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# Internal inconsistencies of the physical quantities approach

*by*
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Marx's value theory, based on the determination of value by labor-time, has been widely held to be riddled with errors and internal inconsistencies, and thus in need of correction. Partly for this reason, many theorists have instead embraced what has been regarded as a rigorous alternative, "the physical quantities approach" (Steedman, 1977, p. 72, pp. 216-217), according to which physical conditions of production and real wages are the primary determinants of prices and rates of profit and accumulation. Others, including most Marxist economists, have opted for "labor theories of value" in which inputs and outputs are valued simultaneously, and which therefore yield quantitative results differing little, if at all, from other variants of the physical quantities approach.

Yet during the past two decades, all allegations of internal inconsistency in the quantitative dimension of Marx's theory have been refuted by what has come to be known as the *sequential, nondualist* interpretation (Freeman and Carchedi (eds.), 1996) or *the temporal single-system* (TSS) interpretation (Kliman and McGlone, 1999). This interpretation, for instance, vindicates the internal coherence of Marx's account of the transformation of values into prices of production, and of his law of the tendential fall in the profit rate – i.e., the claim that the profit rate can fall due to labor-saving technical change itself. Several other of Marx's theoretical conclusions that had been dismissed as erroneous, but which can be replicated on the basis of the TSS interpretation, are examined in Kliman and McGlone (1999).

Thus, although Marx's physicalist critics, both Marxist and non-Marxist, have prided themselves on the rigorous nature of their refutations of his ideas, it turns out that it is their own interpretations that have been in error, not the texts themselves. Given that their work in this area has been lacking in rigor, it

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is perhaps worthwhile to investigate whether the foundations of their own approach are as rigorous as they have claimed. Some valuable work along these lines has been done by several authors (e.g., Stamatis, 1983, and the papers by Albarracvn, Farjoun, Freeman, Giussani, and Savran in Mandel and Freeman (eds.), 1984). Here I will extend this line of inquiry, employing some insights of TSS research into the dynamic determination of value.

The first aim of this paper is to show the self-contradictory nature of the dominant, physicalist versions of the "labor theory of value," which attempt to amalgamate the determination of value by labor-time and the physical quantities approach. That which is determined by use-values cannot be determined also by labor-time, and vice-versa. The contradiction is readily apparent once use-value and labor-time grow at different rates.

Second, the paper seeks to show that the physical quantities approach itself leads to arbitrary and indeterminate results. I first show that if use-values *-numéraires* or money commodities- are to substitute for value, then the level of the profit rate can depend on the specific good that serves as measure of value, even in the long run. This demonstration is followed by one that shows that a given change in relative prices can cause the average profit rate either to rise or fall, depending on the money commodity or *numéraire.* Just as use-values themselves are heterogeneous, so are the profit rates measured in terms of them.

Relative price changes also produce larger or smaller profit rate differentials under different money commodities or *numéraires.* Hence, as my final demonstration shows, when capital flows in response to profit rate differentials, use-value valuation implies that one and the same economy can either converge to equilibrium or explode.

## 1. *The Contradiction between Value and Use-Value*

Those who have tried to amalgamate the determination of value by labor-time with the physical quantities approach, by means of simultaneous valuation, have often asserted three important propositions which, taken together, are incompatible. Within the space of a few pages, for instance, Laibman (1997) asserts all of them. The three propositions are

1. Commodities' values fall as labor productivity rises (Laibman 1997:28-29). This proposition follows from the determination of value by labor-time.
2. In a one-commodity world, the value rate of profit and the physical "rate

of profit" (physical surplus divided by physical input), are identical (Laibman 1997:23). This proposition is crucial to the attempt to assimilate the determination of value by labor-time to the physical quantities approach.
3. The profit rate is capital's "potential rate of self-expansion" (Laibman 1997:23)[1]. "Potential" in this context means maximum: when all profit is reinvested, the growth rate of capital equals the profit rate. This follows from the identity that the growth rate, or rate of accumulation, of capital is the product of two ratios, the ratio of net investment to profit and the profit rate (the ratio of profit to capital); see Laibman (1997:63-64).

In keeping with these propositions, assume that labor productivity rises, only one commodity exists. Assume also that all profit is reinvested, so capital is growing at its maximum potential rate. Let K stand for capital, $\dot{K}$ for its maximum growth rate, r for the rate of profit, and let the subscripts v and p indicate value and physical measures, respectively.

Since labor productivity is rising, Proposition 1 implies that the commodity's unit value is falling, so $K_v$ is growing more slowly than $K_p$ (see Laibman (1997:28-29)). Hence,

$$\dot{K}_v < \dot{K}_p \ . \tag{1}$$

Proposition 2 implies that

$$r_v \equiv r_p \ . \tag{2}$$

Finally, Proposition 3 implies that

$$\dot{K}_v \equiv r_v \quad \text{and} \quad \dot{K}_p \equiv r_p \ . \tag{3}$$

Relations (2) and (3) together imply that $\dot{K}_v = \dot{K}_p$ , which contradicts (1). Hence, the propositions cannot all be true.

Any two of the propositions are compatible, but then the third must be rejected. As was just shown, for instance, if both (2) and (3) are true, then (1) is false. This means that, despite rising productivity, capital does not become

1. Laibman (1997:23) writes that "The rate of profit, then, is the central measure of the effectiveness of capitalist production from the point of view of capital: its potential rate of self-expansion." "The" rate of profit refers both to the value rate and the physical rate, since he asserts that the two are the same. Hence, it is clear that "potential rate of self-expansion" refers to the self-expansion of both capital-value and the physical capital stock.

cheaper, so value is not determined by labor-time. One may also consistently assert both (1) and (3), but then $r_v < r_p$, so (2) is false. Thus, it follows necessarily from the determination of value by labor-time that the value and physical rates of profit differ. Rising productivity *both* cheapens constant capital *and* makes the value rate of profit fall below the physical rate. The first is impossible without the second[2]. Finally, one may assert both (1) and (2), but then (3) is false. Yet (3) is true by definition. It is therefore impossible consistently to embrace both the physical quantities approach and the determination of value by labor-time.

By denying (3), moreover, one denies that the profit rate is the maximum rate of accumulation in value terms. One thereby severs any necessary link between the profit rate and the rate of accumulation. Given continually rising productivity and falling values, the maximum rate of accumulation remains permanently below the profit rate, perhaps increasingly so. Thus the profit rate, although it may have some influence, no longer governs the rate of accumulation in any fundamental sense.

What is true is that the *physical* profit rate fails to govern the rate of accumulation of capital-*value.* If value is determined by labor-time and productivity is rising, the maximum rate of growth of capital-value *must* be lower than the physical profit rate – and this must be the case *regardless of how the value rate of profit is defined.* Since $\dot{K}_v < \dot{K}_p$, and $\dot{K}_p \equiv r_p$, it follows that $\dot{K}_v < r_p$.

The violation of (3) therefore arises from the attempt to equate the value and physical profit rates. They may indeed be equated, by means of simultaneous valuation or its equivalent, the use of the post-production *replacement cost* of inputs to compute the value rate of profit. Yet since $\dot{K}_v < r_p$, if $r_v = r_p$, then $\dot{K}_v < r_v$. *The replacement cost profit rate is simply not the maximum rate of accumulation of capital-value.* This definition of the profit rate thus severs the link between profitability and accumulation.

2. This conclusion does not contradict the notion that the cheapening of means of production tends to counteract the fall in the rate of profit. Current productivity growth tends to lower the *current* profit rate, while the cheapening of means of production tends to enhance *future* profitability. Continued productivity growth, however, will counteract this latter tendency.

Those who have thought carefully about these and related issues have indeed recognized that replacement cost valuation is incompatible with dynamic analysis based on the determination of value by labor-time. Mirowski (1989:184) writes that "the real-cost [i.e., replacement cost] method, devoid of explicit invariants, can only calculate a sequence of static equilibria in which the labor-value unit is not comparable from one calculation to the next." Likewise, Duménil and Lévy (1997:15-16) –themselves advocates of replacement cost valuation who are widely recognized for their contributions to dynamic analysis– acknowledge that the replacement cost interpretation of the "labor theory of value does not provide the framework to account for disequilibrium and dynamics in capitalism," or for "the theory of crisis or of historical tendencies."

Instead of computing the profit rate on replacement costs, one may define the profit rate as Marx (1971:131) did: "The relation between the value antecedent to production and the value which results from it –capital as antecedent value is capital in contrast to profit– constitutes the all-embracing and decisive factor in the whole process of capitalist production." According to this definition, the denominator of the profit rate is the *capital-value advanced* for inputs ("value antecedent to production"). By this definition, $r_v$ is *identical* to $\dot{K}_v$ , the relative increase in the value which capitalists hold after production over the value they actually advanced beforehand. And since, if value is determined by labor-time and productivity is rising, $\dot{K}_v < r_p$ , it follows from this definition that $r_v < r_p$.

Research in the temporal single-system interpretation of Marx's value theory has shown that the divergence between the two profit rates can persist in the long-run if productivity continues to grow, and that, due to this divergence, $r_v$ can continually fall even though $r_p$ continually rises. The same result holds in multisector economies – if prices in the aggregate fall as productivity rises, then the price rate of profit can fall continually even when the replacement cost profit rate continually rises (see Kliman, 1997). Such demonstrations vindicate the logical coherence of Marx's law of the tendential fall in the profit rate and refute the Okishio (1961) theorem, which had seemed to invalidate that law.

It is important to note, moreover, that continual productivity growth creates an ever-widening divergence between the physical and value rates of accumulation. This conclusion cannot be challenged by assertions that the replacement cost profit rate is the "true" value rate of profit. No matter how the profit rate is defined, the determination of value by labor-time implies that $\dot{K}_v < \dot{K}_p$ when productivity is rising.

It may be helpful to illustrate these points by means of a simple example, which is not intended as a model of any actual economy. Assume a one-good (corn) economy without fixed capital, in which the wage rate is zero, and all output is reinvested. Hence, the output of one period is the same as the seed-corn input of the next. Letting Kp stand for the input and X for the output, $X_t = Kp_{t+1}$. Assume that the growth path of output is given by $X_t = 1 + 2(2)^t$. Given an initial condition $Kp_0 = 2$, the growth path of the seed-corn is thus given by $Kp_t = 1 + 2^t$. Finally, imagine that the per-period expenditure of living labor, L, is $L_t = 8$. Since output grows, while living labor does not, productivity continually rises.

With all output reinvested, capital is growing at its maximum rate, so the physical profit rate and rate of accumulation are the same, $X_t / Kp_t - 1 = \dfrac{2^t}{1+2^t}$. Beginning at 50% in period 0, this rate rises continually and eventually approaches 100%.

If the unit value of corn before production is constrained to equal the unit value after production, i.e., if valuation is simultaneous, then the unit value is determined by $\lambda_t X_t = \lambda_t Kp_t + L_t$, from which it follows that $\lambda_t = L_t / (X_t - Kp_t) =$ $= 8(0.5)^t$. The values of the output and the input are then $\lambda_t X_t$ and $\lambda_t Kp_t$. The resulting value rate of profit, $\lambda_t X_t / \lambda_t Kp_t - 1$, is the replacement cost rate, which is obviously identical to the physical rate. So it, too, rises continually.

Yet the rate of accumulation of capital-value does not. It is given by $Kv_{t+1} / Kv_t - 1 = \lambda_{t+1} Kp_{t+1} / \lambda_t Kp_t - 1 = -\dfrac{0.5}{1+2^t}$, which is *negative* throughout all time! Proposition 3 is violated. The gap between the rate of accumulation and the profit rate, moreover, increases over time. Initially, the profit rate exceeds the rate of accumulation of capital-value by 75 percentage points, but the difference widens continually and approaches 100 percentage points.

According to the TSS interpretation of Marx's value theory, on the other hand, he did not constrain input and output values to be equal. Thus, the unit value is determined, not by $\lambda_{t+1} X_t = \lambda_t Kp_t + L_t$, but by $\lambda_{t+1} X_t = \lambda_t Kp_t + L_t$. $\lambda_t$ is the input value of period t and $\lambda_{t+1}$ is the output value (which is thus the same as the input value of period t + 1). Now, $\lambda_t Kp_t \equiv Kv_t$, and since $X_t = Kp_{t+1}$, it follows that $\lambda_{t+1}X_t = \lambda_{t+1}Kp_{t+1} \equiv Kv_{t+1}$. Hence we obtain a difference equation for the growth path of capital-value, $Kv_{t+1} = Kv_t + L_t$ or $Kv_{t+1} = Kv_t + 8$. Its solution is $Kv_t = Kv_0 + 8(t)$. By assuming that $Kv_0 = 16$, we ensure that the initial level of the profit rate is the same as the replacement cost rate, which facilitates comparison.

The rate of accumulation of capital-value is $Kv_{t+1} / Kv_t - 1 = 1/(2 + t)$, so that it starts off at 50% and falls continually, eventually approaching zero. The value rate of profit is $\lambda_{t+1}X_t / \lambda_t Kp_t - 1 = Kv_{t+1} / Kv_t - 1$. In contrast with the simultaneous valuation case, then, the rates of profit and accumulation are identical, so that Proposition 3 is respected. Instead of rising along with the physical profit rate, moreover, here the value profit rate continually declines. This confirms Marx's claim that labor-saving technical change can itself reduce the profit rate. Because, moreover, the real wage rate in this example did not rise, and the profit rate is always in equilibrium (since disequilibrium requires at least two sectors), the Okishio theorem has been refuted.

Value and use-value thus move in contradictory ways. It is important to emphasize that this contradiction does *not* result from the temporal method of value calculation. Even under the method of simultaneous valuation, the contradiction is present. To be sure, this method succeeds in suppressing the contradiction between the physical and value rates of profit. Yet, as this example has demonstrated, the contradiction between use-value and value then resurfaces in the form of ever-increasing deviations between the rate of profit and the growth rate of capital-value.

## 2. *Why Does It Matter?*

It should now be clear that the physical quantities approach and the determination of value by labor-time are incompatible, and that defining the profit rate in replacement cost terms fails to overcome the incompatibility. The question then becomes: which rates of profit and accumulation matter, the physical rates or the value rates?

Proponents of the physical quantities approach argue that a falling value rate of profit (computed on capital advanced) is irrelevant because it does not impede accumulation of use-values (Laibman 1997:76-77). The very productivity growth that tends to lower the value rate of profit allows capitalists to accumulate ever-more use-values and expand production accordingly.

This argument begs the question, presupposing what it needs to demonstrate, namely that the goal of capitalists is to accumulate use-values instead of value. If, instead, they seek to accumulate value, accumulation in physical terms might indeed be retarded as firms react to the falling tendency of the profit rate. Moreover, if prices fall as productivity rises, not only will the rate of profit tend to decline, but debt service problems, capital losses, and bankruptcies will tend to result.

Proponents of the physical quantities approach seem to have a stronger argument when they contend that a falling value rate of profit (computed on capital advanced) need not impede capital accumulation in *price* terms. They have continually reminded us that what capitalists care about is not value *per se,* but price. If, in a one-good world, the good's price is set equal to 1 (or any constant), then the rates of profit and accumulation in price terms will always match the corresponding physical rates[3].

Note that the appeal to price is an appeal to valuation in terms of a money commodity or *numéraire,* the distinction being immaterial for present purposes. Yet the argument is cast in terms of a one-good world, in which a money commodity or *numéraire* would seem to be redundant. In the multi-good real world, as I will demonstrate in the next three sections, the argument does not hold water. The measurement of profitability in terms of one or another physical good leads to indeterminancy and incoherent dynamics.

Just as physical goods are themselves heterogeneous, so are the profit rates measured in terms of them, and therefore so are the intertemporal paths of economies in which profit rates influence capitalists' behavior. If one wishes to argue that the money commodity-*numéraire* profit rate is what matters, one must then answer the unanswerable question: "which one?"

3. It is important to note that proponents of the temporal single-system interpretation of Marx's value theory do *not* assert the relevance of the value rate of profit as against the price rate. If by "price" one means value as redistributed across sectors, this interpretation holds that the value and price rates are necessarily equal in the aggregate. Whereas prices in this sense fall in the aggregate as productivity rises, prices as measured in terms of a *numéraire* or money commodity do not, *ceteris paribus.*

### 3. *Arbitrariness of the "Arbitrary* **Numéraire"**

Consider an economy in which the input-output matrix, inclusive of wage goods, is

$$A = \begin{bmatrix} .87719 & .08754 & .00100 & .00100 & .00100 \\ 0 & .89286 & .00315 & .02446 & .00147 \\ 0 & 0 & .90909 & .00100 & .00100 \\ 0 & 0 & 0 & .92593 & .00100 \\ 0 & 0 & 0 & 0 & .94340 \end{bmatrix}$$

where rows and columns indicate the sectors that produce and use the inputs, respectively.

If we constrain input prices to equal output prices, and seek to find a set of prices that results in an equalized rate of profit, the solution must satisfy

$$\mathbf{p}\mathrm{A}(1 + \mathrm{r}) = \mathbf{p} \qquad (4)$$

where **p** is the row vector of stationary prices and r is the uniform profit rate. Yet no solution to (4) results in strictly positive prices. There exist five solutions for 1 + r, namely the reciprocals of **A**'s main diagonal elements. Four of them make some prices negative, while the fifth, 1 + r = 1/.94340 = 1.06, makes the relative prices of goods 1-4 (in terms of good 5) equal to zero.

This is an example of an economy in which "self-reproducing non-basics" (SRNBs) exist, Sraffa's (1960:90-91) term for goods that enter into their own production, but, unlike basics, not into the production of all goods. (In the present example, good 1 is the sole basic; the rest are SRNBs.) If, in the associated system of basic goods, the solution for 1 + r that makes all prices positive is greater than the output-input ratio of all SRNBs (the reciprocals of their main diagonal element in **A**) –that is, if the "own-rate of reproduction" of all SRNBs is less than that of the basic system– then no all-positive price solution to (4) exists.

Because an all-positive price solution to (4) is lacking in this case, some Sraffian authors have suggested that profit rates cannot equalize. Sraffa himself, however, stressed that the assumption of stationary prices in his *Production of Commodities* was merely an assumption, the implications of which he was investigating. He therefore noted that when a SRNB with a low own-rate of reproduction (his "beans") exists, there is another possibility, a normal (uniform) profit rate but nonstationary prices. "The 'beans' could however still be produced and marketed so as to show a normal profit if the producer sold them at a higher price than the one which, in his book-keeping, he attributes to them as a means of production" (Sraffa 1960:91).

To study this possibility, assume that the prices which producers "attribute" to means of production are their input prices, i.e., their prices at the time they become inputs. Thus, a uniform profit rate requires that input prices and output prices differ. System (4) then becomes:

$$\mathbf{p}_t \mathbf{A}(1 + r_{[t,t+1]}) = \mathbf{p}_{t+1} \tag{5}$$

In the present 5-good example, (5) consists of 5 equations in 11 unknowns. Specifying initial conditions (5 prices for $t = 0$) leaves 6 unknowns. To eliminate 1 more unknown, and thus make (5) determinate, choose any one good as a *numéraire* or money commodity. Its price equals 1 throughout all time. It then turns out that:

(a) if all prices are initially positive, then they always remain positive – the apparent impossibility of having a uniform profit rate and all-positive prices in this economy is merely an artifact of the stationary price postulate;

(b) the goods' relative prices are not affected by the choice of money commodity;

(c) money prices converge on a moving equilibrium, and the rate of change in good k's price converges on $a_{kk} / a_{mm} - 1$, where $a_{kk}$ and $a_{mm}$ are the "own" input-output ratios of good k and the money commodity. When, in the present example, good 3 is chosen the money commodity, the five prices' rates of change converge on -3.6%, - 1.8%, 0%, 1.9%, and 3.8%.

To avoid unnecessary pedantry, I omit the proof of these propositions, though they are true for all cases of this type[4].

Finally:

(d) the level of the economy's uniform profit rate depends on the choice of money commodity. The uniform profit rates associated with different money commodities differ, moreover, not only during an initial period of

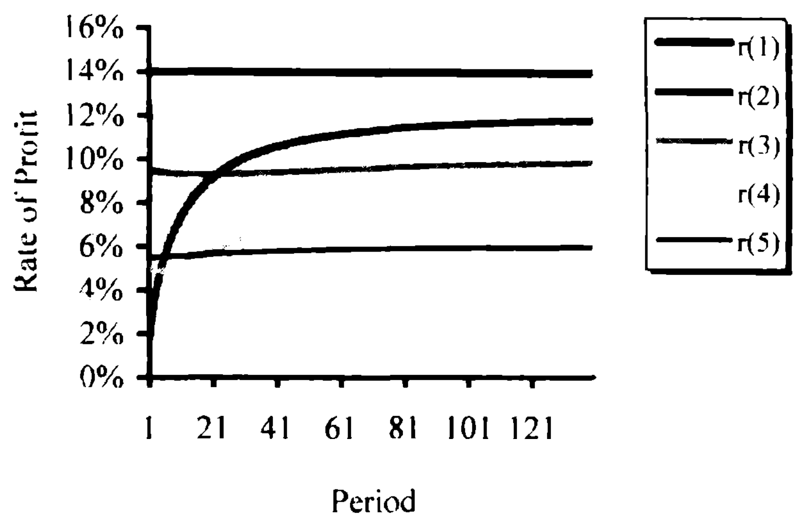
4. By "cases of this type", I mean cases that have the following properties. With goods ranked such that basics are "highest-order" and goods that enter only into their own reproduction are "lowest-order," (i) there is a single good in each group, and (ii) each good has a lower own-rate of reproduction than the next-highest-order good. Where groups contain multiple goods and/or some SRNBs have higher own-rates of reproduction than the next-highest-order good(s), more complex but similar conclusions hold.

"disequilibrium," but throughout all time. This is so even though relative prices are invariant to the choice of money commodity. The level of the uniform profit rate converges on $r_{[t,t+1]} = 1 / a_{mm} - 1$, the money commodity's own-rate of reproduction.

Again, I omit the proof, though the result holds for all such cases. Since there are five possible monies in the present example, there are five different long-run profit rates: 14%, 12%, 10%, 8%, and 6% (see Figure 1)[5].

Figure 1

*Equalized Profit Rates Under Different* Numéraires




Advocates of valuation in terms of a *numéraire* typically claim that the choice of *numéraire* is "arbitrary," by which they mean that, if one chooses a different *numéraire,* one does not alter any significant results. Yet the present example makes clear that the choice of *numéraire* is arbitrary in precisely the opposite sense – it leads to profit rates of arbitrary size!

This conclusion is particularly damaging to any theory that emphasizes the importance of the profit rate to capitalists' behavior and the rate of accumulation. As noted above, the maximum rate of accumulation is given by the profit rate. But which one? Is the maximum rate of capital's self-expansion in the present case 14%?, 12%?, 10%?, 8%?, 6%?

The problem is conceptual as well as quantitative. The physical quantities approach –the rooting of economic theory in input-output relations and *numéraire* or money commodity valuation– compels us to believe that the

5. To generate Figure 1, initial prices of all goods were set equal to 1. This assumption does not affect the levels of the profit rates in the long run.

trajectory of the economy will be altered fundamentally, merely because people (or, perhaps, only the theorists) suddenly decide to think of their coats as being worth y ounces of silver instead of x ounces of gold. After all, it is only a change in the measure of value that renders the uniform profit rate indeterminate. This problem underscores the necessity of an *immanent* measure of value, something which is not itself a commodity but instead "constitutes value" (Marx 1971:155), i.e., is the substance of value.

Yet perhaps this is a tempest in a teapot. Both Sraffa and the Sraffians have been at pains to argue that "beans" are "rare" (Sraffa 1962:426). As Bradley and Howard (1982:248) have noted, however, such claims are made without being supported by any empirical evidence.

Rather, Sraffa offered only the weak, *a priori* argument that it is unlikely that a *single* good could have an own-rate of reproduction lower than that of the basic system, in which *large numbers* of goods are directly and indirectly needed to reproduce the basics. Yet systems of SRNBs, not just single SRNBs, can also exist; sturgeon enter into the reproduction of caviar, and caviar into the reproduction of sturgeon. The number of goods in the basic system, moreover, is quite a poor proxy for its own-rate of reproduction. No matter how many there are, the basics' own-rate must be positive, and productivity growth tends to raise it. Imagine, for instance, that the own-rate of the basic system is 25% per annum. If, on average, every 10 racehorses breed a respectable 2 *net* offspring each year that not only live and are healthy but are able to compete as racehorses, then the economy-wide profit rate will be only 20%, not 25%, if racehorses are chosen as the money commodity or *numéraire.*

In any case, it is not the rarity of "beans" that is at issue here, but their existence. The presence of even one such good is sufficient to demolish the determinacy of all theories of profitability and accumulation based on the physical quantities approach, including all theories based on commodity money. Imagine, then, that only 1% of all commodities are non-basics, only 1% of non-basics are SRNBs, and only 1% of SRNBs are "beans." Then only one commodity in a million is of this last misbehaved type. Yet if 1 million different commodities are produced[6], one "bean" does exist, along with two different uniform physical profit rates.

6. Since even slightly differentiated products have different prices, any theory that seeks to determine prices must count them as distinct commodities. One million then seems to be an extremely conservative estimate of the number of commodities. Farjoun (1984:16) notes that about 60,000 different chemicals alone are produced in the United States.

Hence, even if the probability is extremely low that any *particular* good is a "bean," it seems very probable that *some* good in the economy misbehaves in this way. At the very minimum, the burden of disproof falls on the proponents of physical quantities-based theories of profitability and accumulation.

### 4. *Two Economic "Twins Paradoxes"*

Even in the absence of "beans," it is impossible to construct a dynamic theory of capitalist economies based on physical quantities that is both coherent and passably realistic. One reason it isn't possible is that, as Arrow (1981:140) emphasizes, "The view that only real [i.e., physical] magnitudes matter can be defended only if it is assumed that the labor market (and all other markets) always clear." Once this crucial postulate of neoclassical general equilibrium theory is relaxed, "Shifting from one *numéraire* to another [will] affect the direction in which resources are allocated" (Arrow 1981:141).

Arrow substantiates his claim in two main ways. First, he notes that in the absence of complete and instantaneous market-clearing, the allocation of resources between the present and the future will be affected by the choice of *numéraire* (Arrow 1981:142-44). If one lends by purchasing a bond denominated (and thus payable) in terms of the *numéraire,* and excess demand for other goods exists when the bond matures, then one may not be able to use the proceeds from the bond to purchase the other goods one wants. If an excess supply of the *numéraire* exists, one may be unable to exchange one's proceeds (some quantity of the *numéraire*) for those other goods. Similarly, if the bonds themselves are in excess supply, then one may be unable to sell them before maturity. The greater the prior probability that any of this will occur, the smaller the volume of resources allocated to the future; but the probability depends on which good is the numéraire.

Arrow also offers an economic analogue to Einstein's "Twins Paradox" – although Arrow's (1981:142) really is a paradox. Imagine that excess demand for both gold and silver exist. Given that a good's price rises when excess demand for it exists, then, if gold is the *numéraire,* the price of silver will rise. If, however, silver is the *numéraire,* the price of gold will rise. The relative price of silver in terms of gold thus rises in the first case, but falls in the second! Hence, unless markets clear at every instant, the constantly repeated refrain that the choice of *numéraire* affects only absolute (but not relative) prices is false.

I will now present a somewhat similar paradox, one which, however, does not even preclude complete and instantaneous market-clearing. Even in the absence of "beans," the level of the economy's average profit rate depends on the choice of money commodity and, given different money commodities, one and the same change in relative prices can cause the average profit rate to either rise or fall.

Imagine a two-sector economy, in which profit rates are initially equalized, and prices are stationary. In the next period, inputs are purchased at the prices which equalized profit rates, and the same amount of outputs are produced and sold. At the end of the period, however, the relative price of gold in terms of silver rises. As Table 1 illustrates, when gold is the money commodity (or *numéraire*), the output price of gold is unchanged, and therefore so is the profit rate of the gold industry. The output price of silver declines, and therefore so does the profit rate of the silver industry. Hence, the average profit rate falls as well. If, however, silver is the money commodity, then the output price of gold rises, as does the profit rate of the gold industry. The output price of silver remains unchanged, as does the profit rate of the silver industry. Hence, the average profit rate rises.

Table 1
*Dependence of Profit Rates on* Numéraire

Input-Output Relations

3 ozs. gold + 1 oz. silver → 5 ozs. gold
2 ozs. gold + 2 ozs. silver → 5 ozs. silver

| Period | Output Prices | | | Profit Rates (%) | | | Profit Rate |
|---|---|---|---|---|---|---|---|
| | Relative | Gold | Silver | Gold | Silver | Average | Differential |
| 1 | 1 | 1 | 1 | 25.0 | 25.0 | 25.0 | 0.0% |
| 2 (g) | 1.1 | 1 | 0.909 | 25.0 | 13.6 | 19.3 | 11.4 |
| 2 (s) | 1.1 | 1.1 | 1 | 37.5 | 25.0 | 31.3 | 12.5 |

Note: Input and output prices are equal in period 1. The output prices of period 1 are the input prices of period 2.

Imagine a two-sector economy, in which profit rates are initially equalized, and prices are stationary. In the next period, inputs are purchased at the prices which equalized profit rates, and the same amount of outputs are produced and

sold. At the end of the period, however, the relative price of gold in terms of silver rises. As Table 1 illustrates, when gold is the money commodity (or *numéraire*), the output price of gold is unchanged, and therefore so is the profit rate of the gold industry. The output price of silver declines, and therefore so does the profit rate of the silver industry. Hence, the average profit rate falls as well. If, however, silver is the money commodity, then the output price of gold rises, as does the profit rate of the gold industry. The output price of silver remains unchanged, as does the profit rate of the silver industry. Hence, the average profit rate rises.

Everything is the same in these two cases except the commodity which serves as measure of value, yet the level of the average rate of profit and the direction of its change differ! We see again that the "arbitrary" *numéraire* is arbitrary in a sense diametrically opposite to that intended by those who use it as a substitute for a value theory. This paradox, moreover, has important implications for any theory of accumulation which holds that the rate of accumulation depends on the levels of profit rates. *Numéraire* or money commodity valuation implies that one and the same change in relative prices will cause the rate of accumulation to accelerate or decelerate, depending upon which good serves as the measure of value.

### 5. *Incoherent Cross-Dual Dynamics*

Much research in "cross-dual" dynamics has been conducted during the past two decades in order to ascertain whether, and under what conditions, profit rates tend to equalize. The cross-dual models commonly assume that capital flows are directly related to the size of intersectoral profit rate differentials. Yet a further important implication of the paradox just considered is that, as Table 1 illustrates, the "choice" of *numéraire* or money commodity affects the size of the differential. Hence, *numéraire* or money commodity valuation implies that that convergence or divergence of profit rates depends upon which good serves as the measure of value!

To demonstrate this, I will examine a two-firm cross-dual model presented in Duménil and Lévy (1993, pp. 84-88). Good 1 is a capital good and good 2 is a consumption good. The (equalized) wage rate is paid *ex post,* and the firms employ identical technologies, each using $\alpha$ units of the good 1, and the same amount of labor, per unit of output. Demands (D) for the goods are given by

$$D_t^1 = \alpha(Y_t^1 + Y_t^2)$$

$$D_t^2 = \frac{p_t^1 Y_t^1 + p_t^2 Y_t^2 - p_t^1 D_t^1}{p_t^2} , \tag{6}$$

where p and Y indicate price and supply; supplies depend on the profit rate differential:

$$Y_{t+1}^j = Y_t^j(1 + \gamma[r_t^j - r_t^k]) , \tag{7}$$

where $r_t^j$ is the profit rate of enterprise j in period t.

The authors assume that both firms adjust their prices in reaction to excess supply:

$$p_{t+1}^j = p_t^j\left(1 - \beta\frac{Y_t^j - D_t^j}{Y_t^j}\right) . \tag{8}$$

Here, however, the price of the money commodity will be set equal to 1 throughout time, so (8) applies solely to the other good's price.

Finally, profit rates are measured as

$$r_t^j = \frac{p_t^j D_t^j - (p_t^1 \alpha + p_t^2 w) Y_t^j}{p_t^1 \alpha Y_t^j} , \tag{9}$$

where w is the wage bill per unit of output.

To simulate system (6)-(9), I have selected $\alpha = 0.1$, $w = 0.85$, $\beta = 0.1$, and $\gamma = 0.104$. The simulation begins from an initial static equilibrium. Input prices equal output prices, and the both goods' prices equal 1. Supplies equal demands, and 100 units of good 1 and 900 units of good 2 are produced. The profit rate is equalized at 50%. At time t = 2, the supply of good 1 is perturbed slightly, to 99.9. (Much smaller perturbations could be introduced; simulations indicate that the long-term evolution of the economy is invariant to the size of the perturbation.) How does the economy respond to this supply shock?

Again, the answer depends on which good serves as the money commodity. When the price of good 2 is set equal to 1, supplies, demands, profit rates, and relative prices fluctuate, but only very slightly. The fluctuations become dampened and eventually the variables all converge to equilibrium. Profit rates and the price of good 1 converge on their original equilibrium levels, while supplies and demands of the two goods converge on equilibrium levels (approximately 99.95 and 899.53) slightly below their original ones. When, however, good 1 is the money commodity, although the variables again fluctuate, the fluctuations are explosive, and supply-demand imbalances grow over time. The economy crashes at time t = 90, the output of good 1 having become negative. Figures 2 and 3 illustrate some of these contradictory results.

Figure 2
*Supply Shock Under Different* Numéraires

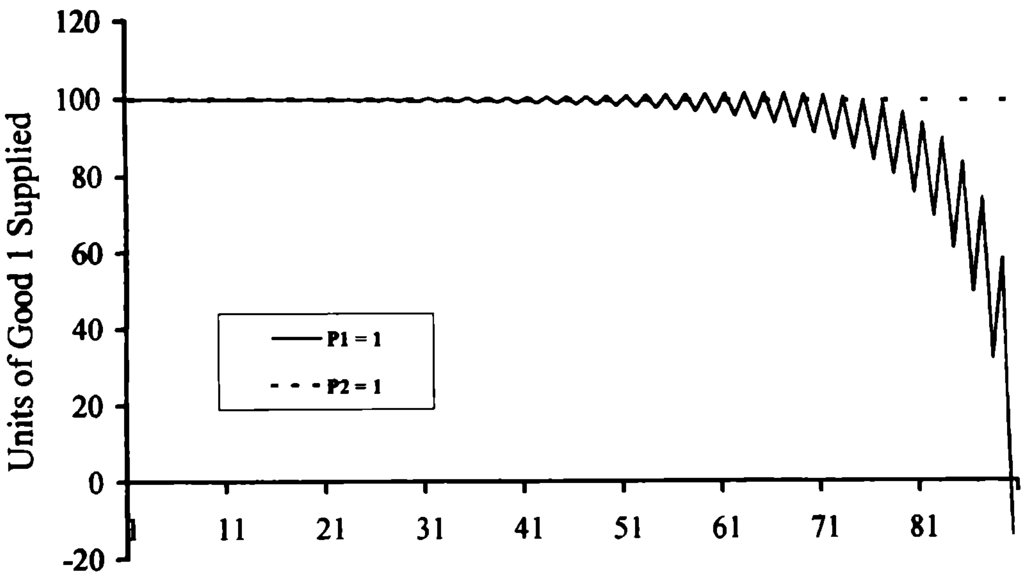
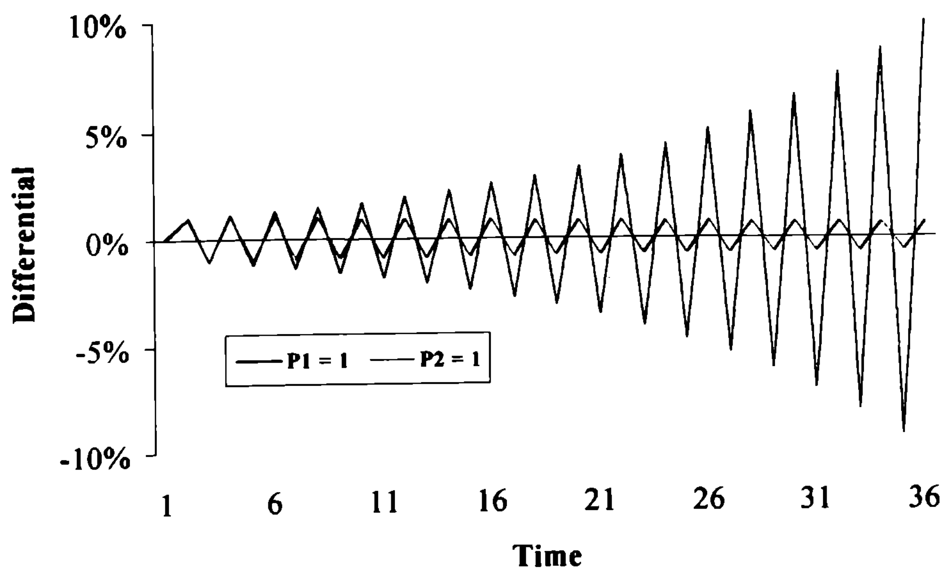



Figure 3
*Profit Rate Differential ($r_1$ - $r_2$) Under Different* Numéraires



For some other values of $\beta$ and $\gamma$, of course, the economy will behave in the same way irrespective of the money commodity, either converging to equilibrium or crashing. A whole range of parameter values, however, give rise to cases in which the economy converges to equilibrium under one money commodity while exploding under the other. This result can be replicated, moreover, for many different values of the remaining parameters ($\alpha$ and w). Nor is the result dependent on this particular model; in Kliman (1998), I obtained the same result on the basis of a somewhat simpler cross-dual model that assumes continual market-clearing.

In all cases, the equations governing the two firms' supplies, prices, and profit rates are symmetrical in every respect. Apart from the switching of the money commodity, the economic system is exactly the same in the two cases. The conclusion seems clear: by shifting from one money commodity or *numéraire* to another, spurious differences in profit rates are created and spurious behavioral reactions follow. Because, however, there is no objective basis for deciding which numéraire is correct or which money is the "real" one –in this sense as well the choice is an arbitrary one– the physical quantities approach to valuation, profitability, and accumulation is fatally flawed. This exercise shows once again that dynamics requires an immanent measure of value, something that is not subject to changes in value, and therefore something other than a commodity.

## 6. *Summary and Conclusions*

This paper has shown, first, that the physical quantities approach to price, profit, and accumulation is incompatible with the determination of value by labor-time. Several aspects of their incompatibility were highlighted: the rate of accumulation of capital-value is not governed by the physical "profit rate"; when labor productivity changes, the value and physical rates of accumulation diverge; and, due to productivity growth, the value rate of profit (computed on capital advanced) can fall while its physical counterpart rises.

The paper has also challenged assertions that the physical rates of profit and accumulation are what matter. Even the physical quantities approach requires a measure of value in a multi-good context, but the physical measures of value –money commodities and *numéraires*– have been shown to be arbitrary measures that yield arbitrary dynamics. To accept that value can be measured by one or another use-value, one must accept that the "choice" of money commodity or *numéraire* can determine: the level of the *equalized* profit rate, even in the long run; the direction of change in the average profit rate when relative prices change; and whether an economy exhibits convergent or explosive behavior.

These results suggest that existing attempts to make "objective" physical quantities the foundation of economic theory suffer from serious internal inconsistencies. To be sure, the physical quantities approach does yield what may seem to be plausible results under some very special conditions – one-good economies, static equilibrium, balanced growth, etc. Yet no general theory can be founded on these results, since it is logically impermissible (though far from uncommon) to use special cases to draw inferences concerning the general case.

These are certainly not new insights. As Sraffa (1951:xxxi-xxxii) noted, Malthus pointed out to Ricardo nearly two centuries ago that the heterogeneity of use-values renders the physical quantities approach incoherent. Ricardo wisely accepted the point and abandoned his attempt to determine "the rate of profits ... independently of value" (Sraffa 1960:93). There is a lesson in this for today.

## References

Arrow, Kenneth J. 1981. "Real and Nominal Magnitudes in Economics," in Daniel Bell and Irving Kristol (eds.), *The Crisis in Economic Theory* (New York: Basic Books).

Bradley, Ian and Michael Howard. 1982. " 'Production of Commodities by Means of Commodities' and the Rehabilitation of Classical and Marxian Political Economy," in Ian Bradley and Michael Howard (eds.), *Classical and Marxian Political Economy: Essays in honor of Ronald L. Meek* (New York: St. Martin's Press).

Duménil, Gérard and Dominique Lévy. 1993. *The Economics of the Profit Rate: Competition, crises and historical tendencies in capitalism* (Aldershott, UK: Edward Elgar).

Duménil, Gérard and Dominique Lévy. 1997. "The Conservation of Value: A rejoinder to Alan Freeman." Submitted to the International Working Group in Value Theory miniconference, Eastern Economic Association conference, Washington, D.C., April. Available from www.gre.ac.uk/~fa03/iwgvt.

Farjoun, Emmanuel. 1984. "The Production of Commodities by Means of What?," in Ernest Mandel and Alan Freeman (eds.), *Ricardo, Marx, Sraffa: The Langston memorial volume* (London: Verso).

Freeman, Alan and Guglielmo Carchedi (eds.). 1996. *Marx and Non-equilibrium Economics* (Cheltenham, UK: Edward Elgar).

Kliman, Andrew. 1997. "The Okishio Theorem: An obituary," *Review of Radical Political Economics* 29:3.

Kliman, Andrew. 1998. "Capitalist Macrodynamics Without Capitalist Value? Presented at the International Working Group in Value Theory miniconference, Eastern Economic Association conference, New York, February. Available from HYPERLINK http://www.gre.ac.uk/~fa03/iwgvt. www.gre.ac.uk/~fa03/iwgvt.

Kliman, Andrew and Ted McGlone. 1999. "A Temporal Single-System Interpretation of Marx's Value Theory", *Review of Political Economy* 11:1 (forthcoming).

Laibman, David. 1997. *Capitalist Macrodynamics: A systematic introduction* (Houndmills and London: Macmillan Press).

Mandel, Ernest and Alan Freeman (eds.). 1984. *Ricardo, Marx, Sraffa: The Langston memorial volume* (London: Verso).

Marx, Karl. 1971. *Theories of Surplus-Value,* Part III (Moscow: Progress Publishers).

Mirowski, Philip. 1989. *More Heat Than Light: Economics as nature's physics, physics as nature's economics* (Cambridge: Harvard Univ. Press).

Okishio, Nobuo. 1961. "Technical Changes and the Rate of Profit," *Kobe University Economic Review* 7.

Sraffa, Piero. 1951. "Introduction" to David Ricardo, *One the Principles of Political Economy and Taxation* (Cambridge: Cambridge Univ. Press).

Sraffa, Piero. 1960. *Production of Commodities by Means of Commodities: Prelude to a critique of economic theory* (Cambridge: Cambridge Univ. Press).

Sraffa, Piero. 1962. Letter to Peter Newman, 4th June. Published in Appendix to Krishna Bharadwaj, "On the Maximum Number of Switches Between Two Production Systems," *Schweizerische Zeitschrift für Volkswirtschaft und Statistik* 106:4, December.

Stamatis, Georg. 1983. *Sraffa und sein Verhältnis zu Ricardo und Marx,* Göttingen.

Steedman, Ian. 1977. *Marx After Sraffa* (London: NLB and Verso).