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Non-basics, the Standard Commodity, and the Uniformity of the Profit Rate

by
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In Sraffa's «Production of Commodities...» we find the proposition that non-basic goods do not enter the standard commodity in a system of single-product industries. In Sraffa's own words: «We may in consequence simplify the discussion by assuming that all non-basic equations are eliminated at the outset so that only basic industries come under consideration» (Sraffa 1960, §35).

This view is, as far as I know, unanimously shared by all those who have written on Sraffa's work and related material. See for example, G. Abraham Frois and E. Berrebi (1976), p. 73, A. Roncaglia (1978), p. 74, L. Pasinetti (1977), p. 109, P. Newman (1962), p. 67, and T. Miyao (1977), p. 152.

More specifically, the first two works accept Sraffa's argument as it stands, Pasinetti makes the required assumption for Sraffa's argument to be correct without justifying it theoretically, and Newman and Miyao simply rule out non-basic goods, assuming that the technical coefficient matrix A is inde-composable. We will return later to these authors. Moreover, in appendix B of Sraffa's «Production of Commodities...», it is held that it may be impossible to equalize profit rates throughout the economy. This is also adopted by all commentators of Sraffa's work. This paper intends to show that the standard commodity may contain non-basics and that the equalization of the profit rate in all sectors is always possible.

We consider an economy producing n goods. Goods 1, ..., m are basics and m+1, ..., n are non-basics. We denote by a_{ij} the quantity of good i needed to produce one unit of good j, i, j = 1, ..., n.

In matrix notation

$$A_{11} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ . & . & \cdots & . \\ . & . & \cdots & . \\ . & . & \cdots & . \\ a_{m1} & a_{m2} & \cdots & a_{mm} \end{bmatrix}$$

$$A_{12} = \begin{bmatrix} a_{1,m+1} & a_{1,m+2} & \cdots & a_{1n} \\ a_{2,m+1} & a_{2,m+2} & \cdots & a_{2n} \\ . & . & \cdots & . \\ . & . & \cdots & . \\ . & . & \cdots & . \\ a_{m,m+1} & a_{m,m+2} & \cdots & a_{mn} \end{bmatrix}$$

$A_{12} = 0$, where A_{12} is a $m\text{X}(n-m)$ matrix,

$$A_{22} = \begin{bmatrix} a_{m+1,m+1} & a_{m+1,m+2} & \cdots & a_{m+1,n} \\ a_{m+2,m+1} & a_{m+2,m+2} & \cdots & a_{m+2,n} \\ . & . & \cdots & . \\ . & . & \cdots & . \\ . & . & \cdots & . \\ a_{n,m+1} & a_{n,m+2} & \cdots & a_{nn} \end{bmatrix}$$

$$A = \begin{bmatrix} A_{11} & A_{12} \\ 0 & A_{22} \end{bmatrix}.$$

We assume that the largest positive eigenvalue $\lambda(A)$ of A is less than unit.

Moreover, we assume that the matrices A_{11} , A_{22} are semi-positive and indecomposable. We begin by calculating the proportions at which goods enter the standard commodity.

Let q,

$$q=\begin{bmatrix} q_1 \\ q_2 \end{bmatrix},$$

be the vector denoting these proportions. Then

$$(1+R)Aq=q,\ q\geq 0$$

$$(1+R)\begin{bmatrix} A_{11} & A_{12} \\ 0 & A_{22} \end{bmatrix}\begin{bmatrix} q_1 \\ q_2 \end{bmatrix}=\begin{bmatrix} q_1 \\ q_2 \end{bmatrix} \text{ i.e}$$

$$A_{11}q_1+A_{12}q_2=\frac{1}{1+R}q_1,\ q_1\geq 0,\ \text{and} \tag{1}$$

$$A_{22}q_2=\frac{1}{1+R}q_2,\ q_2\geqq 0. \tag{1'}$$

We want to find out, if there exists a solution $q_1 \geq 0,\ q_2 \geqq 0$. The solution $q_1 > 0$, $q_2 = 0$ is possible. Indeed, if $q_1 > 0$, $q_2 = 0$, (1) and (1′) reduce to

$$A_{11}q_1=\frac{1}{1+R}q_1,\ q_1>0,\ \text{i.e.}$$

$$R=\frac{1}{\lambda(A_{11})}-1,\ \lambda(A_{11})\in(0,1),$$

where $\lambda(A_{11})$ is the largest positive eigenvalue of A_{11}.
Now if $q_1 \geq 0,\ q_2 \geq 0$, (1) and (1′) yield:

$$R=\frac{1}{\lambda(A_{22})}-1,\ q_2\geq 0. \tag{2}$$

As A_{22} is semi-positive and indecomposable, $\lambda(A_{22})>0$.
Thus

$$R=\frac{1}{\lambda(A_{22})}-1,\ \lambda(A_{22})\in(0,1),\ q_2>0. \tag{2'}$$

By (1) and (2′)

$$A_{11}q_1+A_{12}q_2=\lambda(A_{22})q_1 \Leftrightarrow$$

$$(\lambda(A_{22})I_1 - A_{11})q_1 = A_{12}q_2 \Leftrightarrow$$

$$q_1 = (\lambda(A_{22})I_1 - A_{11})^{-1}A_{12}q_2. \tag{3}$$

The condition $\lambda(A_{22}) > \lambda(A_{11})$ is sufficient for the existence of the inverse of $(\lambda(A_{22})I - A_{11})$ and necessary and sufficient for the positivity of this inverse. Thus, by (3)

$$q_1 = (\lambda(A_{22})I_1 - A_{11})^{-1}A_{12}q_2 > 0 \tag{3'}$$

if, and only if, $\lambda(A_{22}) > \lambda(A_{11})$.

As $\lambda(A_{22}) > 0$, $\lambda(A_{11}) \in (0, \lambda(A_{22}))$ is an entirely imaginable case. Hence, by (2′) and (3′), the Standard commodity may contain *all* the non-basic commodities in positive proportions, as well as the basic ones in positive proportions too, provided that $\lambda(A_{22}) > \lambda(A_{11})$. The following numerical example will clarify this argument.

Let $A = \begin{bmatrix} 2/4 & 1/4 \\ 0 & 3/4 \end{bmatrix}$, then

$$(1+R)Aq = q \Leftrightarrow \begin{bmatrix} 1/2 & 1/4 \\ 0 & 3/4 \end{bmatrix}\begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = \frac{1}{1+R}\begin{bmatrix} q_1 \\ q_2 \end{bmatrix} \text{ i.e.}$$

$$\frac{1}{2}q_1 + \frac{1}{4}q_2 = \frac{1}{1+R}q_1,\ q_1 > 0,$$

$$\frac{3}{4}q_2 = \frac{1}{1+R}q_2,\ q_2 > 0.$$

These equations imply that

$$\left.\begin{array}{l} R = \frac{1}{3},\ q_2 > 0 \\ \frac{1}{2}q_1 + \frac{1}{4}q_2 = \frac{3}{4}q_1,\ q_1 > 0 \end{array}\right\} \Leftrightarrow$$

$$\left.\begin{array}{l} R = \frac{1}{3},\ q_2 > 0 \\ q_2 = q_1 > 0 \end{array}\right\} \Leftrightarrow \left.\begin{array}{l} q_1 = q_2 > 0 \\ R = \frac{1}{3} \end{array}\right\} \Leftrightarrow \left\{\begin{array}{l} q = a\begin{vmatrix} 1 \\ 1 \end{vmatrix},\ a > 0 \\ R = \frac{1}{3}. \end{array}\right.$$

We are now able to discuss the closely related problem of the uniformity of the profit rate throughout the economy. Let p, $p = (p_1, p_2)$, be the price vector and L, $L = (L_1, L_2) > 0$ the labour input vector, w the nominal wage rate and r_1, r_2 the profit rates, then, the price equations are:

$$p_1 = (1 + r_1) p_1 A_{11} + wL_1,$$

$$p_2 = (1 + r_2)(p_1 A_{12} + p_2 A_{22}) + wL_2.$$

Let $\hat{p} = \frac{1}{w} p$, $w > 0$, then

$$\hat{p}_1 = (1 + r_1) \hat{p}_1 A_{11} + L_1 \tag{4}$$

$$\hat{p}_2 = (1 + r_2)(\hat{p}_1 A_{12} + \hat{p}_2 A_{22}) + L_2. \tag{4'}$$

By (4) and (4′)

$$\hat{p}_1 = L_1 (I_1 - (1 + r_1) A_{11})^{-1} > 0, \text{ if (and only if) } r_1 < \frac{1}{\lambda(A_{11})} - 1, \tag{5}$$

$$\hat{p}_2 = ((1 + r_2)\hat{p}_1 A_{12} + L_2)(I_2 - (1 + r_2) A_{22})^{-1} > 0, \text{ if (and only if)}$$

$$r_2 < \frac{1}{\lambda(A_{22})} - 1 \tag{5'}$$

Thus $r = r_1 = r_2$ and $p = (1+r)pA + wL > 0$ only if

$$r \in [0, \min(\frac{1}{\lambda(A_{11})} - 1, \frac{1}{\lambda(A_{22})} - 1)). \tag{6}$$

Thus by restricting the profit rate in the appropriate interval, a uniform profit rate and strictly positive prices are always possible.

At this point, we can ask ourselves which is the root of the misunderstanding.

It is clear from the discussion above that both problems appear when

(i) non-basic commodities are used to produce non-basic commodities,

(ii) $\lambda(A_{22}) > \lambda(A_{11})$, i.e., the physical own rate of reproduction of basics exceeds that of non-basics. All writers accept that (i) is generally a valid assumption. So, we can discuss case (ii).

Pasinetti (1977, p. 109) takes it for granted that «our knowledge of the economic System permits us to accept the assumption $\lambda(A_{22}) < \lambda(A_{11})$».

This knowledge, however, is not incorporated in Sraffa's model, reproduced by Pasinetti. To take for granted something which the model ought to explain is not, I think, methodologically correct. The same methodological error is commited by Sraffa himself in his correspondence with Newman, which is reproduced in Bharadwaj (1970). He asserts that «in a real system, however, there is not one but a large number of basic products, and the ratio R resulting from the system which they form is *practically* certain to be much smaller than the own ratio of anyone separate non-basic (or any of such small groups of interconnected non-basics as may exist)». Note that R is, in this paper's terminology, $\frac{1}{\lambda(A_{11})} - 1$, so Sraffa and Pasinetti share the same opinion on this matter.

Why have all writers rejected case (ii) as unrealistic? Because it destroys the predominance of the basic sector over the non-basic one. To have a uniform profit rate, when (ii) is true, we must lower it until it becomes smaller than $\frac{1}{\lambda(A_{22})} - 1$.

Hence, it is the non-basic sector which determines the range of the profit rate, and, in consequence, the range of all other magnitudes. Given the purpose of these writers to «rehabilitate» classical economists, who are supposed to have given the priority to some kind of «basic sectors», it is easy to understand why the case (ii) has been named «unrealistic», and set aside. In the context of Sraffa's model, however, as well as in the context of linear systems in general, such a case is to be treated in exactly the same way as the «normal» one.

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