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# A Note on Foley's Article
## *«The value of money, the value of labour power and the marxian transformation problem»*

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In his article *The value of money, the value of labour power and the marxian transformation problem* Foley (1982) presents a conception of the relation between labour values and prices of production. According to Foley, although production prices deviate from corresponding labour values, the ratio of the labour value of the total net product to the production price of the total net product equals to the ratio of the labour value of the total labour power to the production price of the total labour power, i.e. to the ratio of the labour value of the total real wages to the production price of the total wages. This obviously implies a proportionality between the production prices of the net product, the real wages and the surplus product and the labour values of the net product, the real wages and the surplus product respectively, although the production price of each individual commodity is not equal or/and proportional to its labour value.

We are going to prove that, although Foley asserts for the opposite, i.e. that the production prices are not proportional to the labour values, his model entails *implicitly* the assumption that all production prices are proportional to the corresponding labour values! We are going to prove, also, that in only one single case the production prices are not proportional to the labour values and, nevertheless, the production price of the net product, the production price of the real wages and the production price of the surplus product are proportional to the corresponding labour values. This is the case when the net product, the real wages and the surplus product consist of the same commodity basket.

In order to present precisely our argument we assume that a single production system [A, $l$, Y] is given, where A, A $\geq$ 0, is the square, nxn, indecomposable matrix of the inputs in means of production per unit of commodity produced, $l$, $l$ > 0, is the 1xn vector of the inputs in direct, homogeneous labour per unit of commodity produced and Y, $Y \geq 0$, is the nX1 vector of the system's net product. Let X be the corresponding nX1 vector of the system's gross product. We further assume that the production technique $[A, \ell]$ used by the system is productive, which means that the maximum eigenvalue of matrix A, $\lambda_m^A$, is less than unity:

$$(0 <)\, \lambda_m^A < 1\,. \qquad (1)$$

Because of the indecomposability of A relation (1) implies

$$(I - A)^{-1} > 0\,. \qquad (2)$$

According to the definition of the net product it holds

$$Y = X - AX = (I - A)\,X\,, \qquad (3)$$

where AX represents the means of production that are used up in the production process.

Taking into account (2) we get from (3)

$$X = (I - A)^{-1}\, Y \qquad (4)$$

Because of $Y \geq 0$ and (2) we get from (3):

$$X > 0\,. \qquad (5)$$

Let ω be the 1Xn vector of the labour values. According to the definition of ω it holds

$$\omega = \omega A + \ell\,. \qquad (6)$$

Equation (6) implies

$$\omega = \ell (I - A)^{-1}\,. \qquad (7)$$

From (3) and (7) we get for the labour value $\omega Y$ of the net product Y:

$$\omega Y = \ell (I - A)^{-1} (I - A)\, X \;\Rightarrow$$

$$\omega Y = \ell X\,. \qquad (8)$$

Equation (8) means that the labour value of the net product Y, i.e. the direct and indirect labour required for the production of the net product Y, equals the direct labour required for the production of the corresponding gross product X. Obviously this holds not only for the labour value of the total net product, but for the labour value of every net product as well. So, for example, it holds for the labour value $\omega Y^i$ of the net product $Y^i$, which corresponds to the gross product $X^i$

$$X^i = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \\ \vdots \\ 0 \\ 0 \end{bmatrix} \begin{matrix} 1 \\ \vdots \\ \vdots \\ i \\ \vdots \\ \vdots \\ n \end{matrix}, \quad \text{for every } i,\ i = 1,2,...,n$$

consisting of one unit of commodity i, i=1, 2, ..., n. For we get from (8)

$$\omega Y^i = \ell X^i = \ell_i, \ i = 1, 2, ..., n$$

i.e. that the labour value $\omega Y^i$ of $Y^i$ as net product is equal to the direct labour required for the production of the corresponding gross product $X^i$, which consists of one unit of commodity i.

Let $\varpi$ denote the labour value of one unit of labour power, where

$$0 < \varpi < 1. \tag{9}$$

Thus, wage workers earn the fraction $\omega Y \varpi (= \ell X \varpi)$ of the labour value $\omega Y (= \ell X)$ of the net product Y and capitalists the fraction $\omega Y(1-\varpi)[= \ell X(1-\varpi)]$. This does not only hold for the labour value of the total net product, but for the labour value of each component of the total net product as well. For example, the labour value $\omega Y^i$ of the net product $Y^i$, which corresponds to the gross product $X^i$ consisting of one unit of commodity i, is divided into the fraction $\omega Y^i \varpi$, which is appropriated by the wage workers, and the fraction $\omega Y^i(1-\varpi)$, which is appropriated by the capitalists.

However, the commodities are not exchanged at their labour values $\omega$, but at market prices p, as it is correctly stated by Foley, and these prices are not necessarily proportional to the labour values. For convenience sake we suppose that market prices are equal to production prices.

We assume that all constant capital is used up during the production period and that all variable capital is paid in advance at the beginning of the production period. Thus, it holds for the nx1 vector p of the prices

$$p = (1 + r)(pA + w\ell), \tag{10}$$

where r is the uniform profit rate and w is the nominal wage rate (=money wage rate).

For every r, $0 \leq r < \frac{1-\lambda_m^A}{\lambda_m^A} (= r_{max} = r_{(w=0, p>0)})$,

the maximum eigenvalue of the matrix (1+r)A is positive and less than unity.

Thus, given the indecomposability of A and hence of (1+r)A, it holds

$$[I - (1 + r)A]^{-1} > 0 \tag{11}$$

Assuming $w > 0$ and taking into account (11), we get from (10)

$$p[I - (1 + r)A] = (1 + r)w\ell \Rightarrow$$
$$p = (1 + r)w\ell[I - (1 + r)A]^{-1}. \tag{12}$$

Given the profit rate r, equation system (12) determines uniquely the *relative* prices p of all produced commodities and the *relative* price w of the commodity «labour power».

The determination of the *absolute* prices p and w requires the introduction of a *normalization equation.* The general form of a normalization equation is

$$yp = c,$$

where y, $y \geq 0$, is the nx1 vector of the normalization commodity and c is a positive scalar.

We can now present the thesis of Foley. Foley introduces the term *value of money.* Namely he demands the validity of the relation

$$\frac{\omega Y}{pY} = d, \quad d>0, \; d=\text{constant}. \tag{13}$$

Equation (13) is an arbitrary normalization equation. It can be written as

$$pY = \frac{1}{d}\omega Y, \tag{13a}$$

where, given the fact that d, ω and Y are given, $\frac{1}{d}\omega Y$ is a given positive constant.

At the same time Foley demands that equation (13) or (13a) to hold not only for the total net product Y, but for *every net product.* For the net product $Y^i$, so that the following relation holds:

$$\frac{\omega Y^i}{pY^i} = \frac{\omega_i}{p_i} = \frac{\omega Y}{pY} \Rightarrow$$

$$p_i = \frac{1}{d}\omega_i = \frac{1}{d}\omega Y \Rightarrow \tag{14}$$

$$pY^i = \frac{1}{d}\omega Y^i = \frac{1}{d}\omega Y, \ \forall i, \ i = 1,2,\ldots,n$$

Observe that equation (14) implies that production prices $p_i$, $i=1,2,\ldots,n$, are proportional to labour values $\omega_i$, $i=1,2,\ldots,n$. Thus by demanding that the ratio of the labour value to the price of every net product should be equal to the same constant 1/d, Foley unconsciously sets production prices proportional to labour values.

Thus, Foley's assertion, that the production price of every gross product, i.e. the production price of every produced commodity, deviate from the corresponding labour value and simultaneously all production prices of the net products incorporated to commodities are proportional to the corresponding labour values, is impossible. For, if the assertion, that all production prices of the net products incorporated to commodities are proportional to the corresponding labour values, is valid for all net products, then, given the fact that any single commodity or commodity basket may be a net product, it is also valid for every single commodity or commodity basket. Hence it is also valid for every gross product, since any single commodity or commodity basket can be a gross product. Consequently all production prices are proportional to the corresponding labour values. Because of this is also the production price of any commodity basket proportional to the corresponding labour value. For example, it holds for the commodity basket $\Psi$

$$\frac{\omega\Psi}{p\Psi} = \frac{\omega Y}{pY} = d, \tag{15}$$

where $\Psi$ is the real wage basket, i.e. the workers consumption basket, and for the commodity basket $\Phi$

$$\frac{\omega\Phi}{p\Phi} = \frac{\omega Y}{pY} = d, \tag{16}$$

where $\Phi$ is the surplus product. Obviously it holds

$$\Psi + \Phi = Y.$$

If, however, equation (14) is not valid, i.e. if production prices are not proportional to the corresponding labour values, then *in the general case* –even if we normalize the price vector p as Foley does using (13) or (13a)– the prices

$p\mathrm{Y}$, $p\Psi$ and $p\Phi$ of Y, $\Psi$ and $\Phi$ are not proportional to the corresponding labour values $\omega\mathrm{Y}$, $\omega\Psi$ and $\omega\Phi$.

If we demand the validity both of (13) and (15) [and hence the validity of (13), (15) and (16)], then obviously the equation system consisting of (12), (13) and (15) is *overdetermined*, since it has two price normalization equations, and in the general case has no solution. Hence, a discussion on a proportionality between $p\mathrm{Y}$, $p\Psi$ and $p\Phi$ on the one hand and $\omega\mathrm{Y}$, $\omega\Psi$ and $\omega\Phi$ on the other hand is not feasible.

However, there exists one case, where the equation system consisting of (12), (13) and (15) is not overdetermined. This occurs when Y, $\Psi$ and $\Phi$ consist of the same commodity basket and consequently

$$\Psi = (1-\alpha)\,\mathrm{Y}, \quad 0 < \alpha < 1,$$

and hence

$$\mathrm{Y} = \Phi + \Psi = (1-\alpha)\,\mathrm{Y} + \Phi \;\Rightarrow\; \Phi = \alpha\mathrm{Y}.$$

Thus (13) and (15) are linearly depended equations and hence the equation system consisting of (12), (13) and (15) is not overdetermined. In this case we have proportionality between the prices $p\mathrm{Y}$, $p\Psi$ and $p\Phi$ and the corresponding values $\omega\mathrm{Y}$, $\omega\Psi$ and $\omega\Phi$ (see also Giussani 1991-92, pp. 73-75).

This case, however, does not describe the real economic system but a quasi-one-good-economy, which using homogeneous labour and a *composite commodity* AX as means of production produces *one and only one composite commodity* Y (of the same or of a different composition with its means of production AX). Therefore the thesis of Foley is only either in the case of a quasi-one-good-economy or in the trivial case, in which all production prices are proportional to the labour values.

## References

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