POLITICAL ECONOMY - 13 - AUTUMN 2003 - p.p. 71 - 97

Evidence for Mixed non-Linearity in Daily Stock Exchange Series[1]

by

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It is a common use in practice to assume that linear models can describe the structure of time series. Indeed most of the theoretical models in finance need the linear assumption. This is the case for the CAPM, APT models... However, many aspects of economic behaviour may be non-linear: investors' attitudes toward risk and expected return, the terms of certain contracts like options and other derivatives securities, the strategic interactions among market participants, the way that information is incorporated into stock prices etc.

A large collection of non-linear models can be ranged into two broad categories. Models that are non-linear in mean and hence depart from the martingale hypothesis and models that are non-linear in variance and hence depart from independence including GARCH models. Many empirical studies have found statistical evidence for non-linearity in financial series. Examples include Engle (1982), Tong (1983), Hinich and Patterson (1985), Tsay (1986), Ashley and Patterson (1989), Kyrtsou et al. (2003), Kyrtsou and Terraza M.,(2002, 2003), Kyrtsou and Terraza V., (2000), Afonso and Teixeira (1998), Chauveau, Damon, and Guégan (1999), and Barnett et al. (1995) among others.

So, in view to model real data, questions arise about the stationarity and the linearity of the data, particularly if we decide to transform them before adjusting a specified model. The problem is that any transformation leads to lose information. In this paper, we adopt the approach of Ashley and Patterson (2000). Once any linear dependence is removed from the returns

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1. We would like to thank the professors Mm Estella Dagum (University of Bologna, Italy) and Mr Michel Terraza (University of Montpellier I, France) for their helpful comments.

series via a prewhitening model, any remaining serial dependence must be due to a non-linear generating mechanism. Tests for non-linearity are applied to the residuals of an AR(p) model. Then, we compare different non-linear tests in order to investigate non-linear behaviour on a given time series.

This methodology is applied to 12 international stock exchange series and two French stock series: France-Telecom and Alcatel. These are the most traded stocks on the French market during the period that we use in our empirical analysis. By comparison, several research papers are shown that the stocks are more volatile and asymetric than indices, but there are not sufficient studies to test the existence of non-linearities.

The plan of the paper is as follows. In section 2 we present the non-linearity tests while in section 3 we report and discuss the obtained empirical results. Section 4 concludes the article.

2. Research Methodology

2.1. *The BDS and Dechert tests*

Brock et al. (1987), Brock et al. (1996) and Dechert (1995) have proposed two tests of the i.i.d hypothesis based on the correlation integral. These tests compare the null hypothesis that a series is i.i.d against the alternative hypothesis that a series is linearly or non-linearly correlated. It is based on the statistics W^{BDS} (for the Brock et al. test) and W^{D} (for the Dechert test) as defined below:

$$W^{BDS} = \sqrt{T}\,\frac{D_{m,T}(\varepsilon)-(D_{1,T}(\varepsilon))^{m}}{\sigma_{m,T}(\varepsilon)} \xrightarrow{d} N(0,1)$$

$$W^{D} = \sqrt{T}\,\frac{S_{m,T}(\varepsilon_1,\varepsilon_2)}{\sigma_{T}(\varepsilon_1,\varepsilon_2)} \xrightarrow{d} N(0,1)$$

which means that W^{BDS} and W^{D} converge in distribution to N(0,1), where $S_{m,T}(e_1,e_2) = C_{m,T}(e_1,e_2) - C_T(e_1)C_T(e_2)$. Here $\sigma^2_{m,T}(e)$, $\sigma^2_T(e_1,e_2)$, and $D_{m,T}(.)$, $C_{m,T}(.)$ are reciprocally the asymptotic variances and the correlation integrals, given by Brock et al. (1987), Brock et al. (1996) and Dechert (1995). In our application we vary the embedding dimension m from 2 to 20, and we use $e = 0.5\sigma, 1\sigma, 1.5\sigma, 2\sigma$ for the BDS test and $\varepsilon_1 = \varepsilon$, and $\varepsilon_2 = 2\varepsilon_1$ for the Dechert test. σ is the standard deviation of the returns series.

Whatever the choice of m and given the value of ε, ε_1, and ε_2, we calculate the W^{BDS} and W^{D} statistics. The obtained values of $|W^{BDS}|$ and $|W^{D}|$ are to be compared with the theoretical value 1.96 of the normal distribution at the 5% level. If the estimated value is higher than 1.96, then the null hypothesis of independence in X is rejected. This rejection can result from:

1. Either a structure of dependence resulting from a stochastic linear process (e.g. ARMA), or
2. A structure of dependence issued from a nonlinear stochastic process (e.g. TAR, NMA, ARCH, GARCH, EGARCH), or
3. A structure of dependence issued from a nonlinear deterministic process (e.g. Hénon map, logistic equation, Mackey-Glass equation).

2.2. *The White neural network test*

White's neural network test for neglected non-linearity (White (1989, 1990)), uses a single hidden layer feed-forward neural network with additional direct connections from inputs to outputs (figure 1). The null hypothesis of interest specifies linearity in the mean relative to an information set. The performance of the test depends on the following M statistic:

$$M_T = \left(\left(T^{-1/2} \sum_{t=1}^{T} \Phi_t \hat{e}_t \right) \hat{W}_T^{-1} \left(T^{-1/2} \sum_{t=1}^{T} \Phi_t \hat{e}_t \right) \right)$$

where $\hat{e}_t$ are the estimated residuals of the linear model, $\Phi_t = (\Psi(\tilde{x}_t' \Gamma_1), \ldots\ldots, (\Psi(\tilde{x}_t' \Gamma_q))'$, where Ψ is an activation function. In this case, the logistic $\Psi(\lambda) = (1+e^{-\lambda})^{-1}$, $\lambda \in \Re$, is used. $\Gamma = (\Gamma_1, \ldots, \Gamma_q)$ (hidden unit activations vector) is chosen a priori, independently of the sequence $\{x_t\}$, for given $q \in N$ (Lee et al. (1993)). $\hat{W}_t$ is a consistent estimator of $W^* = \mathrm{var}\left(T^{-1/2} \sum_{t=1}^{T} \Phi_t e_t^* \right)$.

Implementing the test as a Lagrange multiplier test requires the following hypothesis formulation:

$$H_0 : E(\Phi_t e_t^*) = 0 \quad \text{vs} \quad H_\alpha : E(\Phi_t e_t^*) \neq 0$$

For the case where M_T is asymptotically $\chi^2(q)$ under the null as $T \to \infty$, Bonferroni bounds provide an upper limit on the p-value. If $p_1, \ldots, p_k$ denote the ascending-ordered p-values corresponding to k draws from Γ, then the simple Bonferroni implies rejection of a linear null at the $100\alpha\%$ level if $p_i \leq \alpha/k$ for all i, so that, in the limit, the simple Bonferroni p-value is given by $\alpha = kp_k$. Hochberg (1988) suggests a modification to the Bonferroni method, which allows consideration of the p-values rather than just the largest, which may have led to a loss of power. The modified Hochberg-Bonferroni limit is given by $\alpha = \min_{i=1,\ldots,k}(k-i+1)p_i$, so that H_0 is rejected if there exists an i such as $p_i \leq \alpha/(k-i+1)$, $i = 1, \ldots, k$.

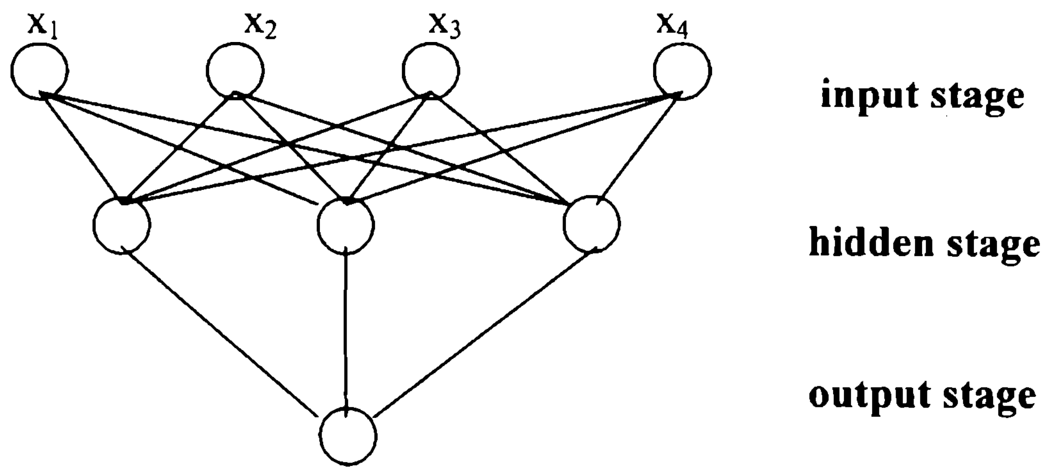

Figure 1
Single hidden-layer feedforward network.

2.3. *Hinich bicovariance test*

This test assumes that X_t is a realisation from a third-order stationary stochastic process and tests for serial independence using the sample bicovariances of the data. The (r, s) sample bicovariance is defined as:

$$C_3(r,s) = (T-s)^{-1} \sum_{t=1}^{T-s} X_t X_{t+r} X_{t+s} \quad \text{with } 0 \leq r \leq s, \text{ and t the number of observations}$$

The sample bicovariances are thus a generalisation of a skewness parameter. The $C_3(r,s)$ are all zero for zero mean, serially i.i.d. data. One would expect non-zero values for the $C_3(r,s)$ from data in which X_t depends on lagged crossproducts, such as $X_{t-i}X_{t-j}$ and higher order terms.

Let $G(r,s) = (T-s)^{0.5} C_3(r,s)$ and define X_3 as

$$X_3 = \sum_{s=2}^{\zeta} \sum_{r=1}^{s-1} [G(r,s)]^2$$

Under the null hypothesis that X_t is a serially i.i.d. process, Hinich and Patterson (1995) show that X_3 is asymptotically distributed $\chi^2(\zeta[\zeta-1]/2)$ for $\zeta < T^{0.5}$. They recommend using $\zeta = T^{0.4}$. Under the assumption that $E(X_t^{12})$ exists, the X_3 statistic detects non-zero third order correlations. It can be considered as a generalisation of the Box-Pierce portmanteau statistic.

2.4. *Hinich bispectrum test*

The Hinich bispectrum test is used to estimate the bispectrum of a stationary time series and provides a direct test for non-linearity and also test for Gaussianity[2]. If the process generating the data is linear then the skewness of the bispectrum will be consatnt. If the test rejects constant skewness then a non-linear process is implied.

Linearity and Gaussianity can be tested using a sample estimator of the skewness function $\Gamma(w_1, w_2)$ with:

$$\Gamma^2(w_1,w_2) = \frac{\left|B_{xxx}(w_1,w_2)\right|^2}{S_{xx}(w_1)\,S_{xx}(w_2)\,S_{xx}(w_1+w_2)}$$

where $S_{xx}(w)$ is the spectrum of X_t at frequency w. The bispectrum at frequency pairs (w_1, w_2) is defined as:

$$B_{xxx}(w_1,w_2) = \sum_{r=-\infty}^{\infty} \sum_{s=-\infty}^{\infty} C_{xxx}(r,s)\, e^{[-i2\pi(w_1 r + w_2 s)]}$$

in the principal domain given by
$\Omega = \{(w_1, w_2): 0 < w_1 < 0.5, w_2 < w_1, 2w_1 + w_2 < 1\}$.

The null of the Hinich linearity test is actually given by:

H_0 = flat skewness function, absence of third order non-linear dependence,
H_1 = non-linear dependence.

2. Details of the bispectrum estimation method can be found in Hinich and Patterson (1985), Ashley and Patterson (2000) and also in Barnett et al. (1995).

H_0 is rejected if the standard normal test statistic Z is superior to 1.96 for a significance level $\alpha = 5\%$. When the null is Gaussianity the related test statistic is denoted by H and is also a standard normal random variate under the null. The Hinich bispectral test has the property that it is unaffected by the application of a linear filter to X_t. This follows from the fact that the squared skewness function $\Gamma^2(w_1, w_2)$ is invariant to linear filtering.

2.5. *McLeod-Li and Engle tests*

The McLeod and Li (1983) portmanteau test for non-linear dependence is conducted by examining the Box-Ljung Q statistic of the squared residuals after filtering with an ARMA process. Instead of using the residuals from a linear representation, the raw data can be examined through the use of the k autocorrelation coefficients for $\{X_t\}$, $\{|X_t|\}$ and $\{X_t^2\}$. The Q statistic for each of these three transformed data series can be used to examine the presence of serial correlation. For example, Granger and Newbold (1986) have suggested that if $\rho(k) = \rho^2(k)$ for all k, then the time series X_t is linear.

Under the null hypothesis that the prewhitened series X_t is an i.i.d process McLeod and Li (1983) show that, for a fixed L:

$$T^{0.5}\rho^2(k) = (\rho^2(1), \ldots\ldots\ldots, \rho^2(L))$$

is asymptotically a multivariate unit normal. Consequently, for L sufficiently large, the usual Box-Ljung statistic

$$Q = T(T+2)\sum_{i=1}^{L}\frac{[\rho^2(k)]^2}{T-i}$$

is asymptotically $\chi^2(L)$ under the null hypothesis of a linear generating mechanism for the data.

Engle (1982) proposed a LaGrangian multiplier test that explicitly examines for non-linearity in the second moment. This test is presented below:

We perform a regression on X_t and calculate the residuals e_t. Then we perform the second regression, of the following form for a selected value of p:

$$e_t^2 = a_0 + \sum_i a_i e^2_{t-i} + u_t, \quad \text{for } i = 1, \ldots, p$$

Under the null hypothesis of a linear generating mechanism for X_t, TR^2 for this regression is asymptotically distributed $\chi^2(p)$.

2.6. *Tsay test*

The Tsay (1986) test is a generalization of the Keenan (1985) test. It explicitly looks for quadratic serial dependence in the data. While the Engle test examines evidence for non-linearity in the variance the Tsay test checks for non-linearity in the mean. The procedure to compute the test proposed by Tsay is as follows:

Let the $K = k(k-1)/2$ column vectors $V_1 \ldots V_K$ contain all the possible crossproducts of the form $X_{t-i} X_{t-j}$, where $i \in [1, k]$ and $j \in [i, k]$. Also let $v^*_{t,i}$ denote the projection of $v_{t,i}$ on the subspace orthogonal to $X_{t-1}, \ldots, X_{t-k}$, i.e. the residuals from a regression of $v_{t,i}$ on $X_{t-1}, \ldots, X_{t-k}$.

The parameters $\gamma_1 \ldots \gamma_K$ are then estimated by applying OLS to the regression equation:

$$X_t = \gamma_0 + \sum_{i=1}^{K} \gamma_i v^*_{t,i} + \eta_t$$

So long as p exceeds K, this projection is unnecessary for the dependent variable X_t since it is prewhitened using an AR(p) model. The Tsay test statistic is then the usual F statistic for testing the null hypothesis that $\gamma_1 \ldots \gamma_K$ are all zero, where it is assumed that $E(X_t^8)$ exists.

3. Empirical results

In our empirical analysis we use daily stock and index series (Table 1). We make preliminary transformation working with returns. Even though the results are not reported here, we find that asymmetric and leptokurtic distributions characterise our returns. As in Ashley and Patterson (2000) in order to test the robustness of Hinich test, we routinely bootstrap the significance levels, as well as computing them based on asymptotic theory. After prewhitening, we draw 1000 T samples at random from the empirical distribution of the observed T-sample of data. The Hinich, McLeod-Li, Engle and Tsay tests are implemented in Toolkit, a Windows-based computer program presented in Patterson and Ashley (2000).

The BDS test results, reported in Tables 3-16 reveal that for all the returns series the null of i.i.d is rejected with a $|W^{BDS}| > 1.96$. In order to apply BDS as test for non-linearity the data are prewhitened by Box Jenking estimation of an AR(p) model, as a means of removing linear dependence. In our implementation we choose the AR(p) model for which p minimizes the Schwartz (SC) criterion. The results are given in Table 2.

Table 1
Stock and Index series

Stock exchange	Index/Stock	Period	Sample
London	Ftse100	01/02/87 – 03/14/01	3703
Francfort	Dax30	01/02/87 – 03/14/01	3703
New York	SP500	01/02/87 – 03/14/01	3703
Milan	Mib30	10/21/94 – 03/14/01	1668
Helsinki	Hex	01/02/87 – 03/14/01	3703
Stockholm	Affwall	01/02/87 – 03/14/01	3703
Zurich	Swissmi	07/01/88 – 03/14/01	3313
Amsterdam	Eoe	01/02/87 – 03/14/01	3703
Madrid	Madridi	01/02/87 – 03/14/01	3703
Tokyo	Nikkey	01/02/87 – 03/14/01	3703
Paris	CAC40	09/11/87 – 03/30/01	3525
Athens	Giase	10/17/86 – 04/25/01	3577
Paris	France Telecom	10/20/97 – 02/10/00	763
Paris	Alcatel	10/20/97 – 02/10/00	763

Table 2
Model selection for the returns series

Series	Returns	Model
Ftse100	dlftse	AR(1)
Dax30	dldax	-
SP500	dlsp500	-
MIB30	dlmib	-
Hex	dlhex	AR(1)
Affwall	dlaff	AR(1)
Swissmi	dlswiss	-
Eoe	dleoe	-
Madridi	dlmadrid	AR(1)
Nikkey	dlnikkey	AR(2)
CAC40	dlcac40	-
Giase	dlgiase	AR(2)
France Telecom	dlfrt	AR(3)
Alcatel	dlcac	-

After filtering, the BDS results do not change[3]. The $|W^{BDS}|$ statistic is widely superior to 1.96 for $\alpha = 5\%$. Concerning the Dechert test (Tables 17-30), the conclusions are identical with those of BDS test. Nevertheless, the value of the W^D statistic remains significant but it is less high. This is because the Dechert test is more robust when noise is present in the data.

The White test results displayed in Table 31, provide clear evidence against the hypothesis of linearity in the mean for all of the returns series except the France-Telecom returns series. The value of the test statistic exceeds significantly the critical value for $\alpha = 5\%$ (i.e. $\chi^2(4) = 9.48$). The modified Hochberg-Bonferroni limit confirms these findings. Only for the France Telecom returns series the zero hypothesis (linearity in mean) is accepted.

Tables 32-45 report the results of the Hinich, McLeod-Li, Engle and Tsay tests with prewhitening. We remind that the Hinich bispectral test has the property that it is unaffected by the application of a linear filter to returns series. Except the case of the Hinich bispectral (for all series) and the Tsay tests (only for the France Telecom returns series[4]), the tests appear to have high power to detect non-linearity in the data. Ashley and Patterson (2000) provided evidence for the differential power of these tests to detect non-linearity of the various forms (ARCH/GARCH, TAR, Markov Switching etc.) proposed in the literature. They also found that except for the Hinich bispectral test, all of the tests reject linearity in the data.

4. Implications

The implications of our findings in favour of mixed non-linearity are very interesting, especially in the context of financial markets. Taking the complex behaviour in stock markets into account, Kyrtsou and Terraza (2003) have shown that it is more robust than the traditional stochastic approach to model the observed data by a non-linear chaotic model disturbed by dynamic noise. Such a model, having almost no autocorrelations in returns but significant picks in absolute and squared returns, may provide a good explanation of the unpredictability of the first moment of asset returns and the remaining

3. The results are available upon request
4. This does not mean that the France Telecom returns series is linear. Taking into account the Mcleod-Li and Engle tests results we can conclude in favour of a non-linear in variance underlying process.

structure of the second moment. In fact, they construct a mixed non-linear model having negligible or even zero autocorrelations in the conditional mean (corresponding to the deterministic component), but a rich structure in the conditional variance (characterising the stochastic component). The model is a noisy Mackey-Glass equation with errors that follow a GARCH(p,q) process (MG-GARCH(p,q)). It permits us to capture volatility-clustering phenomena, according to which stock-price fluctuations are characterised by episodes of low volatility with small price changes irregularly mixed by episodes of high volatility with large price changes.

APPENDIX

Table 3

BDS test results for dlftse

ε/σ	**0.5**	**1**	**1.5**	**2**
m=2	7.9348	6.0187	5.9395	5.8467
m=3	10.1481	8.6053	8.3062	8.6318
m=4	16.370	11.298	9.9417	10.768
m=5	17.858	12.909	11.595	12.518
m=10	-	-	30.262	22.189
m=15	-	-	-	-
m=20	-	-	-	-

Table 4

BDS test results for dldax

ε/σ	**0.5**	**1**	**1.5**	**2**
m=2	10.403	10.212	10.228	10.387
m=3	12.814	13.559	13.785	14.232
m=4	19.169	17.463	16.372	17.242
m=5	35.613	21.980	19.558	20.612
m=10	-	-	122.30	82.181
m=15	-	-	-	1450.30
m=20	-	-	-	6974.50

Table 5

BDS test results for dlsp500

ε/σ	**0.5**	**1**	**1.5**	**2**
m=2	4.2908	4.7334	5.4367	5.9502
m=3	6.4704	7.6142	8.4962	9.1871
m=4	7.9410	10.331	11.011	11.592
m=5	9.4256	13.497	14.109	14.450
m=10	-	37.075	43.327	37.233
m=15	-	-	168.30	120.90
m=20	-	-	288.85	515.17

Table 6

BDS test results for dlmib

ε/σ	**0.5**	**1**	**1.5**	**2**
m=2	4.0774	3.8545	4.159	4.2944
m=3	6.4896	8.2429	6.9509	6.8541
m=4	11.208	14.766	8.915	9.094
m=5	-9.5453	13.348	10.835	11.329
m=10	-	-	-3.429	-3.7315
m=15	-	-	-	-
m=20	-	-	-	-

Table 7

BDS test results for dlhex

ε/σ	**0.5**	**1**	**1.5**	**2**
m=2	25.794	26.201	25.570	24.901
m=3	39.840	37.664	34.802	32.478
m=4	60.219	52.875	45.582	40.267
m=5	101.16	79.396	61.805	50.618
m=10	4639.10	1334.30	471.95	215.02
m=15	-	6131.20	7838	1609.30
m=20	-	42083	19694	16972

Table 8

BDS test results for dlaff

ε/σ	**0.5**	**1**	**1.5**	**2**
m=2	16.728	16.705	16.545	16.456
m=3	21.488	21.490	21.076	21.058
m=4	26.745	27.565	26.503	26.133
m=5	35.171	38.417	34.868	33.848
m=10	-4.422	-5.0730	205.18	170.09
m=15	-	-	-	1487.10
m=20	-	-	-	-

Table 9

BDS test results for dlswiss

ε/σ	**0.5**	**1**	**1.5**	**2**
m=2	11.227	9.3314	9.337	9.6204
m=3	16.686	12.337	12.162	12.263
m=4	18.620	13.861	14.034	14.057
m=5	21.211	16.317	15.730	15.709
m=10	-4.4680	-5.2019	8.7861	29.485
m=15	-	-	-	-
m=20	-	-	-	-

Table 10

BDS test results for dleoe

ε/σ	**0.5**	**1**	**1.5**	**2**
m=2	9.6861	9.7098	10.108	10.843
m=3	12.252	12.734	13.307	14.387
m=4	14.185	15.014	15.632	17.076
m=5	21.251	18.034	18.150	20.100
m=10	-4.3155	118.82	31.609	48.156
m=15	-	-	-	247.19
m=20	-	-	-	1487.20

Table 11

BDS test results for dlmadrid

ε/σ	**0.5**	**1**	**1.5**	**2**
m=2	10.169	11.148	11.077	11.762
m=3	14.107	15.752	15.793	16.455
m=4	18.704	21.440	22.513	22.973
m=5	20.112	28.091	31.530	31.111
m=10	-3.1017	-4.0908	283.70	205.10
m=15	-	-	-	5863.60
m=20	-	-	-	-

Table 12

BDS test results for dlnikkei

ε/σ	**0.5**	**1**	**1.5**	**2**
m=2	6.9656	8.9068	9.7783	10.539
m=3	11.111	12.938	14.353	15.032
m=4	14.761	16.648	18.130	18.536
m=5	16.540	22.024	23.493	23.526
m=10	-	132.97	111.66	92.844
m=15	-	-	1090.60	687.59
m=20	-	-	14296	7683.80

Table 13

BDS test results for dlcac40

ε/σ	**0.5**	**1**	**1.5**	**2**
m=2	4.6131	3.9939	4.3996	4.430
m=3	7.0507	5.8115	6.0041	5.7346
m=4	7.8408	6.1944	7.1198	7.0897
m=5	14.207	7.1279	8.5488	8.6159
m=10	-3.7210	-4.9410	-5.6387	15.20
m=15	-	-	-	-
m=20	-	-	-	-

Table 14

BDS test results for dlgiase

ε/σ	**0.5**	**1**	**1.5**	**2**
m=2	25.921	26.029	26.281	26.044
m=3	39.575	37.941	35.890	33.651
m=4	57.033	51.199	45.458	40.377
m=5	83.803	69.074	56.987	47.691
m=10	877.18	468.23	236.82	129.35
m=15	-	4898.90	1440.60	467.85
m=20	-	55123	10619	2050

Table 15

BDS test results for dlfrt

ε/σ	**0.5**	**1**	**1.5**	**2**
$m=2$	5.8421	4.9595	4.9739	4.4185
$m=3$	9.0889	7.1735	7.9207	6.8869
$m=4$	14.622	10.905	11.116	9.0452
$m=5$	13.184	14.362	12.901	10.072
$m=10$	-	-	79.110	33.204
$m=15$	-	-	165.87	162.34
$m=20$	-	-	-	-

Table 16

BDS test results for dlalc

ε/σ	**0.5**	**1**	**1.5**	**2**
$m=2$	5.0289	5.1744	5.2661	5.6723
$m=3$	5.2910	5.8051	5.8278	6.1005
$m=4$	6.5017	7.0537	6.7223	6.7693
$m=5$	8.2433	8.4695	7.6879	7.3226
$m=10$	26.650	20.322	12.008	8.8560
$m=15$	-	60.083	18.463	10.298
$m=20$	-	339.25	32.803	12.940

Table 17

Dechert test results for dlftse

$\varepsilon_1=\varepsilon/\sigma$ **and** $\varepsilon_2=2\varepsilon/\sigma$	**0.5**	**1**	**1.5**	**2**
$m=2$	9.1095	10.096	6.7342	13.289
$m=3$	8.3352	9.9210	8.0381	10.863
$m=4$	7.8867	9.5805	7.2245	9.7281
$m=5$	7.4632	8.9116	8.0012	9.1722
$m=10$	9.0166	9.5361	7.1356	8.2134
$m=15$	7.9476	8.6170	5.0748	6.4836
$m=20$	6.6523	6.3599	4.5084	4.890

Table 18

Dechert test results for dldax

$\varepsilon_1=\varepsilon/\sigma$ and $\varepsilon_2=2\varepsilon/s$	**0.5**	**1**	**1.5**	**2**
m=2	13.073	13.650	13.028	13.174
m=3	13.741	12.452	10.175	9.1223
m=4	11.151	11.713	11.134	10.360
m=5	14.096	13.708	12.188	9.9765
m=10	11.837	11.684	8.6866	5.8333
m=15	10.544	10.481	9.4476	8.1910
m=20	9.7017	9.4957	7.0496	4.9224

Table 19

Dechert test results for dlsp500

$\varepsilon_1=\varepsilon/\sigma$ and $\varepsilon_2=2\varepsilon/\sigma$	**0.5**	**1**	**1.5**	**2**
m=2	9.0852	10.689	10.602	12.145
m=3	9.1332	9.6329	9.5986	9.7447
m=4	9.5769	9.2213	8.9540	9.4738
m=5	12.038	11.786	10.879	10.995
m=10	10.807	10.177	8.7826	7.5752
m=15	9.2631	9.8362	8.8881	6.7835
m=20	9.0168	9.2459	8.3415	7.1454

Table 20

Dechert test results for dlmib

$\varepsilon_1=\varepsilon/\sigma$ and $\varepsilon_2=2\varepsilon/\sigma$	**0.5**	**1**	**1.5**	**2**
m=2	7.4503	8.9260	10.089	10.952
m=3	6.5499	7.2650	7.7752	7.8379
m=4	5.6956	6.5665	6.6934	7.6723
m=5	5.7974	7.0002	9.1231	11.697
m=10	4.4986	6.0048	8.2760	10.795
m=15	3.2086	5.4348	6.6807	8.600
m=20	2.3289	3.1897	3.6856	4.3885

Table 21

Dechert test results for dlhex

$\varepsilon_1=\varepsilon/\sigma$ and $\varepsilon_2=2\varepsilon/\sigma$	**0.5**	**1**	**1.5**	**2**
m=2	19.114	18.507	15.095	10.740
m=3	18.688	17.904	14.864	10.550
m=4	20.435	16.768	12.865	10.500
m=5	17.520	16.909	13.620	10.175
m=10	17.389	17.952	15.427	12.881
m=15	16.432	16.713	15.237	15.799
m=20	16.693	15.725	15.103	15.828

Table 22

Dechert test results for dlaff

$\varepsilon_1=\varepsilon/\sigma$ and $\varepsilon_2=2\varepsilon/\sigma$	**0.5**	**1**	**1.5**	**2**
m=2	13.191	13.620	12.091	10.941
m=3	13.832	15.608	14.131	12.506
m=4	14.333	14.147	12.209	11.878
m=5	15.178	15.161	12.692	12.028
m=10	13.573	14.553	12.142	8.7593
m=15	12.133	12.881	10.887	9.9617
m=20	10.357	10.486	7.8863	6.2571

Table 23

Dechert test results for dlswiss

$\varepsilon_1=\varepsilon/\sigma$ and $\varepsilon_2=2\varepsilon/\sigma$	**0.5**	**1**	**1.5**	**2**
m=2	10.423	12.564	12.552	11.033
m=3	10.320	11.710	10.283	8.0765
m=4	10.631	12.040	11.603	9.7438
m=5	10.604	11.974	11.766	10.051
m=10	8.9359	9.920	9.0335	7.0034
m=15	8.7422	9.3883	8.3628	7.6596
m=20	6.6068	6.6507	5.6589	5.0977

Table 24

Dechert test results for dleoe

$\varepsilon_1=\varepsilon/\sigma$ and $\varepsilon_2=2\varepsilon/\sigma$	**0.5**	**1**	**1.5**	**2**
$m=2$	14.123	16.061	15.269	15.811
$m=3$	11.979	14.144	13.518	11.948
$m=4$	14.202	16.663	15.283	13.856
$m=5$	10.785	12.507	12.917	11.604
$m=10$	9.7320	11.752	11.834	10.293
$m=15$	10.487	11.936	11.312	10.106
$m=20$	10.732	11.192	9.1406	6.2572

Table 25

Dechert test results for dlmadrid

$\varepsilon_1=\varepsilon/\sigma$ and $\varepsilon_2=2\varepsilon/\sigma$	**0.5**	**1**	**1.5**	**2**
$m=2$	10.853	11.092	11.103	11.042
$m=3$	11.285	12.514	11.145	8.2924
$m=4$	10.432	10.885	11.053	11.504
$m=5$	11.518	10.860	9.8947	8.5121
$m=10$	11.025	10.636	8.9384	8.3388
$m=15$	9.5529	9.6069	8.3229	6.3864
$m=20$	9.8836	8.8481	7.1179	5.4532

Table 26

Dechert test results for dlnikkei

$\varepsilon_1=\varepsilon/\sigma$ and $\varepsilon_2=2\varepsilon/\sigma$	**0.5**	**1**	**1.5**	**2**
$m=2$	12.807	12.079	9.9370	7.5471
$m=3$	10.254	10.550	10.322	8.0106
$m=4$	13.067	12.495	9.5907	8.0548
$m=5$	10.540	10.519	9.2866	8.6706
$m=10$	10.167	9.3215	7.7832	7.8718
$m=15$	9.9135	9.2182	6.9242	6.3426
$m=20$	7.6435	7.8424	7.4130	7

Table 27

Dechert test results for dlcac40

$\varepsilon_1=\varepsilon/\sigma$ and $\varepsilon_2=2\varepsilon/\sigma$	**0.5**	**1**	**1.5**	**2**
m=2	5.4674	8.4732	10.998	12.021
m=3	7.3813	8.8087	9.6432	9.6037
m=4	5.6117	6.0981	7.2572	8.1902
m=5	5.9699	6.9813	8.3678	9.3453
m=10	8.3517	10.521	12.222	13.338
m=15	6.6209	8.5405	9.8366	11.788
m=20	5.6597	6.3164	7.5656	10.114

Table 28

Dechert test results for dlgiase

$\varepsilon_1=\varepsilon/\sigma$ and $\varepsilon_2=2\varepsilon/\sigma$	**0.5**	**1**	**1.5**	**2**
m=2	21.870	17.913	13.943	11.128
m=3	19.859	16.890	11.659	8.3956
m=4	18.142	14.539	10.654	7.9839
m=5	15.809	12.982	9.6133	6.6264
m=10	12.900	9.7775	6.8964	6.4029
m=15	11.683	8.8552	5.7033	6.5078
m=20	11.517	9.5223	6.3241	3.6148

Table 29

Dechert test results for dlfrt

$\varepsilon_1=\varepsilon/\sigma$ and $\varepsilon_2=2\varepsilon/\sigma$	**0.5**	**1**	**1.5**	**2**
m=2	6.3524	6.4601	5.9109	5.9248
m=3	4.6269	2.6792	3.3970	4.8857
m=4	3.5640	2.9633	2.0979	3.2664
m=5	5.9398	4.6907	3.6145	3.3535
m=10	3.2837	2.8894	3.8777	7.4701
m=15	3.8939	2.8411	2.3922	4.0278
m=20	4.6001	4.1205	6.1459	10.966

Table 30
Dechert test results for dlalc

$\varepsilon_1=\varepsilon/\sigma$ **and** $\varepsilon_2=2\varepsilon/\sigma$	**0.5**	**1**	**1.5**	**2**
m=2	2.1804	2.0157	1.7181	3.0321
m=3	3.3994	2.4835	2.5110	4.0035
m=4	4.6425	3.2447	3.1157	4.3362
m=5	4.5761	3.7393	4.4437	5.8064
m=10	4.0872	3.2749	2.9184	4.2318
m=15	4.5351	3.9095	4.9463	6.6647
m=20	1.7930	1.6675	2.4842	4.0005

Table 31
White test results

Series	**White test**	**H-B limit**
Dlftse	161.80219	0.000
Dldax	39.78820	0.000
Dlsp500	58.43802	0.000
Dlmib	21.09503	0.000
Dlhex	98.38109	0.000
Dlaff	71.49888	0.000
Dlswiss	47.27658	0.000
Dleoe	101.7685	0.000
Dlmadrid	52.36863	0.000
Dlnikkei	48.29899	0.000
Dlcac40	19.17260	0.001
Dlgiase	61.91074	0.000
Dlfrt	2.84862	0.590
Dlalc	78.08702	0.000

Table 32

Significance levels for the Hinich, McLeod-Li, Engle and Tsay tests (dlftse)

Tests	Bicovariance (ζ=26)	(M=138) Bispectral Gaussianity	Linearity	McLeod-Li (L=24)	Engle (p=5)	Tsay (k=5)
Bootstrap	0.000	0.962	0.324	0.000	0.000	0.000
Asymptotic	0.000	0.000	0.000	0.000	0.000	0.000

Table 33

Significance levels for the Hinich, McLeod-Li, Engle and Tsay tests (dldax)

Tests	Bicovariance (ζ=26)	(M=138) Bispectral Gaussianity	Linearity	McLeod-Li (L=24)	Engle (p=5)	Tsay (k=5)
Bootstrap	0.000	0.983	0.274	0.000	0.000	0.001
Asymptotic	0.000	0.000	0.000	0.000	0.000	0.000

Table 34

Significance levels for the Hinich, McLeod-Li, Engle and Tsay tests (dlsp500)

Tests	Bicovariance (ζ=26)	(M=138) Bispectral Gaussianity	Linearity	McLeod-Li (L=24)	Engle (p=5)	Tsay (k=5)
Bootstrap	0.000	0.986	0.613	0.006	0.001	0.000
Asymptotic	0.000	0.000	0.000	0.000	0.000	0.000

Table 35

Significance levels for the Hinich, McLeod-Li, Engle and Tsay tests (dlmib)

Tests	Bicovariance (ζ=19)	(M=86) Bispectral Gaussianity	Linearity	McLeod-Li (L=24)	Engle (p=5)	Tsay (k=5)
Bootstrap	0.000	0.002	0.008	0.000	0.000	0.000
Asymptotic	0.000	0.000	0.000	0.000	0.000	0.000

Table 36

Significance levels for the Hinich, McLeod-Li, Engle and Tsay tests (dlhex)

Tests	Bicovariance (ζ=26)	(M=138) Bispectral Gaussianity	Linearity	McLeod-Li (L=24)	Engle (p=5)	Tsay (k=5)
Bootstrap	0.000	0.869	0.360	0.000	0.000	0.000
Asymptotic	0.000	0.000	0.000	0.000	0.000	0.000

Table 37

Significance levels for the Hinich, McLeod-Li, Engle and Tsay tests (dlaff)

Tests	Bicovariance (ζ=26)	(M=138) Bispectral Gaussianity	Linearity	McLeod-Li (L=24)	Engle (p=5)	Tsay (k=5)
Bootstrap	0.000	0.864	0.459	0.000	0.000	0.000
Asymptotic	0.000	0.000	0.000	0.000	0.000	0.000

Table 38

Significance levels for the Hinich, McLeod-Li, Engle and Tsay tests (dlswiss)

Tests	Bicovariance (ζ=25)	(M=129) Bispectral Gaussianity	Linearity	McLeod-Li (L=24)	Engle (p=5)	Tsay (k=5)
Bootstrap	0.000	0.911	0.246	0.000	0.001	0.000
Asymptotic	0.000	0.000	0.000	0.000	0.000	0.000

Table 39

Significance levels for the Hinich, McLeod-Li, Engle and Tsay tests (dleoe)

Tests	Bicovariance (ζ=26)	(M=138) Bispectral Gaussianity	Linearity	McLeod-Li (L=24)	Engle (p=5)	Tsay (k=5)
Bootstrap	0.000	0.983	0.375	0.000	0.000	0.000
Asymptotic	0.000	0.000	0.000	0.000	0.000	0.000

Table 40

Significance levels for the Hinich, McLeod-Li, Engle and Tsay tests (dlmadrid)

Tests	Bicovariance (ζ=26)	(M=138) Bispectral Gaussianity	Linearity	McLeod-Li (L=24)	Engle (p=5)	Tsay (k=5)
Bootstrap	0.000	0.822	0.369	0.000	0.000	0.000
Asymptotic	0.000	0.000	0.000	0.000	0.000	0.000

Table 41

Significance levels for the Hinich, McLeod-Li, Engle and Tsay tests (dlnikkei)

Tests	Bicovariance (ζ=26)	(M=138) Bispectral Gaussianity	Linearity	McLeod-Li (L=24)	Engle (p=5)	Tsay (k=5)
Bootstrap	0.000	0.816	0.374	0.000	0.001	0.000
Asymptotic	0.000	0.000	0.000	0.000	0.000	0.000

Table 42

Significance levels for the Hinich, McLeod-Li, Engle and Tsay tests (dlcac40)

Tests	Bicovariance (ζ=26)	(M=134) Bispectral Gaussianity	Linearity	McLeod-Li (L=24)	Engle (p=5)	Tsay (k=5)
Bootstrap	0.000	0.842	0.359	0.000	0.000	0.000
Asymptotic	0.000	0.000	0.000	0.000	0.000	0.000

Table 43

Significance levels for the Hinich, McLeod-Li, Engle and Tsay tests (dlgiase)

Tests	Bicovariance (ζ=26)	(M=136) Bispectral Gaussianity	Linearity	McLeod-Li (L=24)	Engle (p=5)	Tsay (k=5)
Bootstrap	0.000	0.915	0.425	0.000	0.000	0.000
Asymptotic	0.000	0.000	0.000	0.000	0.000	0.000

Table 44

Significance levels for the Hinich, McLeod-Li, Engle and Tsay tests (dlfrt)

Tests	Bicovariance (ζ=14)	(M=54) Bispectral Gaussianity	Linearity	McLeod-Li (L=24)	Engle (p=20)	Tsay (k=5)
Bootstrap	0.008	0.400	0.339	0.017	0.021	0.513
Asymptotic	0.000	0.000	0.006	0.000	0.001	0.549

Table 45

Significance levels for the Hinich, McLeod-Li, Engle and Tsay tests (dlalc)

Tests	Bicovariance (ζ=14)	(M=54) Bispectral Gaussianity	Linearity	McLeod-Li (L=1)	Engle (p=1)	Tsay (k=5)
Bootstrap	0.015	0.852	0.141	0.015	0.020	0.008
Asymptotic	0.000	0.000	0.000	0.047	0.051	0.000

References

Afonso, A., and Teixeira, J., (1998): Non-linear tests of weakly efficient markets: evidence from Portugal, *Working Paper*, Instituto Superior de Economia e Gestao, Portugal.

Ashley, R., and Patterson, D.M., (2000): A non-linear time series workshop: a toolkit for detecting and identifying nonlinear serial dependence , Kluwer Academic Publishers: Norwell.

Ashley, R., and Patterson, D.M., (1989): Linear versus nonlinear macroeconomics, *International Economic Review*. 30, pp. 685-704.

Barnett, W.A., Gallant, A.A., Hinich, M.J., Jungeilges, J., Kaplan, D., and Jensen M.J., (1995): Robustness of nonlinearity and chaos tests to measurement error, inference method, and sample size, *Journal of Economic Behavior and Organization*, 27, pp. 301-320.

Brock, W.A, Dechert, W.D., and Scheinkman, J.A., (1987): A test for independence based on the correlation dimension, *Working Paper*, University of Wisconsin.

Brock, W.A, Dechert, W.D., and Scheinkman, J.A., and LeBaron, B., (1996): A test for independence based on the correlation dimension, *Econometric Review*, 15, pp. 197-235.

Brock, W.A, Hsieh, D.A., and LeBaron, B., (1992): *Nonlinear dynamics, chaos and instability*, MIT Press, Cambrige, 328 pages, second edition.

Chauveau, T., Damon, J. and Guégan, D., (1999): Study of Alcatel and France-Telecom stocks regarding to non-linearity, *Working Paper*, Caisse des Dépôts et Consignations, Paris.

Dechert, W.D., (1995): An application of chaos theory to stochastic and deterministic observations, *Working Paper*, University of Houston.

Denker, G., and Keller, G., (1986) Rigorous statistical procedures for data from dynamical systems, *Journal of Statistical Physics*, 44, pp. 67-93.

Engle, R.F., (1982): Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation, *Econometrica* 50, pp. 987-1007.

Girerd-Potin, I., and Taramasco, O., (1994): Les rentabilités à la Bourse de Paris sont-elles chaotiques?, *Révue Economique*, Vol. 45, n°2, pp. 215-238.

Granger, C.W.J., and Newbold, P. (1986): *Forecasting Economic Time series*, 2nd edition, New York: Academic Press.

Hinich, M,. and Patterson, D.M., (1985): Evidence of nonlinearity in daily stock returns, *Journal of Business and Economic Statistics* 3, pp. 69-77.

Hinich, M,. and Patterson, D.M., (1995): Detecting epochs of transient dependence in white noise, *Working Paper*,, University of Texas at Austin.

Hochberg, Y., (1988): A sharper Bonferroni procedure for multiple tests of significance, *Biometrica*, 75, pp. 800-802.

Hsieh, D.A., (1989): Testing nonlinear dependence in daily foreign exchange rates, *Journal of Business*, 62, No 3., 339-368.

Keenan, D.M., (1985): A Tukey nonadditivity-type test for time series non-linearity, *Biometrica* 72, pp. 39-44.

Kyrtsou, C., W. Labys, and Terraza, M., (2003): Noisy chaotic dynamics in commodity markets, forthcoming in *Empirical Economics*.

Kyrtsou, C., and Terraza, M., (2002): Stochastic chaos or ARCH effects in stock series? A comparative study, *International Review of Financial Analysis*, 11, pp. 407-431.

Kyrtsou, C., and Terraza, M., (2003): It is possibly to study chaotic and ARCH behaviour jointly? Application of a noisy Mackey-Glass equation with heteroskedastic errors to the Paris Stock Exchange returns series, *Computational Economics*, 21, pp. 257-276.

Kyrtsou, C., and Terraza, V.,(2000): Volatility behaviour in emerging markets: a case study of the Athens Stock Exchange, using daily and intra-daily data, *European Research Studies*, Vol. 3, issue 3-4, pp. 3-15.

Lee, T.-H., White, H., and Granger, C.W.J., (1993): Testing for neglected nonlinearity in time series models, *Journal of Econometrics*, 56, pp. 269-290.

Liu, T., Granger, C.W.J, Heller, W.P., (1992): Using the correlation exponent to decide whether an economic series is chaotic, *Journal of Applied Econometrics*, Vol. 7, pp. 25-39.

McLeod, A.I., and Li, W.K., (1983): Diagnostic checking ARMA time series models using squared-residuals autocorrelations, *Journal of Time Series Analysis*, 4, pp. 269-273.

Pollard, D., (1984): *Convergence of stochastic processes*, Springer-Verlag, New York.

Tsay, R.S., (1986): Nonlinearity tests for time series, *Biometrica* 73, pp. 461-466.

Tong, H., (1983): *Threshold models in non-linear time series analysis*, Springer-Verlag: New York.

White, H., (1989): Some asymptotic results for learning in single hidden layer feedforward networks models, *Journal of the American Statistical Association*, Vol. 84, pp. 1003-1013.

White, H., (1990): Connectionist nonparametric regression: multilayer feedforward networks can learn arbitrary mappings, *Neural Networks*, 3, pp.535-550.

Abstract

We show that a battery of well-known non-linearity tests can give us valuable model identification information. The methodology is applied to 12 international stock exchange series and two French stock series: France-Telecom and Alcatel. The implications of our findings in favour of mixed non-linearity are very interesting, especially in financial markets where the interactions between heterogeneous investors can produce complex dynamics.