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On the Significance of S. Vassilakis's Article *«Non-basics, the Standard Commodity, and the Uniformity of the Profit Rate»*

by
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It is known that in Sraffa's model of single production, when the maximum rate of profit of the non-basic subsystem is smaller than that of the basic subsystem, negative as well as tending to infinity commodity prices appear (Sraffa 1960, Appendix B). This is neither an exception to the rule nor a case without practical significance, as Sraffa considers it to be (Sraffa 1962, pp. 425-426), but rather a very common case. For it appears, when (a) non-basic commodities enter the production of non-basic commodities in such proportions that (b) the maximum rate of profit of the non-basic subsystem is smaller than that of the basic one.

A non-basic commodity, like any other commodity, can enter its own production as well as the production of other non-basic commodities even in such proportions that the maximum rate of profit of the non-basic subsystem is smaller than that of the basic subsystem. For such a case to occur, it suffices that one non-basic commodity enters its own production in such a proportion that the maximum rate of profit of this non-basic commodity, and hence the maximum rate of profit of the non-basic subsystem as a whole, is smaller than the maximum rate of profit of the basic subsystem.

In his article, Vassilakis shows that in this case negative commodity prices can be ruled out. This can be done by limiting the uniform rate of profit of the actual system as a whole to the maximum rate of profit of the non-basic subsystem. Of course this is trivial. For, as known, negative commodity prices appear in this case, only for values of the uniform rate of profit of the actual production system as a whole greater than the maximum rate of profit of the non-basic subsystem; thus if one limits the uniform rate of profit of the actual production system in this way, then naturally negative commodity prices do not appear. Not at all trivial is however the way in which the above constraint of the uniform rate of profit of the actual production system as a whole can be introduced; for when the uniform rate of profit of the actual production system is equal to the maximum rate of profit of the non-basic subsystem, then the nominal wage rate must be equal to zero, because, due to the above constraint of the uniform rate of profit of the actual production system, the maximum rate of profit of the non-basic subsystem is also the maximum rate of profit of the actual production system. Vassilakis shows that this constraint is introduced by measuring prices, using as a measure a standard commodity, consisting of all the basic and certain non-basic commodities in positive quantities. In showing that such a standard commodity exists, Vassilakis shows at the same time that –contrary to Sraffa's claim, according to which the standard commodity consists only of all the basic commodities in positive quantities (Sraffa 1960, 36ff)– a standard commodity consisting of positive quantities of commodities only, consists also of non-basic commodities, exist when the maximum rate of profit of the non-basic subsystem is smaller than that of the basic subsystem. Vassilakis offers a mathematical example. His example describes a production system with two commodities, in which, for the production of one unit of commodity 1 0.5 units of commodity 1 are required, and for the production of one unit of commodity 2 0.25 units of commodity 1 and 0.75 units of commodity 2 are required. Obviously, commodity 1 is a basic commodity, while commodity 2 is a non-basic commodity, which enters its own production.

As an addition to his example, we assume that for the production of one unit of commodity 1 as well as of commodity 2, 0.5 units of labour are required. One can easily find out that the maximum rate of profit of the basic subsystem (R_b) is equal to (1–0.5):0.5 (=1) and the maximum rate of profit of the non-basic subsystem is equal to (1–0.75):0.75 (=1/3). The Sraffian standard system uses 1 unit of labour and 1 unit of commodity 1 as means of production and produces a gross standard product equal to 2 units and a net standard product equal to 1 unit of commodity 1. This standard system has a maximum rate of profit equal to (2–1):1 (=1). Its maximum rate of profit is therefore equal to the maximum rate of profit R_b of the basic subsystem.

If we normalize prices according to Sraffa by setting the price of the net standard product equal to one,

$$p_1 = 1, \qquad (1)$$

then from equation (1) and the following equations of price determination

$$p_1 = 0.5p_1(1 + r) + 0.5w \qquad (2)$$

$$p_2 = (0.25p_1 + 0.75p_2)(1 + r) + 0.5w \tag{3}$$

we get the following relation between the nominal wage rate and the uniform profit rate of the actual system:

$$w = 1 - \frac{1}{R_b} r = 1 - r. \tag{4}$$

The standard system of Vassilakis uses one unit of labour, 0.75 units of commodity 1 and 0.75 units of commodity 2 and produces a gross product consisting of one unit of each commodity, and hence a net product consisting of 1–0.75 (=0.25) units of each commodity. This standard system has a maximum profit rate equal to (1–0.75):0.75 (=1/3). Its maximum rate of profit is therefore equal to the maximum rate of profit of the non-basic subsystem R_n.

If we normalize prices by setting the price of the net standard product of Vassilakis equal to one,

$$0.25p_1 + 0.25p_2 = 1\,, \tag{5}$$

then, from equations (2), (3) and (5), we obtain for the w-r-relation:

$$w = 1 - \frac{1}{R_n} r = 1 - 3r. \tag{6}$$

According to the sraffian normalization, i.e. using equation (1), the price of the non-basic commodity 2, p_2, is for w, $2/3 < w \leq 1$, and hence, as can be seen from equation (4), for r, $0 \leq r < R_n$ (=1/3), positive. For $w = 2/3$ and hence for $r = 1/3$, p_2 tends to infinity, and for w, $0 \leq w < 2/3$, and hence for r, $1/3 < r \leq R_b$ (=1), it is negative.

That in the last case p_2 is negative, is a consequence on the one hand of the postulate that a uniform rate of profit should exist, and on the other hand of the fact that in this case there cannot exist a uniform profit rate of the actual system as a whole, for values of r greater than 1/3, i.e. greater than the maximum profit rate of the non-basic subsystem.

Price p_2 being negative in the aforementioned interval, means therefore that: only if the price of commodity 2 could be negative, in other words if it were possible for the buyer of a commodity not to pay the seller, but rather to be paid by him, only then could there be a uniform profit rate of the actual system as a whole for values of r greater than 1/3 and smaller than 1 – which, because naturally in reality there is no such thing as a negative commodity price, means that, for nominal wage rates smaller than 2/3, a uniform profit rate for the actual system as a whole does not exist.

According to Vassilakis's normalization, i.e. using equation (5), the commodity prices are, for every possible value of the nominal wage rate, that is for w, $0 \leq w \leq 1$, non-negative. Non-negative, and not positive, because for $w = 0$ and hence for $r = 1/3$ we have $p_1 = 0$.

Vassilakis presumes from his proof that, when the maximum profit rate of the non-basic subsystem is smaller than that of the basic subsystem, both the uniform rate of profit and the prices of the basic commodities –contrary to the generally accepted claim of Sraffa, according to which these magnitudes depend solely on the production conditions of the basic commodities– also depend on the production conditions of the non-basic commodities, a result which is formally correct.

Vassilakis's proof implies several very important results, which he himself does not derive. We just mention them in brief, starting with an evaluation of his price normalization.

This normalization is a sufficient, but not a necessary condition for the non-negativity of prices in the case where the maximum profit rate of the non-basic subsystem is smaller than that of the basic subsystem. The necessary and sufficient condition lies in the measurement of prices, using as a measure a single or a composite commodity, which contains at least one commodity, produced in the irreducible subsystem with the smallest maximum profit rate.

In the numerical example given by Vassilakis, one can therefore ensure the non-negativity of commodity prices by normalizing prices with

$$ap_2 = \alpha, \qquad \alpha, a\text{: positive constants,} \qquad (7)$$

or with

$$bp_1 + cp_2 = \alpha, \qquad \alpha, b, c\text{: positive constants}^1 \qquad (8)$$

By comparing in our numerical example the w-r-relation resulting from Sraffa's normalization with the one resulting from Vassilakis' normalization, we observe that the w-r-curve of a given actual system –its shape, but also the points where it cuts the two axes– changes with the price normalization. We also observe that in the sraffian normalization the prices of basic commodities and the uniform rate of profit depend exclusively on the production conditions of the basic commodities and not on these of the non-basic commodities as

1. See Stamatis 1988, pp. 71-84.

well, while in Vassilakis's normalization they depend on the production conditions of the basic commodities but also on those of the non-basic ones.

What is the explanation for this? The w-r-relation, and hence the uniform rate of profit, are not in fact the w-r-relation and the uniform rate of profit of the given actual system but rather of the, in each case given, normalization subsystem. Normalization subsystem we call the subsystem which produces as its net product the single or composite commodity that functions as a measure of prices, that is the normalization commodity. In the sraffian normalization, for example, this subsystem contains only sector 1, here the basic subsystem, while in Vassilakis's normalization, or the normalization using equation (7) or (8), it contains sectors 1 and 2, i.e. it contains here the non-basic subsystem as well.

The w-r-relation and the uniform profit rate of the normalization subsystem then hold also as the w-r-relation and the uniform profit rate of the given actual system, for the following reasons: The normalization subsystem uses for the production of the commodities which it produces the same technique used by the actual system; due to the postulate that a rate of profit, equal for all the sectors of the actual system, exists, the uniform profit rate of the normalization subsystem holds –for a nominal wage rate given and the same for the two systems (the normalization subsystem and the actual system)– also for those sectors of the actual system, which are not represented in the normalization subsystem, and therefore holds for the actual system as a whole[2].

The uniform rate of profit is thus primarily the uniform rate of profit of the normalization subsystem and then holds also, for the aforementioned reasons, for the actual system. *This is exactly the reason why it only depends on the production conditions of the commodities contained in the gross product of the normalization subsystem, i.e. produced by the normalization subsystem:* When prices are normalized according to Sraffa or, in general, by using a single or composite *basic* commodity as a normalization commodity, the uniform rate of profit depends *only* on the production conditions of the basic commodities, while when prices are normalized according to Vassilakis, or by using equation (7) or (8), it *also* depends on the production conditions of the non-basic commodities. Consequently, the prices of basic commodities depend, when prices are normalized according to Vassilakis, or with equation (7) or (8), on

2. The fact that the activity levels of the processes of the normalization subsystem are different from those in the actual system is not important, because the production technique is linear.

the production conditions of the non-basic commodities as well, while when they are normalized according to Sraffa, they depend only on the production conditions of the basic commodities.

What has been said of the uniform rate of profit holds also for the relative prices: they also are primarily the prices of the normalization subsystem and then hold also, due to the postulate that there exists only one price for each commodity, for the given actual system. Thus they may vary with the price normalization, if the composition, and hence the production conditions, of the normalization commodity also varies with the price normalization.

Since therefore the w-r-relation is in fact the w-r-relation of the normalization subsystem, the capital intensity, in terms of prices, of the actual system has nothing to do with the slope of the w-r-relation. This slope also depends, apart from the production technique and the nominal wage rate (or the uniform profit rate), on the composition of the product of the normalization subsystem and thus on the capital intensity, in terms of prices, of the normalization subsystem, and not on the composition of the product of the given actual system and thus not on the capital intensity, in terms of prices, of the given actual system[3]. The latter becomes obvious by the fact that, when, given the technique and the nominal wage rate (or the profit rate), the composition of production and hence the capital intensity, in terms of prices, of the actual system vary, the slope of the w-r-curve remains unchanged[4].

Since, as we have said already, it is possible for the ranking of the nominal wage rates of given techniques, given the common rate of profit, to vary with the price normalization, the unambiguous ranking of the given techniques is in general impossible (even when the change of price normalization does not alter the maximum profit rates of these techniques). Because a technique A, which,

3. The slope of the w-r-curve is always equal to

$$-\frac{d\pi_n}{dr}=-\frac{dk_n}{dr}r-k_n,$$

where π_n the profit per unit of labour and k_n the capital intensity (in terms of prices) *in the normalization subsystem* (Stamatis 1988, pp. 22f and 28-31 and Vouyiouklakis and Mariolis 1992, pp. 144-74).

4. With the exception, of course, of the case of equal capital intensity among sectors. In this case, the capital intensity, in price terms, of the normalization subsystem is always equal to that of the actual system – and, in absolute value, to the slope of the w-r-curve. Also, in this case, the capital intensity, in price terms, of the actual system remains unchanged when the output composition of the system changes (Stamatis 1988, pp. 53ff).

for a given price normalization and a given rate of profit, is superior to another technique B, may become, for another price normalization and the same rate of profit, inferior to technique B, is the abovementioned ranking of given techniques in the general case impossible (Stamatis 1984, Ch. IV, Stamatis 1988, pp. 100ff, Stamatis 1993, 1998 and 1998a).

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