# Some remarks upon the article of Alan Freeman «Marx without equilibrium» 

by<br>Georg Stamatis

In his article «Marx without equilibrium» Alan Freeman (1995) subjects to criticize the «simultaneist concept of determination [of the production prices]» and argues for the «sequential approach [of determination of the production prices]».

In the following we clarify the terms «simultaneist concept» and «sequential approach». Afterwards we make some remarks upon the respective opinions of Alan Freeman.

Suppose the production system $[\mathrm{B}, \mathrm{A}, \ell, \mathrm{x}]$ that uses the linear, square, productive technique $[\mathrm{B}, \mathrm{A}, \ell]$ and operates at the activity levels x , where denote:
$B, \quad B \geq 0$, the $n \times n$ output matrix,
$\mathrm{A}, \mathrm{A} \geq 0$, the $\mathrm{n} \times \mathrm{n}$ matrix of the inputs in means of production,
$\ell, \quad \ell>0$, the $1 \times n$ vector of labour inputs, when the production system operates at the basic activity levels, and
$\mathrm{x}, \quad \mathrm{x}>0$, the nx 1 vector of the activity levels of the system.
Since the given production technique is ex hypothesis productive it holds

$$
\begin{equation*}
(\mathrm{B}-\mathrm{A})^{-1} \geq 0 \tag{1}
\end{equation*}
$$

We suppose that the wages are paid at the end of the production period. Consequently it holds for the $1 \times n$ vector of the uniform production prices $p$, the uniform profit rate r and the uniform nominal wage rate w :

$$
\begin{equation*}
\mathrm{pB}=\mathrm{p}(1+\mathrm{r}) \mathrm{A}+\mathrm{w} \ell, \tag{2}
\end{equation*}
$$

where

$$
\begin{equation*}
w>0 \tag{3}
\end{equation*}
$$

and

$$
\begin{equation*}
r \geq 0 . \tag{4}
\end{equation*}
$$

We also suppose

$$
\begin{equation*}
[\mathrm{B}-(1+\mathrm{r}) \mathrm{A}]^{-1} \geq 0 . \tag{5}
\end{equation*}
$$

Under the assumptions (3), (4) and (5) we get from (2)

$$
\begin{equation*}
\mathrm{p}=\mathrm{w} \ell[\mathrm{~B}-(1+\mathrm{r}) \mathrm{A}]^{-1} . \tag{6}
\end{equation*}
$$

It can be proved that, if the matrix $B+A$ is reducible and its canonical form is

$$
\mathrm{B}+\mathrm{A}=\left[\begin{array}{cc}
(\mathrm{B}+\mathrm{A})_{11} & (\mathrm{~B}+\mathrm{A})_{12} \\
0 & (\mathrm{~B}+\mathrm{A})_{22}
\end{array}\right],
$$

where $(B+A)_{11}$ and $(B+A)_{22}$ are square, irreducible matrices and $(B+A)_{11} \geq 0$, $(\mathrm{B}+\mathrm{A})_{12} \geq 0$ and $(\mathrm{B}+\mathrm{A})_{22} \geq 0$, i.e., if the non-basic commodities enter the production of non-basic commodities, then -depending on the arithmetical value of r - there appear either infinite and negative production prices of the nonbasic commodities or zero production prices of the basic commodities.

In order to exclude production prices of such a kind we suppose (assumption (a)) that matrix $\mathrm{B}+\mathrm{A}$ is either irreducible or, if it is reducible, its canonical form is

$$
\mathrm{B}+\mathrm{A}=\left[\begin{array}{cc}
(\mathrm{B}+\mathrm{A})_{11} & (\mathrm{~B}+\mathrm{A})_{12} \\
0 & 0
\end{array}\right]
$$

This means that either there don't exist non-basic commodities or, if they exist, they don't enter the production of the non-basic commodities.

Taking into account (3), (4), (5) and the above assumption (a) we get from (6)

$$
\begin{equation*}
\mathrm{p}>0 . \tag{7}
\end{equation*}
$$

Thus, if both $w$ and $r$ are given, (6) determines uniquely and fully the production price vector $p$. For this reason the vector $p$ is the vector of the absolute and not that of the relative production prices.

The above determination of the price vector $p$ calls Alan Freeman the «simultaneist concept».

According to the above the «simultaneist concept» does not define always uniquely determined and positive production prices. The production prices defined by this concept are uniquely determined and positive, only if the given production system either does not produce non-basic commodities or, if it
does, the non-basic commodities do not enter the production of non-basic commodities.

In addition, the «simultaneist concept» presupposes nothing about the demanded and the produced quantities of commodities. Consequently this concept says nothing about of the equilibrium or non equilibrium of the commodity market.

Thus, if the «simultaneist concept» defines at all a notion of equilibrium, this notion consists merely in the statement that, under the assumptions (3), (4), (5) and (a) and for given production technique and given $w$ and $r$, the absolute production prices are uniquely determined and positive.

The simultaneous determination of the production prices $p$ requires not necessarily that both $w$ and $r$ are given. It is sufficient that either $r$ or $w$ is given. If only r is given, (6) determines uniquely the vector ( $\mathrm{p}, \mathrm{w}$ ) of the relative production prices of the n produced commodities and of the commodity «labour power». If, on the contrary, only wis given, (6) yields $n$ solutions for $r$ and $p$. There exists always a solution for $r$ and $p$ such that $r \geq 0$ and $p>0$. This is the solution which yields the smallest non-negative arithmetical value for $r$.

The determination of the production prices by Ricardo (on the basis of his «corn model»), by Marx ${ }^{1}$, by Mühlpfort, by Dmitriev, by von Bortkiewicz, by Sraffa et allii is a «simultaneist concept» or rather the «simultaneist concept». This concept of Ricardo, Marx et allii is just a concept of determination of the production prices and the uniform profit rate for given uniform nominal or real wage rate ${ }^{2}$ and it is not a Walrasian general equilibrium model for the determination of both the prices and the produced (=demanded) quantities of the commodities.

John von Neumann, who has known that the Walrasian model does not always yield uniquely determined and positive prices (see Kaldor 1989), has present a general equilibrium model in order to determine uniquely determined and positive equilibrium prices and quantities (see von Neumann 1937). He failed in his attempt. For, first, the prices that his model determines

[^0]are non-zero but arbitrary, second, the commodity quantities that his model determines are not absolute but merely relative quantities and, third, in framework of his model the produced commodities are determined not by the demand (which von Neumann does not take into account) but only by the choice of the production technique that maximizes the maximum real rate of growth (see Stamatis 1998).

Consider now what Alan Freeman with «sequential approach» means. We suppose

$$
\begin{equation*}
\mathrm{B}^{-1} \geq 0 \tag{8}
\end{equation*}
$$

and

$$
\begin{equation*}
\left[\mathrm{I}-(1+\mathrm{r}) \mathrm{AB}^{-1}\right]^{-1} \geq 0 . \tag{9}
\end{equation*}
$$

Taking into account (8) and (9) we get from (2)

$$
\begin{align*}
& \mathrm{p}=\mathrm{p}(1+\mathrm{r}) \mathrm{AB}^{-1}+\mathrm{w} \ell \mathrm{~B}^{-1} \Rightarrow  \tag{10}\\
& \mathrm{p}\left[\mathrm{I}-(1+\mathrm{r}) \mathrm{AB}^{-1}\right]=\mathrm{w} \ell \mathrm{~B}^{-1} \Rightarrow \\
& \mathrm{p}=\mathrm{w} \ell \mathrm{~B}^{-1}\left[\mathrm{I}-(1+\mathrm{r}) \mathrm{AB}^{-1}\right]^{-1} . \tag{11}
\end{align*}
$$

The equation system (11) yields $p>0$. For we get from (11)

$$
\begin{align*}
& \mathrm{p}=\mathrm{w} \ell\left\{\left[\mathrm{I}-(1+\mathrm{r}) A B^{-1}\right] \mathrm{B}\right\}^{-1} \Rightarrow \\
& \mathrm{p}=\mathrm{w} \ell[\mathrm{~B}-(1+\mathrm{r}) \mathrm{A}]^{-1}, \tag{6}
\end{align*}
$$

from which, because of (3), (4), (5) and assumption (a), we get $\mathrm{p}>0$.
One can determine the prices $p$ either «simultaneously» by (6) or «sequentially» starting from (10).

Starting from (10) and putting in the right-hand side of (10)

$$
\begin{equation*}
\mathrm{p}=\mathrm{p}_{0}, \tag{12}
\end{equation*}
$$

where $\mathrm{p}_{0}$,

$$
\begin{equation*}
\mathrm{p}_{0}>0, \tag{13}
\end{equation*}
$$

is an arbitrary positive 1 Xn vector, we get

$$
\begin{equation*}
\mathrm{p}_{1}=\mathrm{p}_{0}(1+\mathrm{r}) \mathrm{AB}^{-1}+\mathrm{w} \ell \mathrm{~B}^{-1} . \tag{14}
\end{equation*}
$$

Putting in the right-hand side of (10)

$$
\begin{equation*}
\mathrm{p}=\mathrm{p}_{1} \tag{15}
\end{equation*}
$$

and taking into account (14), we get

$$
\mathrm{p}_{2}=\mathrm{p}_{1}(1+\mathrm{r}) \mathrm{AB}^{-1}+\mathrm{w} / \mathrm{B}^{-1}=
$$

$$
\begin{align*}
& =\left\{\left[p_{0}(1+r) A B^{-1}+w \ell B^{-1}\right](1+r) A B^{-1}\right\}+w \ell B^{-1}= \\
& =p_{0}\left[(1+r) A B^{-1}\right]^{2}+w \ell B^{-1}(1+r) A B^{-1}+w \ell B^{-1} . \tag{16}
\end{align*}
$$

Putting in the right hand side of (10)

$$
\begin{equation*}
\mathrm{p}=\mathrm{p}_{2} \tag{17}
\end{equation*}
$$

and taking into account (16) we get

$$
\begin{align*}
\mathrm{p}_{3}= & \mathrm{p}_{0}\left[(1+\mathrm{r}) \mathrm{AB}^{-1}\right]^{3}+w \ell \mathrm{~B}^{-1}\left[(1+r) \mathrm{AB}^{-1}\right]^{2}+ \\
& +w \ell \mathrm{~B}^{-1}\left[(1+\mathrm{r}) A B^{-1}\right]+w \ell \mathrm{~B}^{-1} . \tag{18}
\end{align*}
$$

Continuing the above procedure we get

$$
\begin{align*}
\mathrm{p}_{\mathrm{m}}= & \mathrm{w} \ell \mathrm{~B}^{-1}\left\{I+\left[(1+\mathrm{r}) A B^{-1}\right]+\left[(1+\mathrm{r}) A B^{-1}\right]^{2}+\right. \\
& \left.+\left[(1+r) A B^{-1}\right]^{3}+\ldots+\left[(1+r) A B^{-1}\right]^{\mathrm{m}-1}\right\}+ \\
& +\mathrm{p}_{0}\left[(1+\mathrm{r}) \mathrm{AB}^{-1}\right]^{\mathrm{m}} . \tag{19}
\end{align*}
$$

Due to (9) the maximum eigenvalue of the positive or semipositive matrix $(1+r) \mathrm{AB}^{-1}$ is positive and less than unity. Consequently the matrix $(1+\mathrm{r}) \mathrm{AB}^{-1}$ is convergent, i.e., the matrix $\left[(1+r) A B^{-1}\right]^{m}$ converges with increasing $m$ towards the $n \times n$ zero matrix. Hence, also the vector $p_{0}\left[(1+r) \mathrm{AB}^{-1}\right]^{m}$ converges with increasing $m$ towards the 1 xn zero vector. Thus at an enough great m we get

$$
\begin{align*}
\mathrm{p} \cong & \mathrm{w} \ell \mathrm{~B}^{-1}\left\{I+\left[(1+\mathrm{r}) \mathrm{AB}^{-1}\right]+\left[(1+\mathrm{r}) \mathrm{AB}^{-1}\right]^{2}+\ldots+\right. \\
& \left.+\left[(1+\mathrm{r}) \mathrm{AB}^{-1}\right]^{\mathrm{m}-1}\right\} . \tag{20}
\end{align*}
$$

From (11) and (20) and at an enough great $m$ we get

$$
\begin{equation*}
\mathrm{p}_{\mathrm{m}} \cong \mathrm{p} . \tag{21}
\end{equation*}
$$

The above iterative procedure of calculation of $p$ is what Alan Freeman calls «sequential approach». It is the well known approach presented by Paolo Giussani (see Giussani 1991-1992).

This «sequential approach» is merely a computational procedure that has no economic meaning. Nevertheless Alan Freeman conceives this procedure as a procedure which actually takes place in the time so that, according to him, the computational steps from 0 to 1 , from 1 to 2 , from 2 to $3, \ldots$, from $m-1$ to $m$ describe transitions from the time point 0 to the time point 1 , from the time point 1 to the time point 2 , from the time point 2 to the time point $3, \ldots$, from the time point $\mathrm{m}-1$ to the time point m .

Alan Freeman fails to appreciate that this procedure, even if it was a real
process that takes place in the time and not only a computational one, it would be possible only if

$$
\begin{equation*}
\mathrm{B}=\mathrm{I}, \tag{22}
\end{equation*}
$$

i.e. only if the production system is a single and not a joint production system. Otherwise, i.e., if

$$
\begin{equation*}
\mathrm{B} \neq \mathrm{I}, \tag{23}
\end{equation*}
$$

relation (8) does not hold, as presupposed ${ }^{3}$.
Thus, the above expositions on the «sequential approach» finally hold only for $\mathrm{B}=\mathrm{I}$ and consequently for

$$
\begin{equation*}
\mathrm{B}^{-1}=\mathrm{I} . \tag{24}
\end{equation*}
$$

Hence, one must put in (9), (10), (11), (14), (16), (18), (19) and (20) the relation (24).

So, we get from (11)

$$
\begin{equation*}
\mathrm{p}=\mathrm{w} \ell[\mathrm{I}-(1+\mathrm{r}) \mathrm{A}]^{-1} \tag{11a}
\end{equation*}
$$

and from (20)

$$
\begin{equation*}
\mathrm{p}_{\mathrm{m}} \cong \mathrm{w} \ell\left\{I+[(1+r) A]+[(1+r) A]^{2}+\ldots+[(1+r) A]^{m-1}\right\} \tag{20a}
\end{equation*}
$$

where

$$
\begin{equation*}
\mathrm{p}_{\mathrm{m}} \cong \mathrm{p} . \tag{21a}
\end{equation*}
$$

The application of (20a) and (21a) instead of (11a) in order to computer the price vector $p$ is merely an iterative procedure that we necessarily use when we cannot use (11a) because the order of the matrix $[\mathrm{I}-(1+\mathrm{r}) \mathrm{A}]^{-1}$ is so great that we cannot computer this matrix.

Both methods, the one by application of (11a) and the other by application of (20a) and (21a), are equivalent, where the second is of course an iterative one.
3. This happens since a square positive or semipositive matrix $B, B \geq 0$, has a positive or semipositive inverse $B^{-1}, B^{-1} \geq 0$, only if $B$ has in each row and in each column only one positive element, i.e. only if B is diagonal (see Magnan de Bornier 1984). This obviously implies that a square positive matrix $B$ has never a positive or semipositive inverse $B^{-1}, B^{-1} \geq 0$, and that a square semipositive matrix $B$ has a semipositive inverse $B^{-1}$ only if $B$ is diagonal. But if $B$ is diagonal, one can the basic activity levels so define that

$$
\begin{aligned}
& \mathrm{B}=\mathrm{I} \Rightarrow \\
& \mathrm{~B}^{-1}=\mathrm{I}^{-1}=\mathrm{I} .
\end{aligned}
$$

Consequently $\mathrm{B}^{-1}$ is semipositive only if the production system is a single production system.

Nevertheless there exists an additional difference among them: The application of the «simultaneist concept», i.e. of (11a), presupposes that either
(a) only w is given or
(b) only r is given or
(c) wand r are given or
(d) only the real wage rate d is given ${ }^{4}$ or
(e) w and d are given.

The «simultaneist concept» determines in the case (a) the vector $p$ of the relative prices of the n produced commodities and $\mathrm{r}^{5}$, in the case (b) uniquely the vector ( $\mathrm{p}, \mathrm{w}$ ) of the relative prices of the n produced commodities and of the commodity «labour power», in the case (c) uniquely the vector p of the absolute prices of the n produced commodities, in the case ( d ) the vector ( $\mathrm{p}, \mathrm{w}$ ) of the relative prices of the n produced commodities and of the commodity «labour power» and $\mathrm{r},{ }^{6}$ and in the case (e) the vector p of the absolute prices of the n produced commodities and r. ${ }^{7}$

Thus, the application of the «simultaneist concept» presupposes not necessarily single production and that both r and w are given. On the contrary the application of the «sequential approach», i.e. of (20a) and (21a), presupposes necessarily single production and that both r and w are given. Thus, its application confine to the above case (c), i.e. to the case, in which, first, the production system is a single production system and, second, both w and r are given. On the contrary the application of the «simultaneist concept» refers to single and joint production systems and to all the cases (a)-(e).

The «sequential approach» is a computational iterative approach and not an actual economic process which take place in the time. It would be a such process only if each computational step would describe a transition from one point of time to an other point of time. This does not happen. If only because of that the first computational step, at which $\mathrm{p}=\mathrm{p}_{0}$, is arbitrary since $\mathrm{p}_{0}$ is positive but arbitrary.

Consequently one can not identify the «sequential approach» with a nonequilibrium conception of the real economic process. For the same reasons one

[^1]can not identify the «simultaneist concept» with a Walrasian equilibristic conception of the real economic process. Thus, economists like Ricardo and Marx, who use the «simultaneist concept» are of course not for this reason «equilibrists». And economists, who use the «sequential approach» are of course not for this reason «non-equilibrists».

Finally, the «sequential approach» is not quite new (see Brody 1974, pp. 88-91). It was known from the Input-Output-Analysis (see Tu 1994, pp. 130131) and from the soviet planning as an iterative approach of the determination not of the prices but of the commodity quantities to be produced.

The problem of the soviet planners was the following: To find out the gross product X that corresponds to the planned and hence known positive or semipositive net product $\mathrm{Y}, \mathrm{Y} \geq 0$. For the net product Y holds

$$
\begin{aligned}
& Y=B x-A x=(B-A) x \Rightarrow \\
& x=(B-A)^{-1} Y .
\end{aligned}
$$

And for the gross product X holds

$$
\mathrm{X}=\mathrm{Bx}=\mathrm{B}(\mathrm{~B}-\mathrm{A})^{-1} \mathrm{Y}
$$

The planners have suppose single production, i.e. $\mathrm{B}=\mathrm{I}$, and have obtain

$$
\begin{align*}
& \mathrm{X}=\mathrm{Bx}=\mathrm{Ix}=\mathrm{x} \\
& \mathrm{Y}=\mathrm{X}-\mathrm{AX}=(\mathrm{I}-\mathrm{A}) \mathrm{X} \tag{25}
\end{align*}
$$

and, because of $\mathrm{B}=\mathrm{I}$ and (1),

$$
\begin{equation*}
\mathrm{X}=(\mathrm{I}-\mathrm{A})^{-1} \mathrm{Y} \tag{26}
\end{equation*}
$$

The resolution of the problem, i.e. of (25), seems convenient. One merely must computer the matrix $(\mathrm{I}-\mathrm{A})^{-1}$. But matrix $(\mathrm{I}-\mathrm{A})^{-1}$ was of the order of $1200 \times 1200$ and the computers obtainable at that time have need many years at «real time» to computer this matrix. Thus, the planners have apply the iterative approach. They have start from

$$
\begin{align*}
& \mathrm{Y}=\mathrm{X}-\mathrm{AX} \Rightarrow  \tag{25}\\
& \mathrm{X}=\mathrm{Y}+\mathrm{AX} \tag{27}
\end{align*}
$$

and, putting in the right-hand side of (27)

$$
\begin{equation*}
\mathrm{X}=\mathrm{X}_{0}, \tag{28}
\end{equation*}
$$

where $X_{0}$,

$$
\begin{equation*}
X_{0}>0 \tag{29}
\end{equation*}
$$

is a positive but arbitrary $n \times 1$ vector, they have obtained

$$
\begin{equation*}
\mathrm{X}_{1}=\mathrm{Y}+\mathrm{AX}_{0} \tag{30}
\end{equation*}
$$

Afterwards, they have put (30) in the right-hand side of (27) and obtained

$$
\begin{aligned}
X_{2} & =Y+A X_{1}=Y+A\left(Y+A X_{0}\right)= \\
& =Y+A Y+A^{2} X_{0}
\end{aligned}
$$

and so on. Continuing this procedure they obtained

$$
\begin{equation*}
\mathrm{X}_{\mathrm{m}}=\left(\mathrm{I}+\mathrm{A}+\mathrm{A}^{2}+\ldots+\mathrm{A}^{\mathrm{m}-1}\right) \mathrm{Y}+\mathrm{A}^{\mathrm{m}} \mathrm{X}_{0} \tag{31}
\end{equation*}
$$

Because of $A \geq 0, B=I$ and (1), the maximum eigenvalue of the matrix $A$ is positive and less than unity. Consequently the matrix $\mathrm{A}^{m}$ converges with an increasing m towards the nXn zero-matrix and hence also the vector $\mathrm{A}^{\mathrm{m}} \mathrm{X}_{0}$ converges towards the $\mathrm{n} \times 1$ zero vector.

At an enough great $m$ is

$$
\begin{equation*}
\mathrm{X}_{\mathrm{m}} \cong\left(\mathrm{I}+\mathrm{A}+\mathrm{A}^{2}+\ldots+\mathrm{A}^{\mathrm{m}-1}\right) \mathrm{Y} \cong \mathrm{X} \tag{32}
\end{equation*}
$$

So, for computational reasons the soviet planners have apply in order to computer X, instead of (26), (31), i.e., instead of the «simultaneist concept», the «sequential approach» ${ }^{8}$.

This «sequential approach» of the soviet planners is the «dual» of the «sequential approach» of Alan Freeman and vice versa. Consequently it is not correct that Alan Freeman calls his «sequential approach» a «non-dualistic approach».

Finally, the «sequential approach» does not imply at any time different prices for a commodity as input and for the same commodity as output, as Alan Freeman maintains. Suppose that the gross product $X_{t-1}$ of the «period» $t-1$ (i.e. of the computational step $t-1$ ) contains the quantity $\mathrm{X}_{\mathrm{t}-1}^{(\mathrm{i})}$ of the commodity i , the part $\alpha \mathrm{X}_{\mathrm{t}-1}^{(\mathrm{i})}, 0<\alpha<1$, of which has been consumed during the «period» $\mathrm{t}-1$ and the rest $(1-\alpha) \mathrm{X}_{\mathrm{t}-1}^{(\mathrm{i})}$ of which enter as means of production into the production process of the next «period», i.e. of the «period» $t$. At the time point $t$, i.e. at the end of the «period» $t-1$ and at the beginning of the «period» $t$, the price of the commodity is $p_{t}^{(i)}, p_{t}^{(i)} \neq p_{t-1}^{(i)}$. The gross product $X_{t}$ of the «period» $t$ also contains the quantity $X_{t}^{(i)}$ of the commodity i. At the time point $t+1$, i.e. at the end of the «period» $t$ and at the beginning of the «period» $t+1$, the price

[^2]of the commodity i is $\mathrm{p}_{\mathrm{t}+1}^{(\mathrm{i})}, \mathrm{p}_{\mathrm{t}+1}^{(\mathrm{i})} \neq \mathrm{p}_{\mathrm{t}}^{(\mathrm{i})}$. Alan Freeman misconceives this fact, i.e. the fact that $p_{t+1}^{(\mathrm{i})} \neq \mathrm{p}_{\mathrm{t}}^{(\mathrm{i})}$, in other words the fact that the price $p^{(i)}$ of the commodity $i$ changes during the time, as if the price $\mathrm{p}_{\mathrm{t}}^{(\mathrm{i})}$ of the commodity i at the time point $t$ was the price of $i$ as an input and the price $p_{t+1}^{(i)}$ of the same commodity $i$ at the time point $t+1$ was the price of the same commodity $i$ as an output and as if both prices $\mathrm{p}_{\mathrm{t}}^{(\mathrm{i})}$ and $\mathrm{p}_{\mathrm{t}+1}^{(\mathrm{i})}$ hold at the same time point.

In Marx and Non-Equilibrium Economics (Freeman 1996) Alan Freeman carries over his «sequential approach» of determination of the production prices to the determination of the labour values.

What is the matter? As well known, it holds for the 1 ( n vector $\omega$ of the labour value

$$
\begin{equation*}
\omega \mathrm{B}=\omega \mathrm{A}+\ell . \tag{38}
\end{equation*}
$$

We have shown earliest (see Stamatis 1979, 1979a, 1980 and 1983) that in joint production systems the labour values $\omega$ are positive but not uniquely determined. Alan Freeman presupposes single production, i.e. $\mathrm{B}=\mathrm{I}$.

Consequently it holds, instead of (38)

$$
\begin{equation*}
\omega=\omega \mathrm{A}+\ell . \tag{39}
\end{equation*}
$$

We get from (39)

$$
\begin{equation*}
\omega=\ell(\mathrm{I}-\mathrm{A})^{-1}\left[=\ell\left(\mathrm{I}+\mathrm{A}+\mathrm{A}^{2}+\mathrm{A}^{3}+\ldots\right)\right](>0) \tag{40}
\end{equation*}
$$

Thus, taking into account $\mathrm{B}=\mathrm{I}$ and (1), it follows from (40) that the labour values $\omega$ are positive and uniquely determined.

But the computation of $\omega$ presupposes the computation of the matrix $(\mathrm{I}-\mathrm{A})^{-1}$. If the order of this matrix is so great that its computation is impossible or inconvenient, one can $\omega$ computer by using the following iterative procedure.

By starting from (39) and putting in the right-hand side of (39)

$$
\begin{equation*}
\omega=\omega_{0}, \tag{41}
\end{equation*}
$$

where $\omega_{0}$,

$$
\begin{equation*}
\omega_{0}>0, \tag{42}
\end{equation*}
$$

is a positive but arbitrary $1 \times n$ vector, one get from (39)

$$
\begin{equation*}
\omega_{1}=\omega_{0} \mathrm{~A}+\ell . \tag{43}
\end{equation*}
$$

Putting in the right-hand side of (39)

$$
\begin{equation*}
\omega=\omega_{1} \tag{44}
\end{equation*}
$$

and taking into account (43) one get from (39)

$$
\begin{align*}
\omega_{2} & =\omega_{1} \mathrm{~A}+\ell \\
& =\left(\omega_{0} \mathrm{~A}+\ell\right) \mathrm{A}+\ell \\
& =\omega_{0} \mathrm{~A}^{2}+\ell \mathrm{A}+\ell . \tag{45}
\end{align*}
$$

A.s.o. until one get

$$
\begin{align*}
\omega_{\mathrm{m}} & =\omega_{0} \mathrm{~A}^{\mathrm{m}}+\left(\ell+\ell \mathrm{A}+\ell \mathrm{A}^{2}+\ldots+\ell \mathrm{A}^{\mathrm{m}-1}\right)= \\
& =\omega_{0} \mathrm{~A}^{\mathrm{m}}+\ell\left(\mathrm{I}+\mathrm{A}+\mathrm{A}^{2}+\ldots+\mathrm{A}^{\mathrm{m}-1}\right) . \tag{46}
\end{align*}
$$

Since, because of $A \geq 0, B=I$ and (1), is $0<\lambda_{m}^{A}<1$, where $\lambda_{m}^{A}$ is the maximum eigenvalue of $A$, the matrix $A^{m}$ converges with increasing $m$ towards the $n \times n$ zero matrix and consequently the vector $\omega_{0} A^{m}$ also converges towards the 1 xn zero vector. Hence at an enough great m is

$$
\begin{equation*}
\omega_{\mathrm{m}} \cong \ell\left(\mathrm{I}+\mathrm{A}+\mathrm{A}^{2}+\ldots+\mathrm{A}^{\mathrm{m}-1}\right) . \tag{47}
\end{equation*}
$$

From (39) and (47) we get that at an enough great $m$ is

$$
\begin{equation*}
\omega_{\mathrm{m}} \neq \omega \tag{48}
\end{equation*}
$$

The above procedure is merely a computational one. As we have shown earlier (see Stamatis, 1988), this computational method has be presented in 1910 by Georg Charasoff (see Charasoff 1910).

Alan Freeman misconceives this computational procedure as an actual economic process which takes place in the real time. Hence the criticism which is exercised by Dyménil and Lévy (1997) upon the inferences of misconceiving the «sequential approach» of the labour value determination as a real economic process by Alan Freeman, is obviously correct.

## References

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[^0]:    1. The fact that one can conceive the incorrect Marxian determination of the production prices as the first computational step of a correct iterative determination (see Stamatis 1995) does not imply that the Marxian determination of the production prices is not a «sumultaneist concept» but now a «iterative», i.e. a «sequential approach».
    2. It is also a concept of determination of the production prices and the nominal wage rate for given profit rate or a concept of determination of the production prices for given nominal wage rate and given profit rate.
[^1]:    4. $d, d \geq 0$, is a $n \times 1$ column vector.
    5. In this case (11a) has $n$ solution. One choices the solution with $r \geq 0$ and $p>0$. This is the solution with the smallest non-negative arithmetical value for $r$.
    6. See footnote 5 .
    7. See footnote 5 .
[^2]:    8. The application of the above «sequential approach» is of course possible only if $\mathrm{B}=\mathrm{I}$, i.e. only if the production system is a single production system.
