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On negative labour values. A summing up

by
George Sotirchos

In 1975 Steedman caused a lot of inconvenience to the entire marxist scientific community by reinventing the almost forgotten fact, which has been discussed by Sraffa a few years before in *Production of Commodities by means of Commodities*, that in certain joint production systems the prices of production at zero profit rate can be negative. Sraffa, however, never connected directly this result to inconsistencies of the classical or marxian labour value theory. Steedman, instead, have spoken clearly for negative labour values and for negative surplus value. Also, in his work the necessity of abandoning the labour theory of value in order to adopt a more realistic or appropriate theory of production prices is a sine qua non. According to him "any labour theory is necessarily a barrier to the development of a surplus-based theory. Consequently, the frequent indentification of opposition to a 'labour theory of value' with opposition to a 'surplus appropriation' theory of the economic process -an indentification the grew quite naturally out of the work of von Böhm Bawerk and Wicksteed- can now be clearly seen to be mistaken. Rejection of any kind of 'labour theory of value' can, following the work of Dmitrief, von Bortkiewicz, and Sraffa, be rooted firmly *within the surplus approach*" [1].

Labour value theory has been, according to Steedman, from Adam Smith's *Wealth of Nations* till today an obstacle to the development of a surplus approach, i.e. of a meaningfull opposition to and critique of the dominant neoclassical theory. Thus, economists, following Steedman's exhortation, have to get rid the soonest possible of those prescientific notions and concepts, because "ill defined or negative embodied labour quantities can obviously contribute nothing to the analysis" [2]. Steedman based his argument on a very simple counterexample of a joint production technique using two processes

1. Steedman (1981), p. 13.
2. Ibid, p. 15.

and producing two commodities. The following table shows the commodity inputs and outputs when each process employes one unit of homogeneous labour.

Table 1

	Inputs			Outputs	
	Commodity 1	Commodity 2	Labour	Commodity 1	Commodity 2
Process 1	5	0	1	6	1
Process 2	0	10	1	3	12

Table 1 does not need any particular interpretation. It describes a very simple joint production technique with no fixed capital or aged machines. Unlike most neoricardians Steedman considers the real wage basket as exogenously given. Hence real wage, in Steedman's numerical example, consists of 3 units of the first commodity and 5 units of the second commodity paid for 6 units of labour. If we suppose a uniform profit rate the equations that determine the prices of production and the profit rate are:

$$5p_1(1+r)+\frac{1}{2}p_1+\frac{5}{6}p_2=6p_1+p_2, \tag{1}$$

$$10p_2(1+r)+\frac{1}{2}p_1+\frac{5}{6}p_2=3p_1+12p_2. \tag{2}$$

From these equations we can determine the profit rate and the relative production prices. In order to determine the absolute production prices we have to add and arbitrary normalisation equation. Steedman adds the implict normalisation equation:

$$3p_1+5p_2=6. \tag{3}$$

According him, however, (3) is not an arbitrary normalisation equation, but it express the fact that "the real wage bundle which is purchased by 6 units of labour must command 6 units of labour" [3], a crystal clear misinterpretation of the role and meaning of the price normalisation in linear production techniques by Steedman, because real wage is *exchanged* to 6 units of labour

3. Steedman (1975), p. 115.

and consequently the nominal wage is exchanged to the real wage basket, or in other words the nominal price of the real wage basket is the nominal wage.

In order to determine the nominal or absolute price magnitudes of an arbitrary linear production technique, and accordingly of the linear production technique consisting of equations (1) and (2), one has to introduce to the system of price equations a normalisation equation that arbitrarily sets the production price of one commodity, or of a commodity basket, equal to a positive arbitrary constant[4].

Proceeding with the solution of the system of (1), (2) and (3) we obtain the following solutions.

$$r = .20 \text{ or } 20\%, \; p_1 = \frac{1}{3}, \; p_2 = 1.$$

The norminal wage equals one, due to the fact that it is arbitrarily set equal to one by means of the normalisation equation. So far everything in Steedman's counterexample has an economic meaning and interpretation, i.e.

4. This arbitrary determination of a commodity price, or of the price of a commodity basket, is called price normalisation. This price normalisation is accomplished in linear production systems through an exogenously given equation of the price of a single commodity, or of a commodity basket, to a homogeneous extensive thing. The term exogenous used above means that this normalisation equation does not belong to the set of equations that describe the linear production system or technique, and it is added to this system. Additionally it must not be incombatible with these equations. The homogeneous extencive thing involved in the normalisation equation is called, and functions as, fictitious money. The usual form of a normalisation equation is $p \cdot y = b$ where p is the 1xn row vector of production prices, y is the nx1 column vector of the normalisation commodity and b is the positive quantity of the homogenous extensive thing B that functions as fictitious money. Obviously, every extensive and homogenous thing can function as fictitious money and there is no economic reason to presuppose that this normalisation should involve labour commanded or any other magnitude endogeneous or exogeneous to the linear economic system. For more details on the normalisation equation and its role in linear economic systems see the innovative work of Stamatis (1983), (1988) and (1998) and Mariolis (1998). Steedman considers his normalisation equation, through the equation of the price of real wage to the labour that wage basket 'commands', as natural to the system he studies. However, this normalisation is as arbitrary as every other possible normalisation. In other words it does not matter if one equates the price of the wage bundle to the labour commanded by this bundle or to any other homogeneous extensive thing. In the contrary it does matter what commodity prices we normalise. For a detailed exposition of the normalisations and its implications to the relative and absolute prices, see Stamatis (1983), (1988) and (1998).

production prices are positive, profit rate is positive and real and nominal wage are positive.

The quantity side is described by the following set of equations:

$$5x_1(1+g)+d_1=6x_1+3x_2, \tag{4}$$

$$10x_2(1+g)+d_2=x_1+12x_2, \tag{5}$$

where x_1 (x_2) the activity level of production process 1 (2), g the uniform *growth rate* and d_1 (d_2) is the autonomous, exogenously given, demand for commodity 1 (commodity 2). If g, the uniform growth rate, equals to the profit rate, then capitalists do not consume and consequently save and invest all profits. Thus, consumption comes out of the wages only. So d_1 equals $\frac{3}{6}x_1+\frac{3}{6}x_2$ and d_2 equals $\frac{5}{6}x_1+\frac{5}{6}x_2$. Also, one has to add a normalisation equations in order to determine the nominal or absolute activity levels. Steedman introduces implicitly the following equation:

$$x_1+x_2=6, \tag{6}$$

interpreting it as a limitation of labour employed, which must equal 6 units. Although labour supply is a congenital limitation of economic activities and can in some cases function as an upper bound to economic activities in neoricardian theory a labour market is not specified and consequently it is not studied, thus (6) is only an arbitrary normalisation of activity levels, as (3) is an arbitrary normalisation of prices, and consequently (3) has nothing to do with labour employed. Solutions of the system of (4), (5) and (6) are:

$$g=r=.20 \text{ or } 20\%,\ x_1=5,\ x_2=1.$$

The activity levels, and the growth rate, thus, are positive. The system above as defined by Steedman's assumptions is so far a joint production system with normal behaviour of production prices and activity levels. What is wrong then with this production system?

If one turns to the calculation of the uniform labour value magnitudes one has to calculate the direct and indirect labour used in the production of each commodity.

Let w_1 (w_2) be the value of the 1^{st} (2^{nd}) commodity, then the system of the equations that determine the uniform labour values is:

$$5w_1 + 1 = 6w_1 + w_2, \tag{7}$$

$$10w_2 + 1 = 3w_1 + 12w_2. \tag{8}$$

The solutions of the equations above are:

$$w_1 = -1 \text{ and } w_2 = 2.$$

So the labour value of commodity 1 must be negative in order to satisfy the system of equations (7) and (8). How Steedman explains this paradox? According him "the more appropriate way to conceive of value is the *change in employment* resulting from a change in net output from (y_1, y_2) to (y_1+1, y_2) or (y_1, y_2+1) where each output can be produced by some meaningful, positive allocation of labour between the processes" [5]. At this point Steedman do classical and marxian labour value theory a concession saying that labour values can be used, and should be conceived, as analytical tools for the production planning by the capitalist in a capitalist economy and/or by the production planner in a centrally planned economy or by the so-called economic agents in every social organisation of commodity production or of every form of production in general. Consequently and according to Steedman we have to get rid of the theoretical structure of labour value, although we may use value magnitudes as analytical tools in order to plan or even optimize our production. At this point we have to emphasize that production prices at profit rate zero are equal or proportional to labour values depending on the normalisation. Especially, these production prices are equal to labour values in this case because of the particular normalisation that Steedman introduces through (3). Consequently, these production prices are not positive and one of them is strictly negative, situation that does not bother Steedman due to the fact that these "economies... are of only formal and not real interest" [6], and naturally beyond the scope of his positivistic scientific analysis. We are going to return to this question of negative production prices and their significance for the neoricardian theory later and after the analysis of the quantity system which follows.

5. Steedman (1975), p. 119.

6. Ibidem, p. 123.

An interesting question asked by Steedman but never answered in detail is what happens to the quantity system when the growth rate is zero. In order to simplify the answer to this question we suppose that the economy described by Steedman behave so, when the growth rate is zero, as if the profit rate is zero. Although this assumption seems to be odd, it implies the fact that capitalists consume the same basket of commodities as workers do. The supposition of a universal commodity basket consumed by both society classes is a common place in neoricardian analysis. Most of the neoricardians use the assumption of the universal not-flexible consumption basket[7]. Thus we can describe the case where g=0 as if r=0 and hence all net product is paid in the form of fixed commodity baskets to the workers. The equations of the quantity system are written:

$$5x_1 + \xi\left(\frac{1}{2}x_1 + \frac{1}{2}x_2\right) = 6x_1 + 3x_2, \tag{4α}$$

$$10x_2 + \xi\left(\frac{5}{6}x_1 + \frac{5}{6}x_2\right) = x_1 + 12x_2, \tag{5α}$$

7. In the majority of neoricardians there is no analysis of the dual to the profit rate-nominal wage rate relation, i.e. of the growth rate-nominal consumption rate relation. Exceptions are Abraham-Frois/Berrebi (Abraham Frois/Berrebi, 1997). However, these authors presuppose the consumption basket as given and fixed and *independent* of its origin. In other words capitalists and workers consume the same basket which is constant and independent of the level of consumption. In mathematical terms and in the framework of Steedman's counter-example the growth consumption relation is described by the following equations:

$$5x_1(1+g) + \xi(\frac{1}{2}x_1 + \frac{1}{2}x_2) = 6x_1 + 3x_2 \tag{4β}$$

$$10x_2(1+g) + \xi(\frac{5}{6}x_1 + \frac{5}{6}x_2) = x_1 + 12x_2 \tag{5β}$$

$$x_1 + x_2 = 6 \tag{6}$$

where x_1, x_2 are the activity levels of the system, g is the growth rate and ξ is the level of consumption. From the above equations we determine the *g-ξ* relation and the activity levels as functions of g and ξ. This relation is dual, but not identical, to the well known *w-r* relation. A detailed exposition of the *w-r* relation is in Stamatis (1984), (1988), and (1999) and Mariolis (1998) and from a neoricardian point of view in Abraham Frois / Berrebi (1997). The *g-ξ* relation is not studied thoroughly by the neoricardians. A summing up can be found in Pasinetti (1992).

$$x_1 + x_2 = 6, \tag{6}$$

where ξ is the number of commodity baskets paid to and consumed by the workers. The solutions are:

$$x_1 = \frac{54}{7},\ x_2 = -\frac{12}{7} \text{ and } \xi = \frac{6}{7}.$$

Activity level of process II is negative (!), which is meaningless and in contrary to Steedman's assertion that "a competitive equilibrium will exist with exactly the same prices, quantities and values as given in the text (when g = r = .20 - G.S.)"[8].

Steedman by-pass the above abnormality by relaxing the neoricardian assumption of a uniform and constant consumption basket, allowing capitalists to consume the left overs of net production after the payment of real wages to workers. Although this approach is in general not wrong, it is in contrary to the usual neoricardian approach. As follows, when the system operates at a profit rate equal to 20% and capitalists consume commodity 1 and commodity 2 at a rate of 5/2 then the activity levels are positive at growth rate equal to zero. It can be easily proved that there is a set of commodity baskets that can be consumed by capitalists at a certain profit rate (and at a certain growth rate resp.) which secures positive activity levels in the original Steedman's counterexample[9]. Consequently, the negative activity levels appear and disappear as functions of the growth rate and of the consumption basket of capitalists, if we consider the consumption basket of workers as a given constant[10], independent of the level of the nominal wage or the preferences of workers.

The opposition to Steedman came substantially before the publication of his paper. As it has been mentioned before, the fact that negative labour values can appear in joint production systems was already a well established fact, when Steedman wrote his article. Morishima[11] in *Marx's Economics* describes a case of a used machine that has negative labour value. An analysis, therefore, of negative labour values phenomenon had appeared before even Steedman

8. Steedman, 1975, p. 121.
9. Abraham Frois / Berrebi (1997).
10. See Filippini and Filippini (1982).
11. Morishima (1973), pp. 181-185.

publishes his article. In the following we are going to describe and evaluate Morishima's solution to the problem using the original numerical counter-example of Steedman and recall the context of Morishima's approach to the problem.

According Morishima there are several different definitions of labour value in Marxian work. The first is rooted deeply in the formation of Marxian thought, in the *Poverty of Philosophy*[12] published in french in 1847 as an answer to Prundhon's *Philosophy of Poverty*. In this marxian critique to prundhonian socialism there is the following passage: "It is important to emphasise the point that what determines value is not the time taken to produce a thing, but the *minimum* time it could possibly be produced in, and this minimum is ascertained by competition. Suppose for a moment that there is no more competition and consequently no longer any means to ascertain the minimum of labour necessary for the production of a commodity; what will happen? It will suffice to spend six hours' work on the production of an object, in order to have the right, according to Mr. Prondhon, to demand in exchange six times as much as the one who has taken only one hour to produce the same object. Instead of a 'proportionality relation' we have a disproportional relation at any rate if we insist on sticking to relations good or bad" [13].

It is obvious that Marx in this passage speaks on the definition Prondhon's of labour values and some implications and deficiencies this definition has.

As it stated by Morishima this definition is consistent with the labour value theory in joint production techniques.

Morishima, however, starting from this passage and in order to solve the paradox of negative labour values reformulates the problem of value determination as a problem of minimization of labour time used up in production. The problem now is formulated as linear programming problem of minimization. In line with the definition above society, or the so-called economic agents, minimize the total labour required to produce an exogenously given net product. Obviously the so called economic agents face a linear programming problem of minimization. A well known theorem of linear programming asserts that every minimization problem has its dual linear

12. Our references are from the german translation of the original french text, see Marx (1947).
13. Marx (1947), pp. 67-68.

programming problem of maximization. Labour values are the solutions of the dual problem. Morishima calls those magnitudes 'optimal values'. In reality they are optimal or efficient employment multipliers, and Morishima solution to the problem of negative labour values in joint production system is rooted deeply in the dominant neoclassical (subjective) school of thought. In this particular school economic agents, under certain weak limitations, maximize their utility, their profit etc., minimize their cost, their labour offer etc.

Morishima developes accurately his general model, so we limit our analysis in Steedman's counterexample[14]. Let us consider a problem of minimization of the total amount of labour required to produce a net output $y = (y_1, y_2)^T \geq 0$. The vector of activity levels satisfy the following matrix inequality:

$$Bx \geq Ax + y \tag{7}$$

If we replace matrices A and B with the input and output matrices of Steedman's counterexample we have the following relations:

$$6x_1 + 3x_2 \geq 5x_1 + y_1, \tag{8}$$

$$x_1 + 12x_2 \geq 10x_2 + y_2. \tag{9}$$

The required total labour for the production of the net product $y = (y_1, y_2)^T$ is

$$L = \ell x = 1 \cdot x_1 + 1 \cdot x_2 = x_1 + x_2$$

when production system operates at activity levels $x = (x_1, x_2)^T$.

Hence the linear programming problem is:

$$\left.\begin{array}{l} \min L = \min(x_1 + x_2) \\ \text{subject to} \\ x_1 + 3x_2 \geqq y_1 (\geqq 0), \\ x_1 + 2x_2 \geqq y_2 (\geqq 0), \\ x_1, x_2 \geqq 0 \end{array}\right\} \tag{10}$$

The solution of this problem is[15]:

14. Morishima (1973), p. 181-185.
15. See Abraham Frois / Berrebi (1997) for the analysis of a numerical example.

$$x_1 = 0 \text{ and } x_2 = \max\left(\frac{y_1}{3}, \frac{y_2}{2}\right).$$

The dual problem of the minimization problem above is the following maximization problem

$$\left.\begin{array}{l} \max \phi \cdot y = \max(\phi_1 y_1 + \phi_2 y_2) \\ \text{subject to} \\ \phi_1 + \phi_2 \leqq 1 \\ 3\phi_1 + 2\phi_2 \leqq 1 \\ \phi_1, \phi_2 \geqq 0 \end{array}\right\} \qquad (10a)$$

As it can easily be verified the solution to the dual problem is:

$$\phi_1 = 0, \ \phi_2 = \frac{y_2}{2}, \text{ if } \frac{y_2}{y_1} \geq \frac{2}{3},$$

or

$$\phi_1 = \frac{y_1}{3}, \ \phi_2 = 0, \text{ if } \frac{y_2}{y_1} \leq \frac{2}{3}.$$

If $\frac{y_2}{y_1} = \frac{2}{3}$, then the problem above has no unique solution but we will return to this issue after the analysis of the minimization problem. We must emphasize that magnitudes ϕ_1 and ϕ_2 are not the 'optimal values', but auxiliary variables that allow us to define the 'optimal values' or efficient employment mutlipliers, since ϕ_1, ϕ_2 represent the optimal total labour required for the production of y_1 or y_2 and 'optimal values' is the amount of labour required for the production of *one additional unit* of commodity 1 or 2.

A graphical solution of the primal minimization problem can be found in figure 1, where $\frac{y_1}{3} > \frac{y_2}{2}$. In this case one allocates $\max\left(\frac{y_1}{3}, \frac{y_2}{2}\right) = \frac{y_1}{3}$ units of labour to the second process and one does not operate the first.

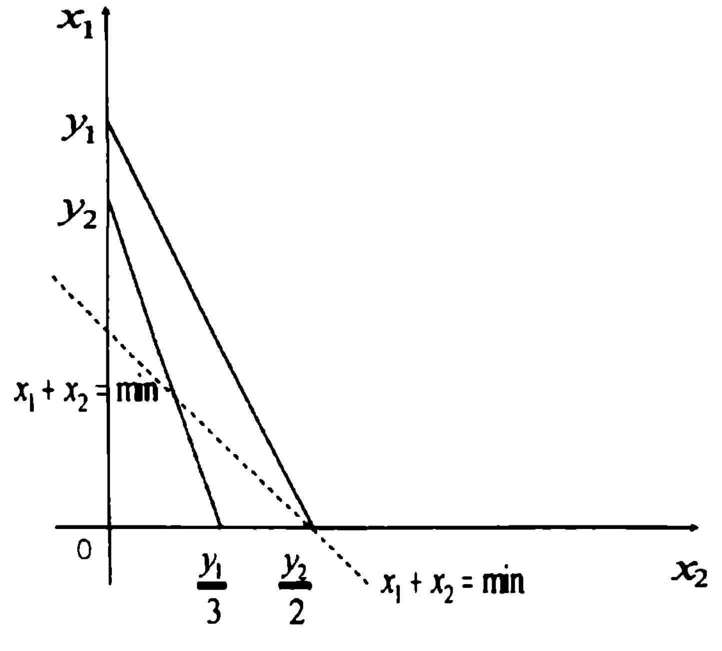

Figure 1

A graphical solution of the dual maximization problem can be found in figure 2.

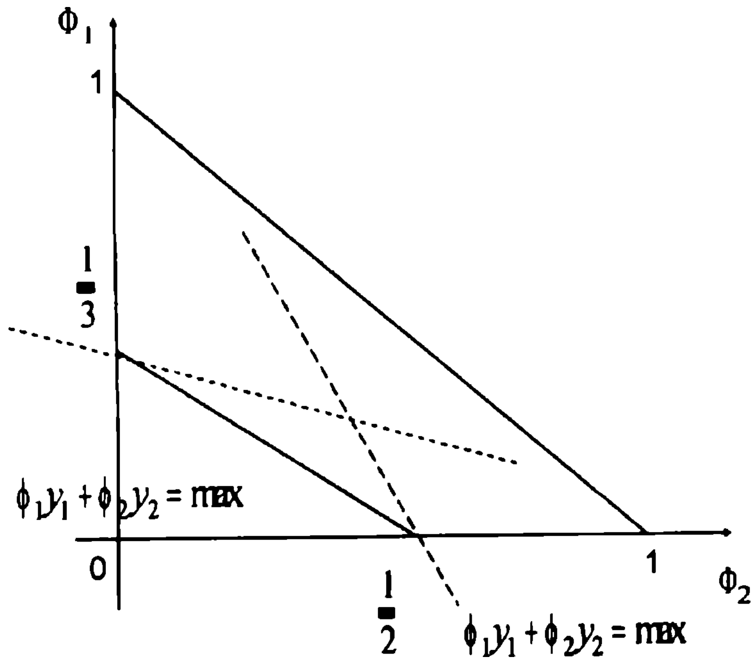

Figure 2

Note that the solution depends on the rate $\frac{y_2}{y_1}$ which is unknown. If $\frac{y_2}{y_1} \geq \frac{2}{3}$, then $\phi_1 = 0$, and if $\frac{y_2}{y_1} \leq \frac{2}{3}$, then $\phi_2 = 0$. When $\frac{y_2}{y_1} = \frac{2}{3}$ then ϕ_1, ϕ_2 are not determined uniquelly.

The system of the inequalities (8) and (9) implies the overproduction or the excess supply of one good, which becomes, by the von Neumann definition of free goods, a free good. Consequently, its 'optimal value', defined as minimum labour required for the production of one additional unit, according Morishima, is equal to zero. The 'optimal value' of the other commodity, which is not a free good is the minimum change of employment when the production increases by one unit of each good[16]. Morishima correctly calls these magnitudes optimal employment multipliers as they represent the minimum quantity needed to increase the production of one commodity by one unit. These optimal values are in any case semipositive, but as Morishima and Catephores explicitly state the property of value additivity is not valid any more under this definition. However, the Fundamental Marxian Theorem which is of 'decisive importance' to Marxian Economics retains its validity[17]. Another property of 'optimal values' questionable is uniqueness. 'Optimal values', defined as efficient employment multipliers, are unique for a certain net product vector $y = (y_1, y_2)^T$. They may be totally different if we consider another output vector $y' = (y_1', y_2')^T$. In our numerical example consider the case when $\frac{y_1}{3} = \frac{y_2}{2}$, then the 'optimal value' or the optimal employment multiplier of good 2 in regard to output vector $(y_1 - 1, y_2 - 1)$ is $\frac{1}{3}$ and of good 2 in regard to the output vector $(y_1 + 1, y_2 + 1)$ is $\frac{1}{2}$ and when the net product vector is $y = (y_1, y_2)^T$ the 'optimal values' are not uniquely determined. The non-uniqueness is easily verified in figure 2 as well, if we allow $\frac{y_2}{y_1}$ to vary arount the value $\frac{2}{3}$. Uniqueness of 'optimal values', unlike Steedman's uniform values, is established only for a given net product vector. Morishima, however, perceives

16. Morishima and Catephores (1978) incorrectly compare the production vector (y_1, y_2) both vectors (y_1+1, y_2) and (y_1, y_2+1). However this is superfluous because the additional labour which is required to produce (y_1+1, y_2) in their numerical example is just the same as the labour required to produce (y_1+1, y_2+1), due to the fact that the addition of one unit to the production of good 1 does not increase employment.
17. Morishima / Catephores (1978), p. 38.

another source of non-uniqueness, the multiplicity of the solutions of linear programming problems. In order to efface this problem Morishima introduces the so-called 'true values', which is nothing but the minimum labour required to produce a certain amount of a commodity, which is unique. In our case 'true value' of one unit of commodity 1 is the minimum labour required for the production of the net product y, $y = (1, 0)$. Respectively the 'true value' of one unit of commodity 2 is the minimum labour required for the production of the net product y, $y = (0, 1)$. Obviously, 'true values' are unique and semi-positive[18].

Despite the deficiences, described above, Morishima's argumentation has a great influence in Marxist economic thought. Consequently, a further evaluation of his argument is needed. Morishima in order to solve the negative labour values problem implicitly assumes that capitalists, allocate the required for the production of an exogenously given net product labour to the individual processes of production in such a manner that this labour is minimized. This ability presupposes the full knowledge of every individual production process and the authority to allocate the available labour to the processes that minimize this labour for a given exogenously net product. Actually it presupposes the existence of a central planner, who requlates every aspect of production and his ultimate goal is the minimization of total labour used. Obviously Morishima deals with a hypothetical society organization and not capitalism, not even a society where production is mediated by exchange, given the fact that a central planner with the authority to allocate every labour unit and with the information to do it optimally does not need the commodity market to sell or to buy, but he can regulate the social production using all technical details of production he already is aware of. On the other hand "the wealth of those societies in which the capitalist mode of production prevails, presents itself as an 'immense accumulation of commodities', its unit being a single commodity. Our investigation must therefore begin with the analysis of a commodity".

Another implication of Morishima's approach to the problem of negative labour values is that what Morishima calls 'optimal or true values' are not actual, and consequently real, magnitudes but the weights of the objective function $\phi \cdot y$ of the dual problem which is to be maximized. The nature of linear programming problems does not imply that they actually are solved,

18. Morishima / Catephores (1978), pp. 36-37 and Morishima (1973), p. 185.

even if they are solvable as the problem in question is. We may, therefore, say that Morishima's 'true or optimal values' are hypothetical tools that can be used by the non-existing in capitalism central planner[19]. As it is pointed out by Stamatis 'true or optimal values' are indirect informations for the production technique and given the fact that demanded net product is given they are indirect informations for the production system[20].

Indirect data of the production system is also what is called net product, since it is not the actually produced net quantity of each good or commodity[21] but the desired or required quantity of each product. The producers Morishima's hypothetical society produce greater quantities of the desired or the required. These produced quantities are the actual or real net product and this product contains commodities in excess supply. The in excess supply produced commodities are the so-called free goods, despite the fact that human labour and commodity inputs are used in their production. Morishima concludes, however, that their 'true or optimal value' is zero, although they are used in the production. The implied close relation to the marginal theory is developed exhaustively in Stamatis (1979).

Roemer has fully accepted the Morishima approach and refined his arguments. A generalisation of Morishima's model can be found in Roemer and Flaschel each one of them examines different aspects of the model[22]. Fujimori has also accepted Morishima's argument but from a critical point of view[23]. The technical character of their work does not permit us to include a detailed exposition of them. However, we are going to return to Fujimori later, because his contribution is not limited to the acceptance of Morishima's thesis.

Worth mentioning is the neoricardian response to Morishima's 'true or optimal values'. According Abraham Frois / Berrebi "the set of activities retained, the technique springing from the minimization of aggregate employment and which allows us to determine 'optimal values', the so called 'true values', may be totally different from the set of activities, or the technique, determining the rate of profit R* and the system of production prices. Thus, in

19. Stamatis (1979), pp. 50-52.
20. Ibidem and Stamatis (1983).
21. As we have seen the term commodity is superfluous, since Morishima's formulation excludes by definition commodity production.
22. See Roemer (1982), Flaschel (1983).
23. See Fujimori (1982). Fujimori adopted partly Kurz's argument as it is expressed in a book review of *Marx after Sraffa* written in 1979.

the treatment of the example due to Steedman, the paradox emphasized by the latter disappeared because the problem has been removed" [24].

Abraham Frois and Berrebi and Steedman in his reply to Morishima's comment correctly point out that the objective function to be minimized, if the problem has to be formalized in minimization terms, is the cost function in price terms, or the objective function to be maximized is the profit function, if the problem has to be formalized in maximization terms. The authors above indirectly show that Morishima's approach to the problem is not appropriate for the prevailing capitalistic mode of production, but it can be used only in hypothetical modes of production.

A striking result for joint production techniques has been proved by Wolfstetter and has been later generalized by Filippini and Filippini and Fujimori[25]. All these authors investigated the conditions under which negative uniform labour values and prices of production appear (and disappear) in joint production techniques. The common result of all authors is that uniquely determined labour values are negative, if and only if inferior processes exist within the technique. The term inferior signify that the process in question produces less net product than another process or more precisely that we can decompose the technique in question in two 'blocks' or sub-techniques in such a manner that a convex combination of one 'block' or sub-technique dominates, or has a greater net product than every other convex combination of the other 'block' or sub-technique. The first, the dominating 'block' or sub-technique is called *superior* and the second, the dominated, 'block' or sub-technique is called *inferior*[26]. As it can be easily proved the inferior process in Steedman's counterexample is process I. Process I produces for one unit of labour used one

24. Abraham Frois / Berrebi (1997), also Steedman (1976α), (1976β).
25. Wolfstetter (1976), Filippini and Filippini (1982) and (1984), and Fujimori (1982). Wolfstetter analysis is limited to the case of 2x2 joint production techniques, Filippini and Filippini instead analyse the general case of nxn techniques. Fujimori writes on mxn techniques, but his results are not applied to production prices. Filippini and Filippini focus on production prices and value analysis is a corollary of their analysis at zero profit rate.
26. Hosoda (1993) and Cottrell (1996) discuss a case where negative uniform labour values appear in a joint production technique where no-inferiority exist, although *superiority exists*. In order to derive this peculiar result Hosoda introduces a definition of inferiority somewhat different from the one given by Filippini and Filippini, Fujimori et al. However, this definition imply an *irrational behaviour* in regard to process comparison, evaluation and classification by capitalists or economic agents in general. See Sotirchos (1998α).

unit of commodity 1 and one unit of commodity 2, process II produces for one unit of labour used three units of commodity 1 and 2 units of commodity 2. If inferiority is defined at a given rate of profit, which is given exogenously, inferior process in a technique can appear and disappear while we increase profit rate from zero to its maximum value. In this case a convex combination a 'block' or sub-technique is called inferior if the r-net product[27] in physical terms of this convex combination is smaller than the convex combination of another 'block' or sub-technique[28]. The decisive difference is the comparison

27. The term r-net product of a process is the vector $b_i - (1+r)a_i$ where b_i denotes the column output vector, a_i denotes the column input vector and r denotes the exogenously given profit rate. Obviously, the r-net product is the net product of a process if we augment inputs by $(1+r)$ and it is obviously less than the net product, which is, according to the latter definition, the 0-net product.
28. In mathematical terms the convex combination of processes $(b_j, a_j, \ell_j)^T$ is called inferior to the convex combination of processes $(b_i, a_i, \ell_i)^T$ at profit rate r if there are α_i and β_j, such that

$$\sum_j \beta_j \begin{pmatrix} \beta_j-(1+r)a_j \\ -\ell_j \end{pmatrix} \leq \sum_i \alpha_i \begin{pmatrix} \beta_i-(1+r)a_i \\ -\ell_i \end{pmatrix}$$

$$\text{and} \quad \sum_i \alpha_i = \sum_j \beta_j = 1, \quad \alpha_i, \beta_j \geq 0.$$

This is the definition given by Filippini and Filippini (1982) and (1984). Wolfstetter (1976) and Fujimori (1982) consider only the case when r=0. For the various definitions of inferiority and their evaluation see Hosoda (1993) and Sotirchos (1998α).

Increasing r from zero to its maximum value, if it exists, inferiority can disappear depending on the structure of production technique. Filippini and Filippini (1984) have examined the original Steedman's counterexample for inferiority increasing r from zero to .445 which is its maximum value. They have easily verified that, when r is in the segment [0, .10), production prices are not positive, or precisely are *of different sign* and when r is in the segment (.10, .445) both prices are positive. At profit rate r=.10 production prices are not determined. It has to be noted that Filippini and Filippini use a different normalisation of the one Steedman uses and thus their analysis of relative price movement differs from the analysis of Steedman's counterexample under the presupposition of his normalisation, when r approaches r=.10. So we exclude this value of r from our analysis because it is far beyond our scope of analysis. However, it can be easily shown that process I in Steedman's example is no more inferior to process II if r>.10, because $[(6, 1)^T - (1+r)(5, 0)^T] = (1-5r, 1)^T$ is not smaller anymore than $[(3, 12)^T - (1+r)(0, 10)^T] = (3, 2-10r)^T$ for r>.10. Consequently process I is inferior to process II for r<.10 because $(1-5r, 1)^T < (3, 2-10r)^T$ for r<.10. Inferiority disappears for r>.10 and negative prices disappear too. For a formal proof see Filippini and Filippini (1982) and Fujimori (1982), who examines a sllightly different case. For the role of the normalization to the formation of relative prices see Stamatis (1984) and (1988).

of processes or sub-techniques at a prevailing rate of profit. Inferiority does not exist in Steedman's example, at the profit rate he chooses, $r = .20$, because the r-net product in physical terms of process I is not comparable to the net product of process II. The r-net product of process I at $r = .20$ is $(6, 1)^T - 1.2\,(5, 0)^T = (0, 1)^T$ and the r-net product of process II at $r = .20$ is $(3, 12)^T - 1.2\,(0, 10)^T = (3, 0)^T$. These vectors of the r-net product of each process are incompatible and they can not be compared or the processes can not be classified as superior and inferior. Summing up the analysis of Wolfstetter, Filippini and Filippini, and Fujimori is the analysis of conditions under which positive uniquely determined labour values exist in joint production. The analysis of Filippini and Filippini is more general because they investigate the conditions under which positive prices of production exist at an exogenously given profit rate. Although this kind of approach is extremely important in the study of the linear joint production techniques, it does not answer the obvious and prevailling question, *why* in certain joint production techniques exist negative uniquely determined labour values. The authors above have answered the question, *how or under which conditions* exist positive uniquely determined labour values. The key word in our question(s) is the word *uniquely determined* and it is the cause of the negativity of labour values in certain 'productive' production techniques. That what Steedman's counterexample shows is that labour productivity in the two processes is different, because one unit of labour employed in process I produces one unit of the 1st commodity and one unit of the 2nd and one unit of labour employed in process II produces three units of commodity 1 and two units of commodity 2. It is obvious that the productivity of the two processes differ in absolute size and it is not appropriate to treat labour values as uniquely determined magnitudes but in regard to each process as individually determined magnitudes. This notion of the individual labour value has adopted almost simultaneously by Hengstenberg and Fay, Stamatis and Flaschel in his earlier work[29]. Flaschel, however, tries even in his earlier work to define uniquely the labour values, although he accepts initially a non unique determination of labour values in joint production systems. The solution to the problem of negative labour values given independentlly by Hengstenberg and Fay and Stamatis will be presented in the following.

29. Hengstenberg and Fay (1980), Stamatis (1979) and Flaschel (1977). Kurz adopted a similar approach in a book review of *Marx after Sraffa*.

Hengstenberg and Fay and Stamatis remark upon the solutions to the problem so far that they do not take into consideration the fact that the individual productivities of each process in Steedman's numerical example are unknown and different, but cardinaly comparable. However, both authors emphasize that in most cases of joint production individual productivities are not even cardinaly comparable. According them the system of equations has to be re-defined in order to include the deviations of individual productivities of labour. Stamatis[30] uses the following equations:

$$5w_{11} + 1 = 6w_{11} + 9w_{21}, \tag{11}$$

$$10w_{22} + 1 = 3w_{12} + 12w_{22}, \tag{12}$$

where w_{ij} is the individual value of commodity i in process j. The inverse of the individual value w_{ij}, $1/w_{ij}$, is by definition the 'partial' or 'individual' productivity[31] of commodity i in process j. Finally the marxian or average values are defined as the weighted sums of the individual values of each commodity

30. Stamatis (1979), pp. 17-40.

31. On the other hand, Kurz defines the individual productivity as an index attached to the labour employed in each sector, thus as a property of the labour employed in each sector and not as a property of labour in regard to a single or composite commodity produced in each sector. The reason is that he could not accept that the same object in the form of a commodity obtains different individual values if it is a product of different sectors operating under different production conditions. As the dominant theory insists on "same use values can not have different values", an approach accepted fully by the neoricardians. Consequently, that what Kurz calls productivity of labour in each sector is not productivity, because it does not correspond to a single commodity or a composite commodity but an implicit reduction coefficient that transforms non-homogeneous labour to homogeneous. Kurz writes the system of equations (7) and (8) as:

$$5w_1 + \pi_1 = 6w_1 + w_2, \tag{7α}$$

$$10w_2 + \pi_2 = w_1 + 12w_2, \tag{8α}$$

where π_i is, according Kurz, the productivity index of labour in process i, i = 1, 2. Obviously system of equations (7a) and (8a) is totally different from the system of equations (11) and (12), although both systems are different from system of equations (7) and (8). System of equations (7α) and (8α) should be re-written as

$$5w_1 + \pi_1 \cdot \ell_1 = 6w_1 + w_2, \tag{7β}$$

$$10w_2 + \pi_2 \cdot \ell_2 = 3w_1 + 12w_2, \tag{8β}$$

where ℓ_i is the labour quantity employment in process i, i = 1, 2. From the normalization of

$$w_1 = \frac{6x_1}{6x_1 + 3x_2} w_{11} + \frac{3x_2}{6x_1 + 3x_2} w_{12}, \tag{13}$$

$$w_2 = \frac{x_1}{x_1 + 12x_2} w_{21} + \frac{12x_2}{x_1 + 12x_2} w_{22}, \tag{14}$$

where x_1, x_2 are the (semi-positive) activity levels of each process. If these activity levels are given exogenously or determined independently of the so-called quantity system we have a system of six unknowns and four equations. Consequently we can not determine unique solutions for individual and average values. However, it has to be shown that this solution is compatible to positive solutions of individual values and thus of average values. Stamatis has shown using numerical examples that if a production technique is productive and has the property of the so-called "profitability", then individual values are always positive[32]. His analysis has been done by means of numerical examples of techniques. He begins with an example of a technique with no inferior subtechniques and proceeds with examples of techniques with various degrees of 'increasing' relative and absolute inferiority and evaluates the consequences of the various degrees of inferiority to the possibilities of the individual values to coincide. We do not hesitate to say that through numerical examples he has shown all the results of Filippini and Filippini on techniques with inferior subtechniques, results that have been derived by these authors years later. Stamatis using the following figure 3 has proved that individual productivities in original Steedman's example can not coincide for positive values of $1/w_{ij}$, i, j=1, 2.

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labour in Steedman's numerical example we have $\ell_1 = \ell_2 = 1$. Kurz multiplies these quantities by π_1, π_2 respectively. If we re-write (7β) and (8β) in matrix form we obtain:

$$(w_1, w_2)\begin{pmatrix} 5 & 0 \\ 0 & 10 \end{pmatrix} + (\pi_1, \pi_2)\begin{pmatrix} \ell_1 & 0 \\ 0 & \ell_2 \end{pmatrix} = (w_1, w_2)\begin{pmatrix} 6 & 1 \\ 3 & 12 \end{pmatrix}, \tag{7γ}$$

where by definition $\ell_1 = \ell_2 = 1$.

It is obvious that in Kurz's approach labour is non-homogeneous and π_1, π_2 are not productivities but labour reduction coefficients.

32. A generalisation of Stamatis arguments can be found in Vassilakis (1986) and Sotirchos (1998b).

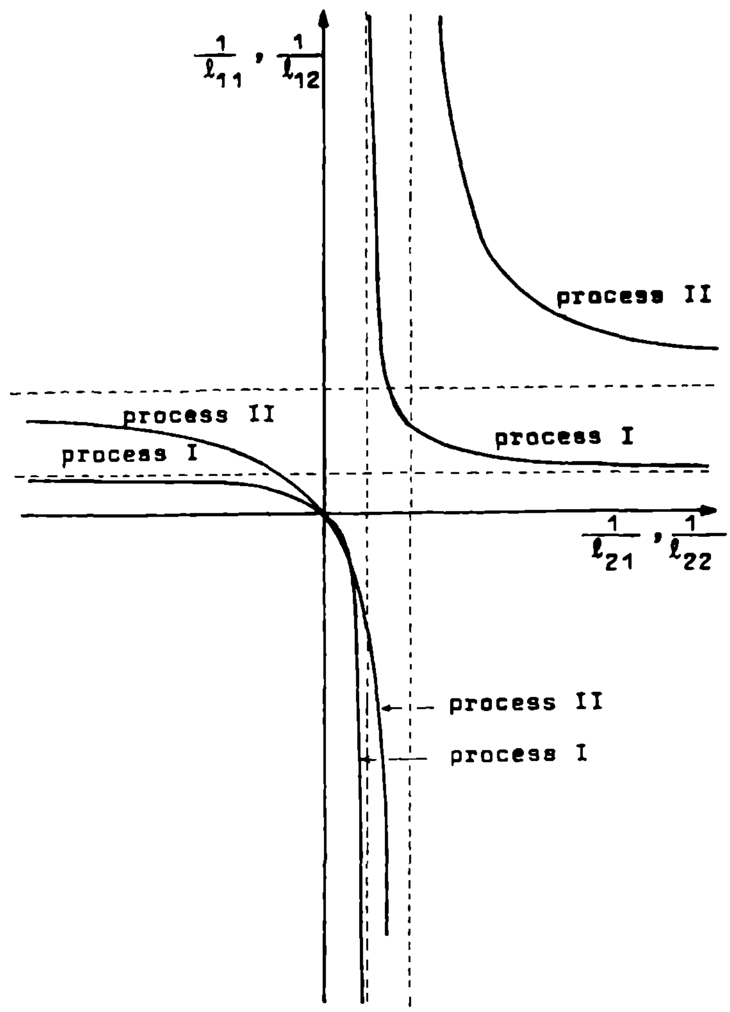

Figure 3

However there exist positive solutions for individual values that do not coincide.

The analysis of individual productivity relations has its consequences for the relations among individual values, due to the fact that individual productivity of commodity i, i=1, 2 produced in sector j, j=1, 2 is the inverse of the individual value of commodity i, i=1, 2 produced in sector j, j=1, 2 [33]. The figure 4 below shows the relations between individual values instead the relations between individual productivities.

33. See Desai (1979), pp. 121-144. Figure 4 adapted from Desai (1979), p. 31.

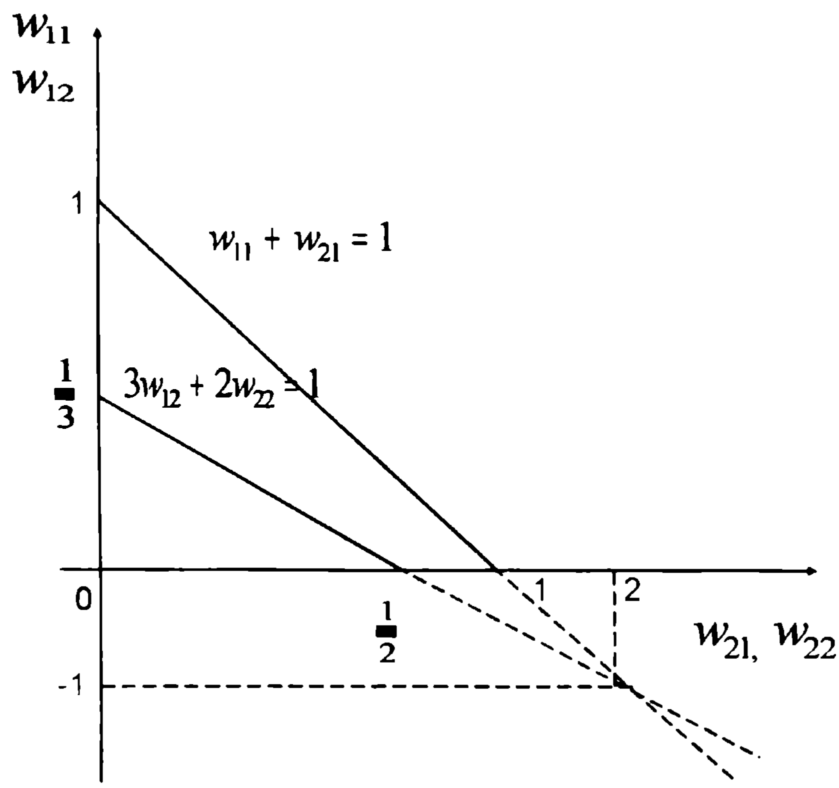

Figure 4

Individual values can not coincide for positive values of both w_1 , w_2 . Although Desai came close to the conclusion that individual values can be non-uniquely defined but positive, or precisely can be non-uniquely defined and obtain positive solutions, never proved this result. Unfortunately he acceptes fully the Morishima 'true values' approach, being attracted by the uniqueness and optimality properties of Morishima's solution to the problem.

Nevertheless, Stamatis' approach to the negative labour value issue can be summed up as follows:

1. There is no such thing as negative value.
2. 'Negative value' is the result of a naive and unconsidered application of mathematics.
3. With joint production systems the (absolute and relative) individual values and hence the absolute and relative average values are not determined – even with square production systems. That the values are not determined here should not be taken as meaning that there are no values. It means only that they cannot be determined because of the lack of information relating to the individual production process. That these unknown values are positive I have already showed when I demonstrated that each of the existing solutions (of which there is an infinitely great number because of

the insufficient information concerning the individual processes) supplies only positive values[34].

This line of argument proves the additivity property of the average of marxian values and the positivity property does not allow one to determine uniquely the labour value magnitudes, an undesired characteristic especially for the positivistic perception in economics and especially the marxian economics. In the following we are going to examine two approaches to the issue that try to determine unique labour values.

Flaschel although accepts initially the deviation of sectoral or individual labour values, he tries to determine them uniquelly. According him the properties of labour values defined by Marx are positivity, additivity, uniqueness and actuality. We have described the first three properties and concluded that we have to abandon one of them in joint production, specifically in joint production techniques with inferior processes at a prevailing rate of profit equal to zero. Why we need the latter property, actuality of value magnitudes? Flaschel answers that "(a) definition of labour values can be given which again is based on the methods of production that are *actually adopted*... (t)he conceptional discrepancy pointed out above between theory and measurement, therefore, can be bridged again, and to be sure, by way of a new interpretation of labour values which applies to the so-called 'sales value method' to the case of labour costs, and through an intimate relationship of this method with the allocation rule proposed for the case of joint products in the system of National Accounts" [35].

The actual values, therefore are derived directly from the data of the economy. In order to give a brief account of Flaschel's approach to the problem of negative labour values we return to his earlier work (Flaschel, 1977) given the fact that his 1981 article is extremely involved. Flaschel starting from Steedman's counterexample derives the following system of individual value equations.

$$5w_{11} + 1 = 6w_{11} + w_{21}, \tag{11}$$

$$10w_{22} + 1 = 3w_{12} + 12w_{22}. \tag{12}$$

He asserts, also, that the given data for the determination of the average labour values are not adequate, and additional conditions on individual labour

34. Stamatis (1983), p. 90.

35. Flaschel, 1983, p. 436.

values are needed. He also asserts that "a further determination of a criterion, determined by production, which can solve this additional problem. If one recalls that in Marx's *Capital* 'exchange value' is the departure point for the determination of 'value', that one departing from the given equalisation of the various commodity types arrives to the content of the exchange process (abstract labour) and that one quantitative definition is attributed (which is necessary but not entirely determined). Then it seems to me that it is from obvious to necessary that where the quantitative definition of labour value strike against freedom degrees, while it is going to be defined and concrete, these freedom degrees to be eliminated by means of the composition of production and exchange, when and if this elimination of the freedom degrees is impossible to be accomblished by means of production only (The way of this elimination depends on the specific characteristics of the already accomblished level of elimination and definition)" [36].

It is obvious from this passage that Flaschel tries to determine the 'actual' labour values from the actual data of exchange between commodities, i.e. their relative market prices. Thus Flaschel determines uniform labour values from the actual data of exchange, using the exchange rates as additional equations in order to add to the system of equations (11) and (12) the following equations that relate the actual exchange data (relative prices) to relative individual values:

$$\frac{6w_{11}}{6w_{11}+w_{21}}=\frac{6p_1}{6p_1+p_2}, \tag{13}$$

$$\frac{w_{21}}{6w_{11}+w_{21}}=\frac{p_2}{6p_1+p_2}, \tag{14}$$

for the production processes I, and

$$\frac{3w_{12}}{3w_{12}+12w_{22}}=\frac{3p_2}{3p_1+12p_2}, \tag{15}$$

$$\frac{12w_{22}}{3w_{12}+12w_{22}}=\frac{12p_2}{3p_1+12p_2}, \tag{16}$$

36. Flaschel, 1979, p. 120.

for the production process II. After elementary manipulations of equations (13) - (16) we derive

$$\frac{w_{11}}{w_{21}}=\frac{p_1}{p_2} \tag{17}$$

and

$$\frac{w_{12}}{w_{22}}=\frac{p_1}{p_2} \tag{18}$$

Obviously Flaschel postulates the proportionality of individual values to market prices. According him equation (13) show that the relative share of the individual value commodity 1 to the net product in process I. Analogously equations (14) - (16) are interpreted. Equations (17) and (18) imply that

$$\frac{w_{11}}{w_{21}}=\frac{w_{12}}{w_{22}}=\frac{p_1}{p_2}, \tag{19}$$

but not, as it is postulated in Steedman's analysis that $w_{11} = w_{12}$ and $w_{21} = w_{22}$. Accordingly, the proportional to market prices labour values are not negative because market prices are not negative. In the linear production techniques that Steedman deals with and under the presupposition of a uniform profit rate market prices are equal or proportional to production prices[37]. Thus the solutions of equations (1) and (2) that determine production prices are $p_1 = 1/3$ and $p_2 = 1$ and as it follows

$$w_{11}=\frac{1}{4}\ w_{21}=\frac{3}{4}\ w_{12}=\frac{1}{9}\ \text{and}\ w_{22}=\frac{1}{3}$$

The marxian or average 'actual' labour values are

$$w_1=\frac{11}{54}\ \text{and}\ w_2=\frac{19}{52}$$

Consequently and always according to Flaschel the property of actuality not only has connected the abstract notion of labour value but it has added information or data to our original system of equations and thus has reduced or even eliminated the freedom degrees and finally has resolved the paradox of

37. Although additional approaches may exist we stick to this assumption. For a treatment of a different approach see Mariolis (1998).

negative labour values. Although the above argument seems consistent it fails to come even close to marxian theory of value form. An exhaustive analysis of the value form can be found in Backhaus (1969). In regard to Flaschel's argument we can only say that he identify the labour value with its forms that appear in reality, market prices. The identification of the labour value with its transformations it has nothing to do with the marxian theory of value and it is related to the opposition to Ricardian Theory of value as it is developed mainly by Bailey. Bailey claimed that because of the fact that market prices are the forms that labour values appear, the latter should be equal and identical to the former. As Marx commented on Bailey that, "... as impossible as it is to 'designate' or 'express' a though except by a quantity of syllables. Hence Bailey concludes that a though is syllables" [38].

A further development of Flaschel's approach exists in his habilitation thesis. Although it is formal and in some cases particularly involved we will precent his main arguments without the proofs by means of the following numerical example

Table 2

	Inputs			Outputs	
	Commodity 1	Commodity 2	Labour	Commodity 1	Commodity 2
Process 1	3	2	1	6	0
Process 2	2	3	1	7	0
Process 3	2	0	1	0	6

The table 2 shows a technology of three single product processes that produce two commodities. Formally this production technique does not belong to the joint production techniques but to a single production with multiple activities for every commodity. In our case commodity 1 is produced by process I and II, these processes are not cardinaly or ordinaly comparable and additionally there are vectors of activity levels, for example $x_1 = (x_1, x_2, x_3)^T = (1, 1, 1)^T$, that verify the productivity property of the technology. Namely if A is the input matrix and B the output matrix, the production technique can produce at least one positive net product $(B - A)x_1$ for activity levels $x_1 = (1, 1, 1)^T$.

38. Marx, K. *Theorien über den Mehrwert*, Teil 3, p. 144.

It is obvious that the labour values can not be determined uniquely given the fact the following system of labour value determination is a system of three equations with two unknowns and thus overdetermined:

$$3w_1 + 2w_2 + 1 = 6w_1, \tag{20}$$

$$2w_1 + 3w_2 + 1 = 7w_1, \tag{21}$$

$$2w_1 + 1 = 6w_2. \tag{22}$$

If one uses the individual value approach as it developed earlier obtains the following set of equations

$$3w_{11} + 2w_{21} + 1 = 6w_{11}, \tag{20α}$$

$$2w_{12} + 3w_{22} + 1 = 7w_{12}, \tag{21α}$$

$$2w_{13} + 1 = 6w_{23}. \tag{22α}$$

The system of equations (20α) - (22α) is similar to the system of equations (11) and (12) and thus requires a similar treatment. The individual labour values are not fully determined and there are degrees of freedom in the determination of these magnitudes. It can be easily verified that individual values are positive but not uniquely determined. Flaschel, in the contrary, and following Murata tries to determine *uniquely* the labour values from the data of the technology described above. He introduces the relative intensities λ_j of process j with respect to the total output of each sector. Sector denotes according to Flaschel the set of processes that produce a single commodity[39]. Obviously sector I is constituted by processes I and II in our example and produce commodity 1. The relative intensities λ_j, j = 1, 2, must fulfill the following conditions

$$\sum_{j=1}^{2} \lambda_j = 1, \ \lambda_j \geq 0, \ j = 1,2.$$

One can define the matrix of relative intensities as:

$$\Lambda = \begin{bmatrix} \lambda_1 & 0 \\ \lambda_2 & 0 \\ 0 & 1 \end{bmatrix}$$

39. Flaschel (1985), p. 289.

If we postmultiply input and output matrix A and B and labour input vector ℓ by Λ we get

$$\bar{A} = A\Lambda = \begin{bmatrix} 3\lambda_1 + 2\lambda_2 & 2 \\ 2\lambda_1 + 3\lambda_2 & 0 \end{bmatrix}, \quad \bar{B} = B\Lambda = \begin{bmatrix} 6\lambda_1 + 7\lambda_2 & 0 \\ 0 & 6 \end{bmatrix},$$

and $\bar{\ell} = \ell = (1,1)$.

Flaschel proves that if technique $(\bar{A}, \bar{B}, \bar{\ell})$ is productive then there exist a uniquely determined positive value vector for every set of exogenously given relative intensities, λ_1 and λ_2[40]. Also Flaschel adds the non necessary condition that $\bar{\ell} > 0$, i.e. labour is indispensable in production[41]. One has to replace this condition with the condition that labour is used in the production of at least one basic good. Average values are according to Flaschel the elements of the following vector

$$\bar{w}\bar{A} + \bar{\ell} = \bar{B},$$

or the solutions of the following system of equations

$$(3\lambda_1 + 2\lambda_2)w_1 + (2\lambda_1 + 3\lambda_2)w_2 + 1 = 6\lambda_1 + 7\lambda_2 , \tag{23}$$

$$2w_1 + 1 = 6w_2 . \tag{24}$$

If relative intensities λ_1 , λ_2 are exogenously given then the system of equations (23) and (24) determines uniquely w_1 and w_2. In order to determine λ_1 , λ_2 Flaschel supposes that are *derived from the actual data* of the economy. He suppose then that λ_1 , λ_2 are the actual relative intensities that individual processes operate within a sector. It can be easily verified tha equations (23) and (24) do not include individual labour values as our initial system of equations (20) - (22) does. Thus it says nothing on the sign of individual values although Flaschel asserts the opposite, emphasing that individual values are necessarily positive. As we have already seen individual values are not necessarily positive but they can obtain strictly positive solutions[42]. Thus Flaschel's striking assertion that individual values are necessarily positive is

40. Flaschel does the procedure above in various stages in order to prove some inconsistencies in Murata's argument. We proceed by excluding this part of his analysis.
41. Flaschel (1985), p. 291.
42. Flaschel (1985) Proposition 1, p. 291.

wrong, because of the fact that the weighted average of a variable can be positive, although some individual values of this variable can be negative or zero.

A similar to Flaschel's approach for the unique determination of the average or weighted labour values introduces Ochoa[43]. His approach to the problem of negative labour values is based on the same principles as those adopted by Stamatis, by Hengstenberg and Fay and by Flaschel –at least initially– Ochoa beginns his argumentation with the simple observation that "... the values of the ... inputs ... are assumed by Steedman to equal the values of the ... output. But why should this be so? If the ... input in years two and three comes from the output of the previous three years period, the value of the ... inputs should be ... the marke value ... (of the input – G.S.)" j and Ochoa continues "even if it were not the market value of the inputs given *prior* to the production ... and can not be determined simultaneously with the value of output" [44]. The system of equations (11) and (12) has to be modified accordingly in order to describe the differentiation of the average input to the individual output values. Thus we obtain

$$5w_1 + 1 = 6w_{11} + w_{21}, \tag{25}$$

$$10w_2 + 1 = 3w_{12} + 12w_{22}. \tag{26}$$

How answer Ochoa the obvious question: how the average or weighted labour values are determined? Ochoa claims that "... this (Steedman's – G.S.) example can be handled by the correct Marxian method (i.e. general value = weighted average of individual values) provided we have a rule for allocating embodied labour to joint products" [45]. At this point Ochoa specifies three alternative ways to approach the allocating rule of the embodied labour in joint production

a. By trivializing the problem and thus by erasing the distrinction of joint production produced in different processes. This is, according to Ochoa, the Steedman's solution, who considers the individual values as equal to each other.

b. By introducing an arbitrary set of numerical weights to the individual

43. Ochoa (1980) and (1981/82).
44. Ochoa (1980), p. 51.
45. Ibid, p. 55.

values w_{ij}, $i, j = 1, 2$, in order to specify the average or weighted labour values. These weights (or ratios as Ochoa calls them) can be determined only by an external criterion and not endogenously. This contradicts one property that, according to Ochoa, average or weighted values should maintain the property to be abstract, i.e. general.

c. The "correct" approach is, according to Ochoa, the equipartioning of labour to each process in such a way that the following conservation principle holds: "That is, the value of each joint production should be the amount of labour required to produce one unit" [46]. Ochoa introduces at this point the notion of *equipartition* of labour to the different types of products[47]. The labour is *equipartioned* to the two products, when labour time is divided equally, irrespective of the process where is employed, between the two products. According to Ochoa, this is the correct Marxian approach to the problem: "that is, the value of each joint product should be the amount of the labour required to produce one unit (of each commodity – G.S.)[48]. Ochoa defines as dL_{ti}, $i = 1, 2$ the required amount of labour in order an additional unit of the commodity i, $i = 1, 2$ to be produced. This labour amount is called, or preferably is defined as, the "relative difficulty" of producing one unit of the commodity i, $i = 1, 2$. This definition of "relative difficulty" leads Ochoa to the pressumption that the unknown social average values of the commodities i, $i = 1, 2$ should be proportional to the relative difficulties to produce an additional unit of this commodity, i.e. $\frac{dL_{t1}}{w_1} = \frac{dL_{t2}}{w_2}$ or equivalently $\frac{dL_{t1}}{dL_{t2}} = \frac{w_1}{w_2}$ (Ochoa denotes labour values with L_i, $i = 1, 2$ – G.S.).

Ochoa applies the equipartition rule to the Steedman's numerical example as follows:

$$25w_1 + 5 = 30w_{11} + 5w_{21}, \tag{25}$$

$$10w_1 + 1 = 3w_{12} + 12w_{22}, \tag{26}$$

and adding (25) to (26) Ochoa gets

$$25w_1 + 10w_2 + 6 = (30w_{11} + 3w_{12}) + (5w_{21} + 12w_{22}) \tag{27}$$

46. Ibid, pp. 59-60.
47. Ibid, p. 55.
48. Ibid, p. 60.

Thus Ochoa considers the value of inputs equal to their social average value of them and the value of the outputs equal to their individual values. He, also, concludes that the following equalities hold:

$$30w_{11} + 3w_{12} = 33w_1 \tag{28}$$

and

$$5w_{21} + 12w_{22} = 17w_2 \tag{29}$$

According to the equipartion rule each additional unit of labour is divided equally in each process to both commodities. Accordingly, it holds:

$$30w_{11} = 5w_{21} = \frac{25w_1 + 5}{2} \tag{30}$$

and

$$30w_{12} = 12w_{22} = \frac{10w_2 + 1}{2} \tag{31}$$

According to (30) and (31) he gets using (28) and (29):

$$33w_1 = 17w_2 = \frac{25w_1 + 10w_2 + 6}{2}. \tag{32}$$

The solution of (32) gives the following numerical values of w_1 and w_2

$$w_1 = 0.278 \quad \text{and} \quad w_2 = 0.540\,.$$

Both labour values are well-defined, i.e. unique and positive. Ochoa adds that individual values can obtain positive solutions as well[49].

The main criticism to Ochoa's contribution to the issue can be found in Stamatis (1980) and Stamatis and Funke (1981). According to the authors above Ochoa introduces and arbitrary equation in order to determine uniquely the social average labour values and namely the equation $\frac{dL_{t1}}{dL_{t2}} = \frac{w_1}{w_2}$.

This exogenous arbitrary determination rule, called *equipartition* rule by Ochoa has no solid foundation in marxian literature and no logical reasoning in marxian work. According to Stamatis (1980) is as arbitrary as Morishima's or Flaschel's attempt to determine uniquely labour values[50].

49. Ibid, pp. 55-57.

50. Stamatis (1980), p. 56.

The solution given by Stamatis to the negative labour values issue has been further elaborated, generalised and evaluated by Vassilakis[51]. In a short note published in greek Vassilakis consides the case of a two commodities, two processes joint production technique and proves that labour values, defined as individual labour values, can obtain a set of strictly positive solutions. These solutions are, however, not uniquely determined. Vassilakis, though, does not only give an algebraic proof in the case of the two commodities, two processes joint production techniques, but considers the case where each production process uses inputs bought from other process, and consequently the input value of each process is a convex combination of the individual values of each commodity produced in the different production processes. This approach differs from Ochoa's approach due to the fact that the latter assumes that the input value of each commodity is the unique social average value, although Vassilakis assumes that the input value of each commodity is an arbitrary convex combination of the individual commodity values and it does not necessarily coinsists with the unique social average value. It is obvious that Vassilakis' approach includes Ochoa's approach as a special case, due to the fact that the unique social average value of a commodity is one of the infinite convex combinations of admissible, i.e. positive, individual values. The original numerical example can be described by the following equations:

$$5(\alpha_{11}w_{11} + \alpha_{12}w_{12}) + 1 = 6w_{11} + w_{21},$$
$$5(\alpha_{11}w_{11} + \alpha_{12}w_{12}) + 1 = 6w_{11} + w_{21},$$

where $\alpha_{11} + \alpha_{12} = \alpha_{21} + \alpha_{22} = 1$ and $\alpha_{ij} \geqq 0,\ i, j = 1,2.$

Now each equation contains three unknowns and a graphical solution is unfortunately not possible in the 2-dimensional plane. Additionally, in the general case each equation contains all four unknowns, w_{11}, w_{12}, w_{21}, w_{22}, and it can not be solved graphically even in the three dimensional space. However, Vassilakis shows that a productive, square 2x2 joint production technique has positive solutions for the individual labour values, although the value magnitudes can not be determined uniquely. His proof is elementary but not simple. A simplification and a generalisation of Vassilakis proof can be found in Sotirchos (1998b) where the separation theorems of convex sets are used, i.e. the proofs are not elementary any more. On the other hand Vassilakis does not

51. Vassilakis (1986).

attempt to determined uniquely the social average labour values as Flaschel, Ochoa et. al. try to do. Social average labour values are positive unknown that can not be uniquely determined due to the fact that the system of equations that determines them has more unknowns than equations.

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