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On the Profitability and the Efficiency of the Techniques in a neo-Ricardian Model of Single Production: A Note

by
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1. Introduction

The purpose of this note is to analyse the profitability and the efficiency of the techniques in a neo-Ricardian model of single production. More precisely, it connects Dmitriev's ([1898] 1974) profitability analysis (the uniform rate of profit is positive when, and only when, «we can obtain a larger quantity of the same product within some finite period of time[1] as a result of the production process», ibid., p. 62) with the notion of vertical integration (Pasinetti (1973)) and it proves that, though the existence of a positive uniform rate of profit and growth can be reduced to (or deduced from) the physical - technical production conditions, the ordering of the available techniques with respect to their profitability and efficiency cannot.

Equivalently, it could be said that we prove the following: Based on Dmitriev's profitability analysis and on the notion of vertical integration, we can construct real indices of the «total capital quantity». However, this construction does not bypass the problem of «capital goods» heterogenity, precisely because the relative commodity prices and the ordering of the techniques cannot be deduced from (or reduced to) the abovementioned indices.

Thus the present note defines a quasi-new method to express the Sraffian critique of the traditional neoclassical theory (of capital and income distribution) and the labor theory of value[2].

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1. As is well known, this proposition was proved within the framework of an «Austrian» model of production, in which the series of dated labor inputs is finite (and hence the maximum rate of profit tends to infinity). See also Sraffa (1960), Appendix D, §3, Kurz and Salvadori (1995), pp. 176-8, Mariolis (2000).
2. In the present note we use the term «labor theory of value» with the following content: a

2. Basic Definitions and Propositions

We assume a linear and *indecomposable* technique of single production $[A, \ell]$. The indecomposable matrix A, $A \equiv [a_{ij}] \geq 0$, symbolises[3] the square nxn matrix of technical coefficients, the element a_{ij} of which represents the quantity of commodity i required to produce one unit of commodity j (as gross product), with $i, j = 1, 2, ..., n$, while the vector ℓ, $\ell \equiv [\ell_j] > 0$ symbolises the 1xn vector of inputs of direct homogeneous labor, the component ℓ_j of which represents the quantity of labor required to produce one unit of commodity j (as gross product).

If we introduce the usual assumptions, as well as the assumption that wages are paid at the beginning of the production period, the 1xn vector of relative commodity prices is determined by the following equations:

$$p = (pA + w\ell)(1 + r), \quad w \equiv p(cd) \tag{1}$$

or

$$p = pHr + w\omega(1 + r), \quad H \equiv A[I - A]^{-1}, \quad \omega = \ell[I - A]^{-1} \tag{2}$$

or

$$p = pBv, \quad B \equiv A + cd\ell, \quad v \equiv 1 + r \tag{3}$$

where p the vector of relative prices, r(w) the by assumption uniform rate of profit (nominal wage rate), d the (positive or semipositive) nx1 vector of the by assumption exogenously given composition of the real wage rate, of which the level is symbolised by the real number c, H the vertically integrated technical coefficients matrix[4], I the nxn identity matrix, ω the vector of the quantities of labor «embodied» in the different commodities (or labor values) and B the augmented matrix of inputs.

Let[5] m_{ij} be the *total* (i.e. direct and indirect) quantity of commodity i

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theory of determining production prices through labor values (i.e. quantities of labor «embodied» in the different commodities).

3. If all elements of a matrix (or vector) A are greater than those of B, we write $A > B$, if they are greater or equal, we write $A \geqq B$; we write $A \geq B$, if $A \geqq B$ and $A \neq B$.

4. See Pasinetti (1973), pp. 2-9.

5. Obviously, the following analysis can analogously be applied to A in order to examine the existence of a positive maximum rate of profit. See also the footnote 11 below.

required to produce one unit of commodity j (as gross product), on the basis of matrix[6] B. Consequently, the quantities m_{ij} are determined by the system:

$$M \equiv \tilde{M}B \tag{4}$$

or (equivalently)

$$[m_{k1}, m_{k2}, \ldots, m_{kn}]\,[I-B] \equiv (1-m_{kk})b_k \;\Leftrightarrow$$

$$[m_{k1}, m_{k2}, \ldots, m_{kn}]\,[I-B_k] \equiv b_k, \quad k = 1, 2, \ldots, n \tag{4a}$$

where $M \equiv [m_{ij}]$, $\tilde{M}$ the nxn matrix, which derives from the nxn matrix M, when we replace all the elements of its principal diagonal with unit, b_k the k-th row of the matrix B, and B_k the nxn matrix, which derives from matrix B, when we replace all the elements of its k-th row with zero. Consequently:

a) If $\sigma_{ii} \neq 0$, it follows that:

$$1 - m_{ii} \equiv (\det[I-B]) / \sigma_{ii} \tag{5}$$

$$m_{ij} \equiv \sigma_{ji} / \sigma_{ii}, \quad i \neq j \tag{5a}$$

where det[I – B] the determinant of [I – B] and σ_{ii}, σ_{ji}, the cofactors of the elements ii and ji (respectively) of the matrix [I – B].

b) If $\lambda \neq 1$, it follows that:

$$[m_{ij} / (1 - m_{ii})] \equiv h'_{ij} \tag{5b}$$

where λ the Perron-Frobenius eigenvalue of B ($\lambda > 0$) and $H' \equiv [h'_{ij}] \equiv B[I-B]^{-1}$ the vertically integrated augmented matrix of inputs.

c) If we use q* to symbolise the right eigenvector of B associated with its largest real eigenvalue λ, then from the equation (4) we obtain:

$$[M - (\lambda \tilde{M}\;)]q^* \equiv 0 \tag{6}$$

The application of the Perron-Frobenius Theorems (for semipositive matrices[7]) to the preceding equations leads to the following results:

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6. When, for example, n = 2 and $b_{ii} < 1$, we have:

$$m_{12} = b_{12}(1 + b_{22} + b_{22}^2 + \ldots) \;\Rightarrow$$
$$m_{11} = b_{11} + b_{21}[b_{12}(1 + b_{22} + b_{22}^2 + \ldots)]$$

7. See, e.g. Kurz and Salvadori (1995), pp. 509-19.

a) From (3), (4a), (6) it follows that[8]:

$$\lambda \leq 1 \Leftrightarrow \omega(cd) \leq 1 \Leftrightarrow \{p > 0, v \geq 1\} \Leftrightarrow \{m_{ij} > 0.\ m_{ii} \leq \lambda, \forall i, j\} \tag{7}$$

$$\{\exists k : m_{kj} > 0, \forall j \text{ and } m_{kk} \gtreqless 1\} \Rightarrow \lambda \gtreqless 1 \tag{8}$$

$$\{\exists k : m_{kj} > 0, \forall j \neq k \text{ and } \lambda > 1\} \Rightarrow m_{kk} > \lambda \tag{9}$$

However:

$$\{0 < m_{ii} < 1, \forall i\} \not\Rightarrow \lambda < 1 \tag{10}$$

Example 1:

$$B = \begin{bmatrix} 4.1 & 0.4 \\ 1 & 1.1 \end{bmatrix} \Rightarrow m_{11} = 0.1, m_{22} \cong 0.97, \lambda > 1.$$

b) From (4a), (7), (4) it follows that:

$$\exists k : m_{kk} = 1 \Rightarrow \{m_{ii} = \lambda = 1, \forall i\} \Rightarrow M[I - B] = 0 \Rightarrow$$

$$\Rightarrow m_{ij} = p_j^* / p_i^*, \forall i,j \tag{11}$$

where p_j^*, p_i^*, the components j and i (respectively) of the left eigenvector p* (known to be positive) of B associated with λ.

c) From (4) it follows that:

$$\exists k : \tilde{m}_k B = \lambda \tilde{m}_k \Rightarrow \lambda = 1 \tag{12}$$

where $\tilde{m}_k$ the k-th row of $\tilde{M}$.

If the inverse of B exists then, from (4), it follows that:

$$\exists k : m_k B = \lambda m_k \Rightarrow \lambda = 1 \tag{13}$$

where m_k the k-th row of M. However, if the inverse of B does not exist, then:

$$\exists k : m_k B = \lambda m_k \not\Rightarrow \lambda = 1 \tag{14}$$

8. As it is easily proven, for a cyclic matrix the relations between λ and m_{ii} can be directly expressed: $\lambda^n = m_{11} = m_{22} = \ldots = m_{nn}$. However, B cannot be cyclic (because $\ell > 0$). Finally, we may note that the condition (7) constitutes a *general profitability condition*, which includes the well-known «Fundamental Marxian Theorem» (see also Mariolis (2000), Part III).

Obviously (see(4a)), in this case b_k is the left eigenvector of B associated with λ:

Example 2:

$$B = \begin{bmatrix} 0.4 & 0.25 \\ 0.5 & 0.3125 \end{bmatrix} \Rightarrow m_1 B = 0.7125 m_1$$

d) From (6) and (7) it follows that:

$$\{\exists k : m_{kk} = \lambda \text{ and } m_{kj} > 0, \forall j\} \Rightarrow \{m_{ij} > 0, m_{ii} = \lambda = 1, \forall i,j\}. \quad (15)$$

However:

$$\exists k : m_{kk} = \lambda \not\Rightarrow \lambda = 1 \quad (16)$$

Example 3:

$$H' = \begin{bmatrix} -1.25 & 0.5 & -0.5 \\ -0.5 & -1.25 & 0.5 \\ 0.5 & -0.5 & -1.25 \end{bmatrix} \Rightarrow B(\equiv H'[I + H']^{-1}) > 0$$

and

$$m_{11} = m_{22} = m_{33} = \lambda = 5,\ m_{12} = m_{23} = m_{31} = -m_{13} = -m_{21} = -m_{32} = -2.$$

Lastly, and if $\lambda \leq 1$ holds, from the systems (3) and (4a) it follows that the «production cost» of *each* commodity can be reduced to the «production cost» of the k-th commodity, as follows[9]:

$$p = [p(B_k v)] + [p_k(b_k v)] \Leftrightarrow$$
$$p = (p_k)[(b_k v)[I - (B_k v)]^{-1}] \Leftrightarrow$$
$$p = (p_k)[m_{k1}(v), m_{k2}(v), \dots, m_{kn}(v)] \Leftrightarrow$$
$$(p_j / p_k) = m_{kj}(v),\ \forall j \neq k \quad (17)$$
$$1 = m_{kk}(v) \quad (17a)$$

9. As is well known, this reduction was first applied by Dmitriev ([1898], 1974), pp. 58-63. See also Duménil (1980), Ch. IV.

and (if we apply the abovementioned reduction for *each* k = 1, 2, ..., n):

$$(p_j / p_i) = m_{ij}(v), \ \forall i, j \tag{18}$$

$$1 = m_{ii}(v), \ \forall i \tag{18a}$$

where (compare with system (4)) $m_{ij}(v)$ the *total* quantity of commodity i required to produce one unit of commodity j (as gross product), *on the basis of the v-augmented matrix of inputs*[10] (Bv). Evidently, p is the left eigenvector of (Bv) associated with its Perron-Frobenius eigenvalue, which is equal with unit (p = p*). Finally, the following also holds:

$$m_{ij}(v) = [\sigma_{ji}(v)] / [\sigma_{ii}(v)], \ i \neq j \tag{18b}$$

where $\sigma_{ji}(v)$, $\sigma_{ii}(v)$, the cofactors of the elements ji and ii (respectively) of the matrix $[I - (Bv)]$.

3. The ordering of techniques with respect to their profitability

We assume that the level of the real wage rate is exogenously given $(c = \bar{c})$, there is an indecomposable augmented matrix B and $\nabla = (1/\lambda) \geq 1$ indicates the rate of profit (determined by the system (3)).

How can we construct an equally or more profitable augmented matrix B′ (indecomposable)?

If the said matrices *differ only in the column* s $(1 \leq s \leq n)$, then from (18b) it follows that:

$$m_{sj}(\nabla) = m'_{sj}(\nabla), \ \forall j \neq s \tag{19}$$

where $m_{sj}(\nabla)$, $m'_{sj}(\nabla)$, the abovementioned quantities computed on the basis of the matrices $(B\nabla), (B'\nabla)$ respectively. Therefore, as may directly be deduced

10. From (2) it follows that:

$$p = \omega\{w(1 + r)[I - (Hr)]^{-1}\} \tag{2a}$$

but this cannot consolidate the existence of a *special* relation between labor values and prices («transformation problem»), because it also holds:

$$p = m_k\{p_k(1 + r)[I - (B_k[I - B_k]^{-1} r)]^{-1}\} \tag{17b}$$

for *each* (because B is indecomposable) k. Thus, there are matrices with which we can «transform» the expended quantities of *any* commodity into prices.

from the equations (7) until (11) and (18), (18a), (19), for these «adjacent» techniques the following hold:

$$\exists k: m'_{kk}(\bar{v}) = 1 \Leftrightarrow \bar{v} = \bar{v}', \bar{p} = \bar{p}' \tag{20}$$

$$\exists k: 0 < m'_{kk}(\bar{v}) < 1 \not\Leftrightarrow \bar{v} = \bar{v}' \tag{20a}$$

$$m'_{ss}(\bar{v}) \gtrless 1 \Leftrightarrow \bar{v} \gtrless \bar{v}' \tag{20b}$$

where $\bar{p}'$ the left eigenvector of B' associated with its Perron-Frobenius eigenvalue λ' and $\bar{v}' = (1/\lambda')$ indicates the associated with technique B' rate of profit. Moreover, from (17), (17a) it follows that (the meaning of the symbols $q'^{*}, B'_k, \bar{p}_k, b'_k, \bar{p}_s, b'_s, B'_s$ is evident):

$$\bar{p} \geq \bar{p}B'\bar{v} \Rightarrow \bar{p}q'^{*} > \bar{p}B'q'^{*}\bar{v} \Rightarrow \bar{v}' > \bar{v} \tag{20c}$$

$$\bar{v}' > \bar{v} \Rightarrow \bar{p}Bq'^{*} > \bar{p}B'q'^{*} \Rightarrow \bar{p}(B - B')q'^{*} > 0 \Rightarrow \bar{p} \geq \bar{p}B'\bar{v} \tag{20d}$$

$$\begin{aligned} \bar{p} \geq \bar{p}B'\bar{v} &\Rightarrow \bar{p}[I - (B'_k\bar{v})] \geq \bar{p}_k(b'_k\bar{v}), \ \forall k \\ &\Rightarrow m_{ij}(\bar{v}) \geq m'_{ij}(\bar{v}), \ \forall i,j \end{aligned} \tag{20e}$$

$$\begin{aligned} m_{sj}(\bar{v}) \geq m'_{sj}(\bar{v}) &\Rightarrow (\bar{p}/\bar{p}_s) \geq (b'_s\bar{v})[I - (B'_s\bar{v})]^{-1} \\ &\Rightarrow \bar{p} \geq \bar{p}B'\bar{v} \end{aligned} \tag{20f}$$

where the equality holds in (20e) for $i = s$ *and*[11,12] $j \neq s$.

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11. Clearly, the preceding analysis can be *analogously* applied when the composition of the real wage rate is unknown, because, in this case, the Dmitriev's reduction leads to the following:

$$p = [p_k(a_k v) + w\ell v][I - (A_k v)]^{-1} \Rightarrow$$
$$p = \{(p_k)[m^A_{k1}(v), m^A_{k2}(v), ..., m^A_{kn}(v)]\} + \{(w\ell v)[I - (A_k v)]^{-1}\}, \ \forall k$$

where $m^A_{ij}(v)$ the abovementioned quantities computed *on the basis of the matrix* (Av) (the meaning of the symbols a_k, A_k is evident) and:

$$0 < m^A_{ii}(v) < 1, \text{ for } 1 \leq v < (1/\lambda^A)$$
$$m^A_{ii}(v) = 1, \text{ for } v = (1/\lambda^A)$$

where λ^A the Perron-Frobenius eigenvalue of A.

12. As is well known, (20c) constitutes the «cost-minimization condition». See, e.g. Bidard (1988), Kurz and Salvadori (1995), pp. 127-35.

If however, the matrices B, B′, differ *with respect to more than one column*, then the condition:

$$\exists k : m'_{kk}(\bar{v}) = 1 \tag{21}$$

is not sufficient for the equality of the techniques with respect to their profitability. It ensures the equality between $\bar{v}$ and $\bar{v}'$, *but not* the one between $\bar{p}$, $\bar{p}'$ (see (11)). In the case of not «adjacent» techniques, as may directly be deduced from (11) (see also (12), (13), (14)), (18), (18a), the following holds:

$$\exists k : m'_{kj}(\bar{v}) = m_{kj}(\bar{v}), \forall j \Leftrightarrow v = \bar{v}', p = \bar{p}' \tag{22}$$

Lastly, as the reader may readily ascertain, in this case analogous conditions to (20c), (20e), (20f) exist and in both cases (i.e. also for not «adjacent» techniques) there is a condition, which ensures the equality with respect to the profitability (with $\dot{z}$ we symbolise the differential of a magnitude z):

$$\bar{p} = \bar{p}B\bar{v} \Rightarrow \bar{p}\dot{B} = 0 \Rightarrow m_k(\bar{v})\dot{B} = 0 \tag{23}$$

Obviously the equation (20) or (22) (and in any case (23)) determines the set of the equally profitable techniques. *However*, the said determination presupposes the calculation of the coefficients $m_{kj}(\bar{v})$ and, consequently, as given the value of the rate of profit. In other words, it is not possible to produce a general rule for ordering the set of available techniques based *directly* on the «physical» data of production (e.g. based on constructed by means of the coefficients $m_{ij}(1)$ indices of the «quantities of capital» or on «quantities of labor embodied»). As is well known, this point forms a «crucial test» (Sraffa (1962), p. 478) for the traditional neoclassical theory and the labor theory of value, in which they fail[13].

Example[14] *4*:

$$A = \begin{bmatrix} 0 & 0.5 \\ 379/423 & 0.1 \end{bmatrix}, \ \ell = [8.9, 0.18], \ d = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

13. In other words, these theories always hold for v = 1. At this value of the rate of profit, the ordering of techniques can be unambiguously based on the coefficients m_{ij} or (equivalently) on the vector ω.

14. Based on Garegnani (1966), pp. 566-7.

For $\bar{c} = 390.572 \times 10^{-4}$ we obtain:

$$B = \begin{bmatrix} 8.9\bar{c} & 0.5 + 0.18\bar{c} \\ 379/423 & 0.1 \end{bmatrix} \Rightarrow M \cong \begin{bmatrix} 0.852 & 0.563 \\ 1.373 & 0.797 \end{bmatrix}, \ \bar{v} = 1.1, \ m_{12}(\bar{v}) \cong 0.627$$

If only the first or only the second column of B changes, then the set of equally profitable techniques is respectively determined by the following equations:

$$\dot{b}_{11} + m_{12}(\bar{v})\dot{b}_{21} = 0 \tag{24}$$

$$\dot{b}_{12} + m_{12}(\bar{v})\dot{b}_{22} = 0 \tag{24a}$$

which must hold simultaneously if we want to construct «non-adjacent» equally profitable techniques. Thus for (for example):

$$B' = \begin{bmatrix} b_{11} - m_{12}(\bar{v})(44/423) & b_{12} - m_{12}(\bar{v})0.4 \\ 1 & 0.5 \end{bmatrix} \Rightarrow v = \bar{v}',$$

we obtain $m_{11} > m'_{11}$, $m_{12} > m'_{12}$, $m_{21} < m'_{21}$, $m_{22} < m'_{22}$ (e.g. the total quantity of capital in terms of commodity 1 (2) is more (less) in B than in B').

Moreover, though it is not possible for the quantity $m_{12}(v)$ (e.g. the relative commodity price) to be repeated over the range of the income distribution variables[15], it is possible that a subset of the equally profitable techniques set to be repeated (*reswitching phenomenon*): B and the techniques that verify (24), (24a) and the following equations (one of them, for «adjacent» techniques and both, for «non-adjacent»):

$$(\dot{a}_{21}/\dot{\ell}_1) = (\dot{a}_{22}/\dot{\ell}_2) = [(\bar{c} - \bar{\bar{c}})/(m_{12}(\bar{\bar{v}}) - m_{12}(\bar{v}))] \tag{25}$$

are equally profitable, for $\bar{c}$ *and* $\bar{\bar{c}}\,(\Rightarrow v = \bar{\bar{v}})$.

Example: $\bar{\bar{c}} = 243.902 \times 10^{-4} (\Rightarrow \bar{\bar{v}} = 1.2)$, the first production method does not change and for the second it holds:

$$\dot{a}_{12} = -1/4, \ \dot{a}_{22} = 38/120, \ \dot{\ell}_2 = 132/100$$

15. For the proof: Mainwaring (1976), pp. 109-13.

4. The efficiency of the techniques

Let f_{ik} be the components of the vector of stocks necessary to support the production of a gross output consisting of f_{ik} units of commodity i, $i \neq k$, and one unit of commodity k. The quantities f_{ij} are determined by the system:

$$F \equiv B\tilde{F} \qquad (26)$$

where $F \equiv [f_{ij}]$ and $\tilde{F}$ the nxn matrix, which derives from the nxn matrix F, when we replace all the elements of its principal diagonal with unit. Consequently, for the quantities f_{ij} *analogously* holds the same that holds for the quantities m_{ij} (compare (26) with (4)) and primarily the following:

$$\lambda \neq 1 \ \Rightarrow \ [f_{ij}/(1-f_{jj})] \equiv g_{ij}' = h_{ij}' \ \Rightarrow \ f_{ij} \equiv (\sigma_{ii}/\sigma_{jj})\, m_{ij} \qquad (27)$$

$$\lambda \leq 1 \ \Leftrightarrow \ \{f_{ij} > 0, f_{ii} \leq \lambda, \forall\, i,j\} \qquad (28)$$

$$\exists\, i : f_{ii} = 1 \ \Rightarrow \ f_{ii} = \lambda = 1, \forall\, i \ \Rightarrow \ f_{ij} = q_i^*/q_j^*, \forall\, i,j \qquad (29)$$

where $G' \equiv [g_{ij}'] \equiv [I-B]^{-1}B = H'$ (because of the single production[16]) and q_i^*, q_j^* the components i, j (respectively) of q^*.

Assume that $\lambda \leq 1$ and consider the well-known growth system associated with the technique B:

$$x = \hat{B}xu, \ \hat{B} \equiv B + \hat{c}d\boldsymbol{\ell}, \ u \equiv 1 + g = (1/\hat{\lambda}) \geq 1 \qquad (30)$$

where x the n-dimensional vector of activity levels per unit of labor employed, $\hat{c}$ the level of capitalists' consumption per unit of labor employed (of which the composition is by assumption uniform for workers and capitalists), g the by assumption uniform rate of growth and $\hat{\lambda}$ the Perron-Frobenius eigenvalue of $\hat{B}(\hat{c} \geq 0 \Rightarrow \hat{\lambda} \geq \lambda)$. Obviously, from (26) until (30), it follows that:

$$x = [(\hat{B}_k^T u)x] + [(\hat{b}_k^T u)x_k], \forall\, k \ \Rightarrow$$

$$(x_i/x_j) = (\hat{q}_i^*/\hat{q}_j^*) = \hat{f}_{ij}(u), \ \forall\, i,j \qquad (31)$$

$$1 = \hat{f}_{ii}(u), \ \forall\, i \qquad (31a)$$

16. See Pasinetti (1973), pp. 17-24.

where $\hat{B}_k^T$ the nxn matrix, which derives from $\hat{B}$, when we replace all the elements of its k-th column with zero, $\hat{b}_k^T$ the k-th column of $\hat{B}$, $\hat{q}^*$ the right eigenvector of $\hat{B}$ associated with $\hat{\lambda}$ and $\hat{f}_{ij}(u)$ the abovementioned quantities f_{ij} computed *on the basis of the matrix* $(\hat{B}u)$.

For a given $\hat{B}$, how can we construct the set of equally efficient matrices? Contrary to the appeared «duality» between (18), (18a) on the one hand and (31), (31a) on the other, the said construction must be exclusively based on the quantities $\hat{m}_{ij}(\bar{u})$ (the meaning of which is evident) and not on the quantities $\hat{f}_{ij}(\bar{u})$. From (18), (18a), (30) it follows that[17]:

$$\hat{B}\bar{x}\dot{u} = [I-(\hat{B}\bar{u})]\dot{x}-\dot{\hat{B}}\bar{x}\bar{u} \Rightarrow$$

$$\hat{m}_k(\bar{u})\hat{B}\bar{x}\dot{u} = -\hat{m}_k(\bar{u})\dot{\hat{B}}\bar{x}\bar{u} \Rightarrow$$

$$\dot{u} = 0 \Leftrightarrow \hat{m}_k(\bar{u})\dot{\hat{B}}\bar{x} = 0 \tag{32}$$

Clearly u does not change if:

$$\hat{m}_k(\bar{u})\dot{\hat{B}} = 0 \tag{32a}$$

or if the vector $\hat{m}_k(\bar{u})\dot{\hat{B}}$ has components with different signs (necessary condition). In the second case, however, if we replace its positive (negative) components with zero, we obtain a more (less) efficient technique. Consequently, the said set is determined by (32a), which is always equivalent with (23) if $\bar{v}=\bar{u}$ (growth path of Charasoff[18] - Neumann), and its determination presupposes the calculation of the quantities $\hat{m}_{kj}(\bar{u})$.

17. As it is easily proven, equally efficient techniques do not necessarily have the same vector of activity levels.
18. For Charasoff's contribution: Kurz and Salvadori (1995), pp. 387-90, Stamatis (1999).

5. Conclusion

Dmitriev's profitability analysis and the notion of vertical integration compose a coherent framework for:

a) The interpretation of the conditions which ensure the existence of a positive uniform rate of profit and growth in techniques à la Leontief-Sraffa.
b) The investigation of the ordering of the available techniques with respect to their profitability and efficiency.
c) The exposition of the Sraffian critique of the traditional neoclassical theory and the labor theory of value.

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