (1) can be written as:

$$[p, w] = [p, w][B' + (B'' r)] \tag{1'}$$

where

$$B' \equiv \begin{vmatrix} A & cd \\ \ell & 0 \end{vmatrix}$$

and thus the labour-power commodity (commodity n+1) appears explicitly as it indeed is: a produced commodity, of which the price does not include profit (the content of the matrix B'' is evident).

Let $m'_{\beta\gamma}$ be the *total* quantity of commodity β required to produce one unit of commodity γ (as gross product), *on the basis of matrix B'* ($\beta, \gamma = 1, 2, \ldots, n+1$). As it is easily proven, the following hold:

$$m'_{\beta\gamma} \equiv m_{ij},\ \beta, \gamma \neq n+1$$

$$m'_{\beta\gamma} \equiv \omega_j,\ \beta = n+1,\ \gamma \neq n+1$$

$$m'_{\beta\gamma} \equiv \omega cd,\ \beta = \gamma = n+1$$

$$\lambda' \leq 1 \Leftrightarrow \{m'_{\beta\gamma} > 0,\ m'_{\beta\beta} \leq \lambda',\ \forall \beta, \gamma\} \tag{25'}$$

$$\{\lambda < \lambda' < 1\} \Leftrightarrow r \in (0, R)$$

where λ' the Perron-Frobenius eigenvalue of B'.

$$p = [p(B_k\pi)] + [p_k(b_k\pi)] \Rightarrow$$
$$p = m_k\{p_k\pi[I-(B_k[I-B_k]^{-1}r)]^{-1}\} \tag{29}$$

equivalently

$$p = (p_k)[(b_k\pi)[I-(B_k\pi)]^{-1}] \Rightarrow$$
$$p = (p_k)[m_{k1}(\pi), m_{k2}(\pi), ..., m_{kn}(\pi)] \Rightarrow$$
$$(p_j / p_k) = m_{kj}(\pi), \quad \forall j \neq k \tag{29a}$$
$$1 = m_{kk}(\pi) \tag{29b}$$

and (if we apply the abovementioned reduction for *each* k = 1, 2, ..., n):

$$(p_j / p_i) = m_{ij}(\pi), \quad \forall i, j \tag{29c}$$
$$1 = m_{ii}(\pi), \quad \forall i \tag{29d}$$

where $\pi \equiv 1+r$, m_k the k-th row of M and $m_{ij}(\pi)$ the *total* quantity of commodity i required to produce one unit of commodity j (as gross product), *on the basis of the π-augmented matrix of inputs ($B\pi$).* As it is easily proven, the following also hold:

$$m_{ij} = m_{ij}(\pi) \Leftrightarrow r = 0 \tag{30}$$
$$m_{ij} < m_{ij}(\pi) \Leftrightarrow r \in (0, R] \tag{30a}$$

Two conclusions are deduced from the preceding generalization:

1. From (1), (2) it follows that:

$$p = \omega\{(pcd)\pi[I-(Hr)]^{-1}\} \tag{31}$$

equivalently

$$p = (pcd)[\omega_1(\pi), \omega_2(\pi), ..., \omega_n(\pi)] \Rightarrow$$
$$(p_j / p_i) = (\omega_j(\pi) / \omega_i(\pi)), \quad \forall i, j \tag{31a}$$
$$1 = \omega(\pi)cd \tag{31b}$$

where $\omega(\pi)$ is the vector of the *π-labour values* (i.e. labour values computed on the basis of the technique[9] [$A\pi$, $\ell\pi$]). However, (31), (31a), (31b) cannot consolidate the existence of a *special relation* between labour values and prices ("transformation problem"), because the equations (29), (29c), (29d) also hold. Thus, there are matrices (linear operators) with which we can "transform" the totally expended quantities of *any* (if A or B are indecomposable)

9. Note that the components of the vector $\omega(\pi)$ are (equivalently) the quantities $m'_{\beta\gamma}$, $\beta = n+1$, $\gamma \neq n+1$, *computed, however, on the basis of the matrix [$B'+(B''r)$]* (see footnote 8).

commodity (included the labour-power commodity) into prices, and the relative prices are not only equal to the relative π-values, but also to the quantities $m_{ij}(\pi)$.

2. The relations (6), (6e) cannot consolidate the existence of a *special relation* between surplus value and profit. From (5), (6), (6e), (25) and (25′) a general profitability condition is deduced, which includes the well-known "Fundamental Marxian Theorem":

$$\begin{aligned}(p,r)>0 \;&\Leftrightarrow\; \lambda<1 \;\Leftrightarrow\; \omega cd<1 \;\Leftrightarrow\; S>0 \;\Leftrightarrow\; \{m_{ij}>0, m_{ii}<1\}\\ &\Leftrightarrow\; \lambda'<1 \;\Leftrightarrow\; \{m'_{\beta\gamma}>0, m'_{\beta\beta}<1\} \;\Leftrightarrow\; U^{*}>(\geq)0 \qquad \text{(G.P.C.)}\end{aligned}$$

where U^{*} ($\equiv (1-\omega cd)d$) is the surplus product of that sub-system (à la Sraffa (1960), Appendix A), which produces the real wage bundle d as its net product.

IV. The inner limits of Dmitriev's approach

Let that the technique is decomposable à la Sraffa (see *Part II, Case 2*) and Dmitriev's profitability condition is satisfied[10]. In contrast with the case of an indecomposable technique, the said condition does not ensure, first, the *unambiguous determination* of p, r (obviously, this point forms a crucial test for the so-called "transformation problem"), and second, the positiveness of p. From (11) and (11a) we obtain the following solutions (see also Egidi (1975), pp. 11-13):

Case 2.1. If $\lambda_I > \lambda_{II}$, then:

i. $p>0,\; 1+r=1/\lambda_I$

ii. $p_I=0,\; p_{II}>0,\; 1+r=1/\lambda_{II}$

10. Suppose that the technique is decomposable à la Sraffa (D_S) and there is *one, and only one*, wage-commodity, e.g. commodity 1. As derived from (22) (or (22a)), the following holds:

$$m_{11} \equiv \omega_1(cd_1) + \eta_{11}^{A}[1-(\omega_1 cd_1)]$$

where η_{11}^{A} the *total* quantity of commodity 1 required to produce one unit of commodity 1 (as gross product), *on the basis of matrix A* (namely, of matrix A_{11}). Consequently, the labour value of the real wage rate is less than the quantity m_{11}, precisely because the wage-commodity enters (directly or indirectly) its own production ($\eta_{11}^{A}>0$). If the technique is decomposable à la Dmitriev (D_D), then $\eta_{ii}^{A}=0$, $\forall i$, and consequently the value of the real wage rate is equal to the *total* quantity of the wage-commodity required to produce one unit of the wage-commodity (as gross product). This point expresses a feature of Dmitriev's model, which, however, does not have further consequences.

Case 2.2. If $\lambda_I = \lambda_{II} = \lambda$, then:

i. $p_I > 0$, system (11a) is incosistent, $1 + r = 1/\lambda$

ii. $p_I = 0, \quad p_{II} > 0, \quad 1 + r = 1/\lambda$

Case 2.3. If $\lambda_I < \lambda_{II}$, then:

i. $p_I > 0, \quad p_{II} \ngeq 0, \quad 1 + r = 1/\lambda_I$

ii. $p_I = 0, \quad p_{II} > 0, \quad 1 + r = 1/\lambda_{II}$

Moreover, as pointed out by Krause (1981), p. 176, the second (ii) solution of the abovementioned *Case 2.1.* implies the following "paradoxical" solution: if $\lambda = \lambda_I = 1$, then zero surplus value (see (10)) coexists with positive profit.

The multiplicity of the solutions[11] and the non-positiveness of p in the case of decomposable techniques are known to be derived (see Sraffa (1960), §35, n.1, §39, n.1 and Appendix B, Pasinetti (1985), pp. 113, 233-4) from the assumption of the existence of a uniform profit rate for the reproductive and non-reproductive processes[12]. If however, the profit rates are different for reproductive (r_I) and non-reproductive (r_{II}) processes, then, first, the determination of r_I, r_{II} presupposes the price vector as exogenously given, second, the *average profit rate* of the system [B, (IX)] depends on the composition of the vector $X \equiv [X_I, X_{II}]^T$ and the production conditions of the non-reproductive commodities[13], and third, Dmitriev's condition is neither necessary nor sufficient for positive profit to exist[14].

Example

Let a decomposable (D_S) technique with $n = 2$ and $b_{22} < 1$:

$$p_I' = p_I' b_{11}(1 + r_I) \Rightarrow \{r_I \gtreqless 0 \Leftrightarrow S \gtreqless 0 \Leftrightarrow \lambda_I = m_{11} = b_{11} \lesseqgtr 1\}$$

11. Dmitriev's approach *always* leads to that solutions, which include $p_I > 0$ (i.e. solutions (i)).
12. Take into account (29c) and note that the quantities $m_{ij}(\pi)$, $i = 1, 2, ..., s$, $j = s+1, s+2, ..., n$, tend to infinity when π tends to $1/\lambda_{II}$.
13. Moreover, note that, on the one hand, the solutions of the system (11), (11a) (as a whole) depend on the production conditions of the non-reproductive commodities (i.e. on the relation between λ_I and λ_{II}) and on the other, there are economically quasi-significant solutions ($p \geq 0$) of the system (11), (11a), which include a profit rate equal to $(1 - \lambda_{II})/\lambda_{II}$.
14. Obviously, if the commodity prices deviate from their production prices, then the same holds even in the case of indecomposable techniques (for a detailed investigation, see Mariolis (1999)).

$$p'_{II} = (p'_I b_{12} + p'_{II} b_{22})(1 + r_{II}) \Rightarrow \{r_{II} \gtrless 0 \Leftrightarrow \tilde{p} \gtrless m_{12}\}$$

where $p' \equiv [p'_I, p'_{II}] (>0)$ a price vector, which deviates from the vector of production prices p $(p' \neq p \Leftrightarrow r_I \neq r_{II})$ and $\tilde{p} \equiv p'_{II}/p'_I$. .

We distinguish two cases:

a) $b_{11} < 1$, $\tilde{p} < m_{12}$ *and* $[(m_{12} - \tilde{p})(X_{II}/X_I)] \geq [(1 - b_{11})/(1 - b_{22})]$:

   S > 0 and *therefore*, the profit of the *reproductive* system is positive, while the profit of the *non-reproductive* system is negative and the profit of the *system as a whole* is non-positive.

b) $b_{11} \geq 1$, $\tilde{p} > m_{12}$ *and* $[(m_{12} - \tilde{p})(X_{II}/X_I)] < [(1 - b_{11})/(1 - b_{22})]$:

   S ≤ 0 and *therefore*, the profit of the *reproductive* system is non-positive, while the profit of the *non-reproductive* system and the profit of the *system as a whole* are positive.

In fact, Dmitriev, by means of equation (15), does not determine the profit rate of the technique as a whole, but the *by assumption uniform* profit rate of that sub-system (à la Sraffa) which produces the real wage bundle d as its net product. Moreover, Dmitriev's condition is simply and only necessary and sufficient condition of the (semi-) positiveness of U^*. Thus, Dmitriev's condition is simply and only necessary and sufficient condition of the positive profit existence in the said sub-system[15] (see also Mariolis (1999)).

15. *Proof*: Let p′ (> 0) a given price vector, which deviates from the vector p (production prices) and r^* the *average* profit rate of the said sub-system. By definition it holds:

$$r^* \equiv (p' U^*)/(p' H d + w' \omega d) \Rightarrow$$

$$r^* \equiv \bar{S}/(E^* + 1), \; E^* \equiv (p' H d)/(w' \omega d) \qquad (32)$$

where $\bar{S}$ the rate of surplus value, E^* the price composition of capital in the sub-system of the real wage bundle and $w' \equiv p'(cd)$. Thus, from (32) it follows that:

$$\forall p' > 0: \{\bar{S} \gtrless 0 \Leftrightarrow r^* \gtrless 0\} \qquad (32a)$$

Now, if we postulate the existence of a *uniform* profit rate r ($\Leftrightarrow p' \overset{!}{=} p$), then the ratio $\bar{S}/(E^* + 1)$ is written as:

$$F(r) \equiv [1 - (\omega c d)] / [(\ell C(r) H + \omega) c d]$$

Thereby, first, we determine the uniform profit rate r as the fixed point of F(r) (r = F(r)) and

V. Conclusions

Three conclusion are deduced from the preceding exposition:

1. Dmitriev's approach can entirely be applied to indecomposable and decomposable techniques, in which, however, there is no non-reproductive commodity that enters (directly or indirectly) its own production. *In any case*, nevertheless, Dmitriev does not determine the profit rate of the technique (or the system) as a whole, but the *by assumption uniform* profit rate of the real wage bundle sub-system[16].
2. In fact, Dmitriev, by means of his general theory of profit, does not prove (as he believed) that the marxian theory of profit is partial. He proves, and this constitutes an *insuperable contribution*, that a theory of profit cannot be founded exclusively on a system of equations (see the G.P.C.).
3. However, Dmitriev's profitability condition is, in the general case, untenable. This entails that the marxian theory of profit is also untenable, *or that the magnitude S does not constitute, in fact, the surplus value* (and that profit is positive when, and only when, *and because* the correctly determined surplus value is positive[17]). Any other case does not exist.

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second, the positiveness of S *is disguised as* necessary and sufficient condition of the positive profit existence in every process and therefore in the system as a whole. Certainly, as we have seen, when the technique is decomposable this "mechanism" does not always work "regularly". Thus, the only thing that $S > (\leq 0)$ ensures (implies), is the positiveness (non positiveness) of the profit in the real wage bundle sub-system *for each* $p' > 0$.

16. It must be pointed out that also G. Charasoff (1910), Ch. X, does not determine the profit rate of the technique (or the system) as a whole, but in fact, the *average* profit rate r^{**} of that system, which produces q (i.e. the right eigenvector of B associated with λ) as its gross product (for a thorough investigation of Charasoff's *standard system* and solution, see Stamatis (1988)):

$$r^{**} \equiv (p'[I-B]q)/(p'Bq) = (1-\lambda)/\lambda, \quad \forall p' > 0$$

$$\lambda < 1 \quad \Leftrightarrow \quad r^{**} > 0 \quad \Leftrightarrow \quad [I-B]q(=U^{**}) > 0$$

Furthermore, he postulates the existence of a uniform profit rate r ($\Leftrightarrow p' \overset{!}{=} p$) and finally, he determines the relative prices from (3). Thus, our critical remarks on Dmitriev's solution, can also be applied to Charasoff's solution (take into account that if the technique is decomposable à la Sraffa, then the following hold: if $\lambda_I \geq \lambda_{II}$, then $q_I > 0$, $q_{II} = 0$ and if $\lambda_I < \lambda_{II}$, then $q > 0$).

17. This, for example, supports the so-called "New Interpretation or Approach" that Duménil (1980) and Foley (1982) have developed. For a different approach, see Mariolis (1998), (1999), (2000).

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