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ABOUT THE INDEPENDENCE OF THE TURNING POINT INDICATOR FUNCTIONS

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In teaching statistics to college and university students very often one is faced with the distribution of the number of turning points in a finite random series of n values $u_1, u_2, \ldots, u_n$. It is well known from time-series that the number of turning points T in the series is simply

$$T = \sum_{i=1}^{n-2} X_i \qquad \text{where}$$

$X_i = 1$, $u_{i-1} < u_i > u_{i+1}$ or $u_{i-1} > u_i < u_{i+1}$ and $X_i = 0$, otherwise.

We require, however, the variation of T to decide when the difference between the observed and the expected number is significant. In order to compute the variance we need to prove the independence of the turning point indicator functions X_i and X_j for $j - i \geq 3$.

Wrong Argument

Some of the solutions submitted up to this point (see, e.g. Kendall 1973, p. 23, Kendall and Stuart 1976, p. 366) refer to the fact that X_i and X_j for $j \geqslant i + 3$ are independent, being at least three terms apart, which means that whether there is a turning point at i, depends only on the numbers in places i-1, i and i+1 and similarly, whether there is a turning point at j, depends only on what happens in places j-1, j, j+1 and hence if these two sets are disjoint there must be independence.

However, since we are dealing with permutations of 1, 2, ..., n, the value of the number we have in place i and the value of the number we find in place j are not independent so one can easily go wrong, if one argues as above, unless one is a bit more precise.

For instance, consider permutations of n-1 0-s and one 1 and let $X_i = 0$ if the i-th number is equal to one of its neighbors and $X_i = 1$ otherwise. Arguing as above we might conclude that X_i is independent of X_j when $j \geqslant i + 3$ but this is not so in the present case, because the events $X_i = 1$ and $X_j = 1$ both have positive probability but they are mutually exclusive so they can not be independent.

A Correct Argument

The number of permutations in which 3 prescribed numbers are in positions i-1, i and i+1 and 3 other prescribed numbers are in positions j-1, j and j+1, while all the other numbers are in some fixed order, is $3!\cdot 3!$. Among these 36 permutations, $4\cdot 3!$ have a turning point at X_i, $3!\cdot 4$ have a turning point at X_j and $4\cdot 4$ have turning points in both places. Consequently, among all permutations of 1, 2, ..., n the proportion of those with a turning point at any one location is $4/3!$ and the proportion of permutations with a turning point in both the i-th and the j-th location is $4\cdot 4/3!\cdot 3!$ which proves the independence of the two events when all permutations are equally probable.

REFERENCES

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